Logic of Visibility in Social Networks

Rustam Galimullin, Mina Young Pedersen & Marija Slavkovik

UNIVERSITY OF BERGEN

Outline

- Context and motivation
- Our social network models
- Static visibility logic
- Example: A malicious agent in the system
- Dynamic extension: Visibility logic

Context and Motivation

- Logic for social networks
- Reasoning about visibility and reachability
- Exploiting our network

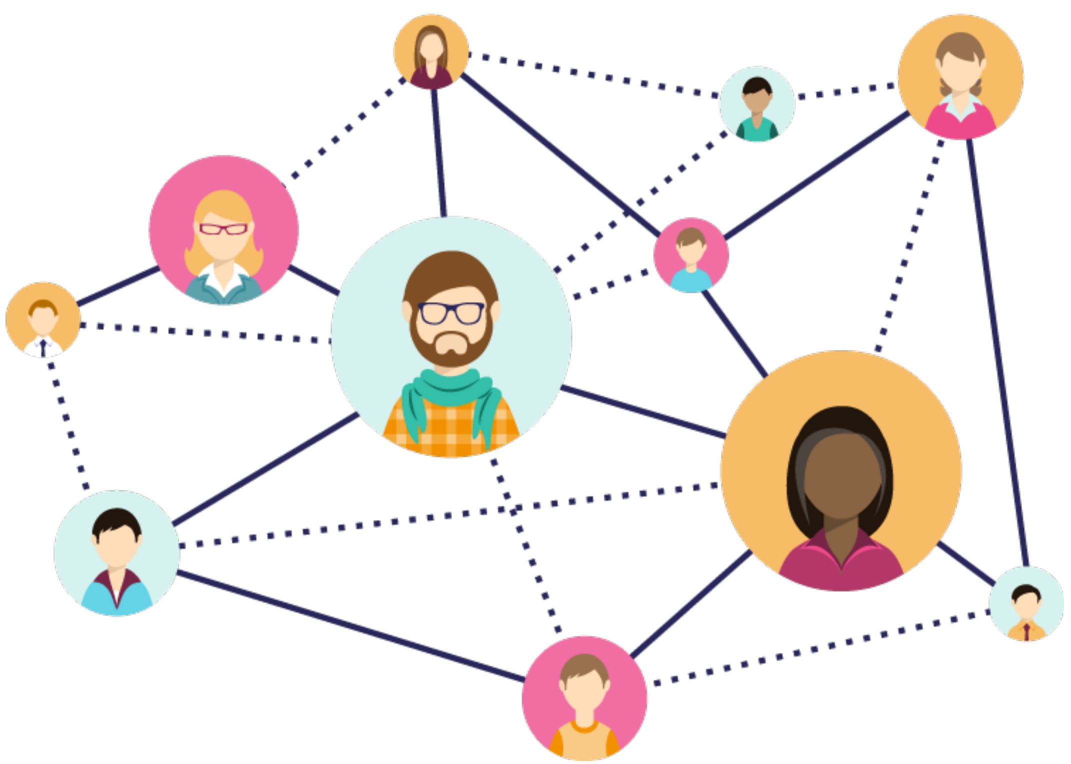


Image: ClipArtKey

Context and Motivation

Prove mathematical results about this system

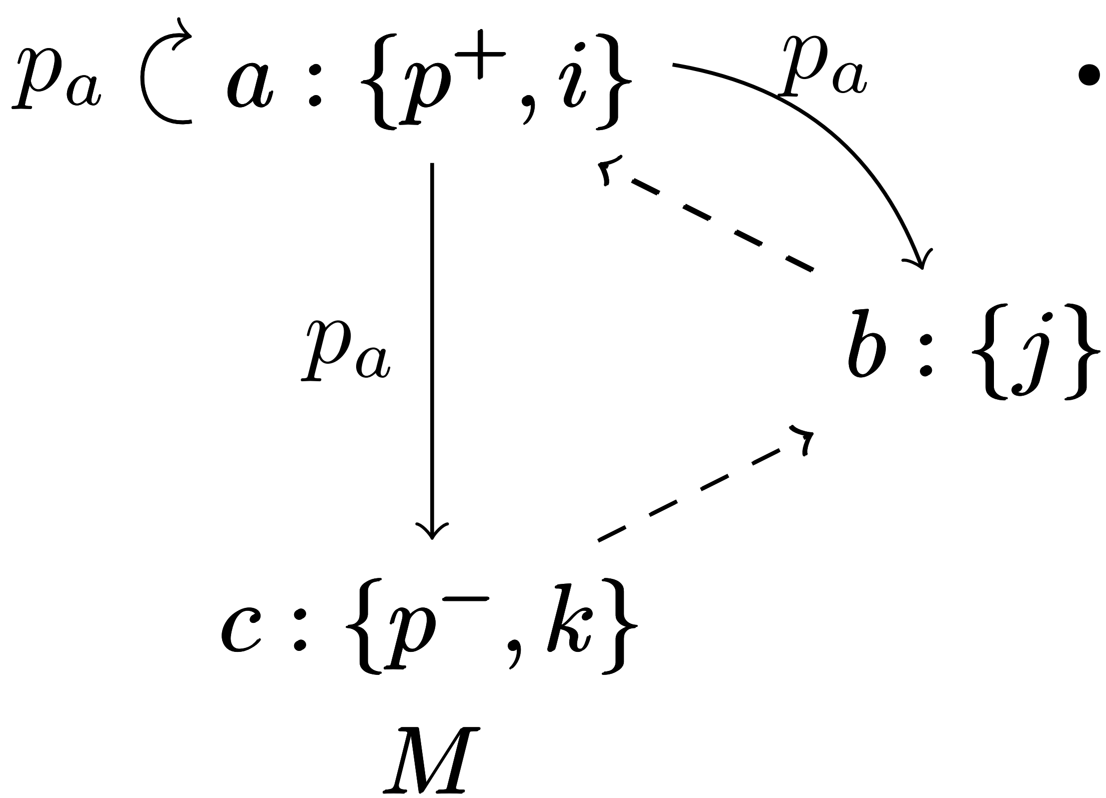
- Outcomes:
  - A new logic to analyze posting and sharing information in a social network
  - Formalize different quantitative and qualitative measures of visibility and reachability
  - Use this logic to understand real-life networks

Our Models

Social network where agents can:

- Post information on a topic
- Share other agents' posts
- Follow and unfollow each other
- Have a *pro* or a *contra* opinion about a topic

Reflexive p_a-arrow: "*a* has posted on *p*"



p_a-arrow from *a* to *b*: "*b* has seen *a*'s post on *p*"

Our Models

- Four rules:

    1. When an agent posts, all her followers can see the post.

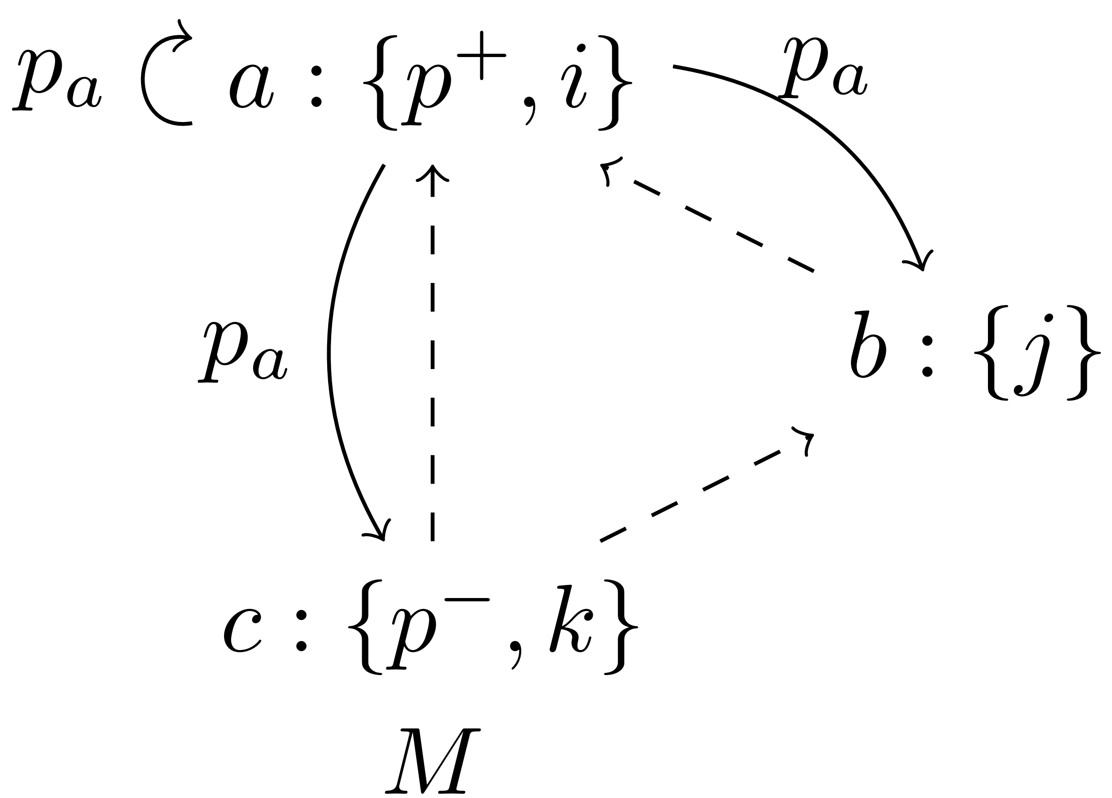
    2. If an agent sees a post on a topic she **likes**, she will **reshare** the post and **follow** the original poster.

    3. If an agent sees a post on a topic she **dislikes**, she does **not reshare** and **unfollows** the agent she saw the post from.

    4. If an agent sees a post on a topic she is indifferent to, she does nothing.

Our Models

- Interpretation: Observe a situation after it has happened



Static Visibility Logic

$$\mathsf{Nom} = \{i, j, k, ...\}$$

Countable set of nominals

$$\mathsf{Top} = \{p, q, r, ...\}$$

Countable set of topics

$$\mathsf{Nom} \cap \mathsf{Top} = \emptyset$$

Syntax

$$\varphi ::= p^+ \mid p^- \mid i \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Diamond_{i:p}\varphi \mid \Diamond_{i:p}^{-1}\varphi \mid \blacklozenge\varphi \mid \blacklozenge^{-1}\varphi \mid @_i\varphi$$

where $p \in \mathsf{Top}$ and $i \in \mathsf{Nom}$.

Static Visibility Logic

Syntax

$$\varphi ::= p^+ \mid p^- \mid i \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Diamond_{i:p}\varphi \mid \Diamond^{-1}_{i:p}\varphi \mid \blacklozenge\varphi \mid \blacklozenge^{-1}\varphi \mid @_i\varphi$$

where $p \in \mathsf{Top}$ and $i \in \mathsf{Nom}$.

$\Diamond_{i:p}\varphi$ “there is an agent satisfying φ who sees the (re)post of agent i on topic p”

$\blacklozenge^{-1}\varphi$ “the current agent is followed by an agent that satisfies φ”

Static Visibility Models

$$M = (A, F, +, -, V, R)$$

A is a non-empty set of agents;

$F : A \to 2^A$ is an irreflexive followership relation;

$+ : A \to 2^{\mathsf{Top}}$ valuation function for *pro* topics;

$- : A \to 2^{\mathsf{Top}}$ valuation function for *contra* topics such that $+(a) \cap -(a) = \emptyset$;

$V : \mathsf{Nom} \to 2^A$ valuation such that for all $i \in \mathsf{Nom}$: $|V(i)| = 1$;

Static Visibility Models

$$M = (A, F, +, -, V, R)$$

$R : \mathsf{Top} \times A \to 2^{A \times A}$ is a visibility relation:

$p \in \mathsf{Top}$ and $a, b, c \in A$

1. If $(a, b) \in R(p, c)$, then $(a, a) \in R(p, c)$.
2. If $(a, a) \in R(p, c)$, then $(a, b) \in R(p, c)$ for all b such that $b \in F(a)$.
3. If $(a, b) \in R(p, c)$, $p \in +(b)$, and $b \neq c$, then $(b, b) \in R(p, c)$ and $b \in F(c)$.
4. If $(a, b) \in R(p, c)$, $p \in -(b)$, and $a \neq b$, then $(b, b) \notin R(p, c)$ and $b \notin F(a)$.
5. If $(a, b) \in R(p, c)$, $p \notin +(b)$, $p \notin -(b)$, and $a \neq b$, then $(b, b) \notin R(p, c)$.

Semantics

M_a pointed visibility model

$$M_a \models p^+ \quad \text{iff} \quad p \in +(a)$$
$$M_a \models p^- \quad \text{iff} \quad p \in -(a)$$
$$M_a \models i \quad \text{iff} \quad a \in V(i)$$

$$M_a \models \neg\varphi \quad \text{iff} \quad M_a \not\models \varphi$$
$$M_a \models \varphi \wedge \psi \quad \text{iff} \quad M_a \models \varphi \text{ and } M_a \models \psi$$

$$M_a \models \Diamond_{i:p}\varphi \quad \text{iff } \exists b, c \in A : (a, b) \in R(p, c) \text{ and } V(i) = \{c\} \text{ and } M_b \models \varphi$$
$$M_a \models \Diamond^{-1}_{i:p}\varphi \quad \text{iff } \exists b, c \in A : (b, a) \in R(p, c) \text{ and } V(i) = \{c\} \text{ and } M_b \models \varphi$$

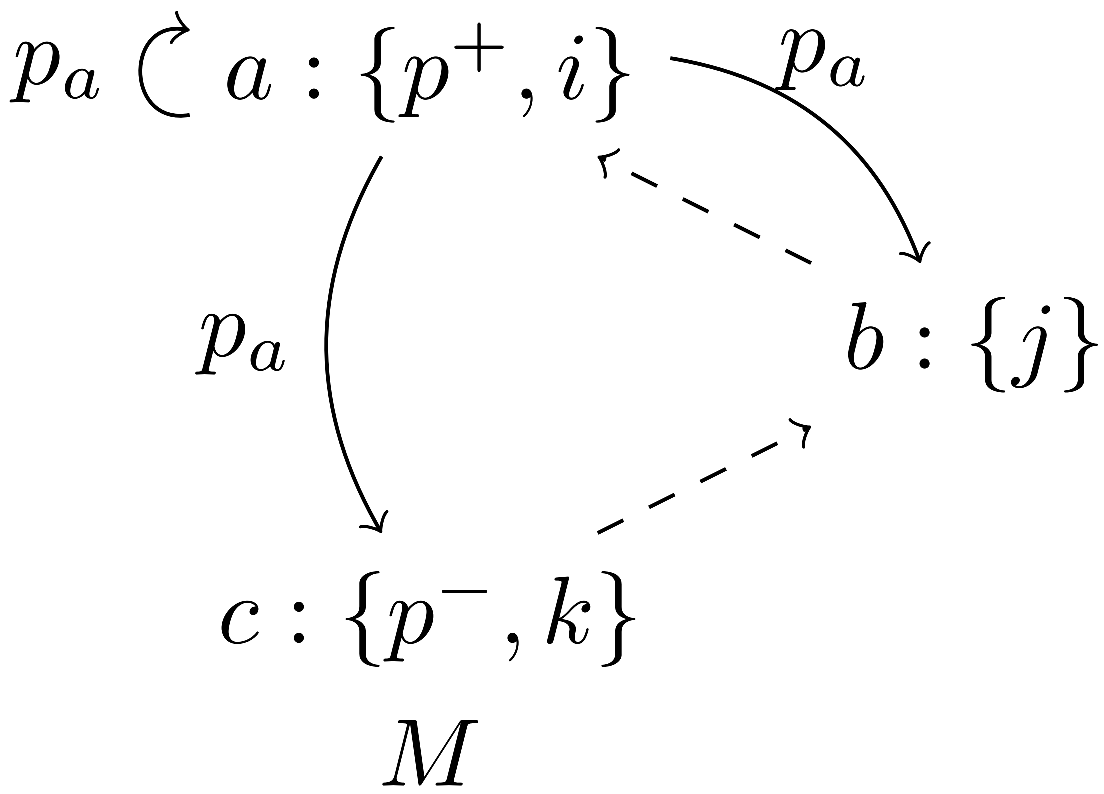
Semantics

M_a pointed visibility model

$$M_a \models \blacklozenge\varphi \quad \text{iff} \quad \exists b \in A : a \in F(b) \text{ and } M_b \models \varphi$$
$$M_a \models \blacklozenge^{-1}\varphi \quad \text{iff} \quad \exists b \in A : b \in F(a) \text{ and } M_b \models \varphi$$

$$M_a \models @_i\varphi \quad \text{iff} \quad M_b \models \varphi \text{ and } \{b\} = V(i)$$

Semantics



$$M_c \models \Diamond_{i:p}^{-1} \top \wedge \blacksquare \neg p^+$$

Visibility

Quantitative (in finite models):

How many agents that are pro p have seen the agent called i's post on p:

$$|\{a \in A \mid M_a \models p^+ \wedge \Diamond_{i:p}^{-1}\top\}|$$

Visibility

Qualitative: $M_a \models \varphi?$

All the followers of the current agent i have shared i's post on p:

$i \wedge \blacksquare^{-1} \Diamond_{i:p} \top$

The current agent i shared a post to a follower j,

but j also saw the post from another source:

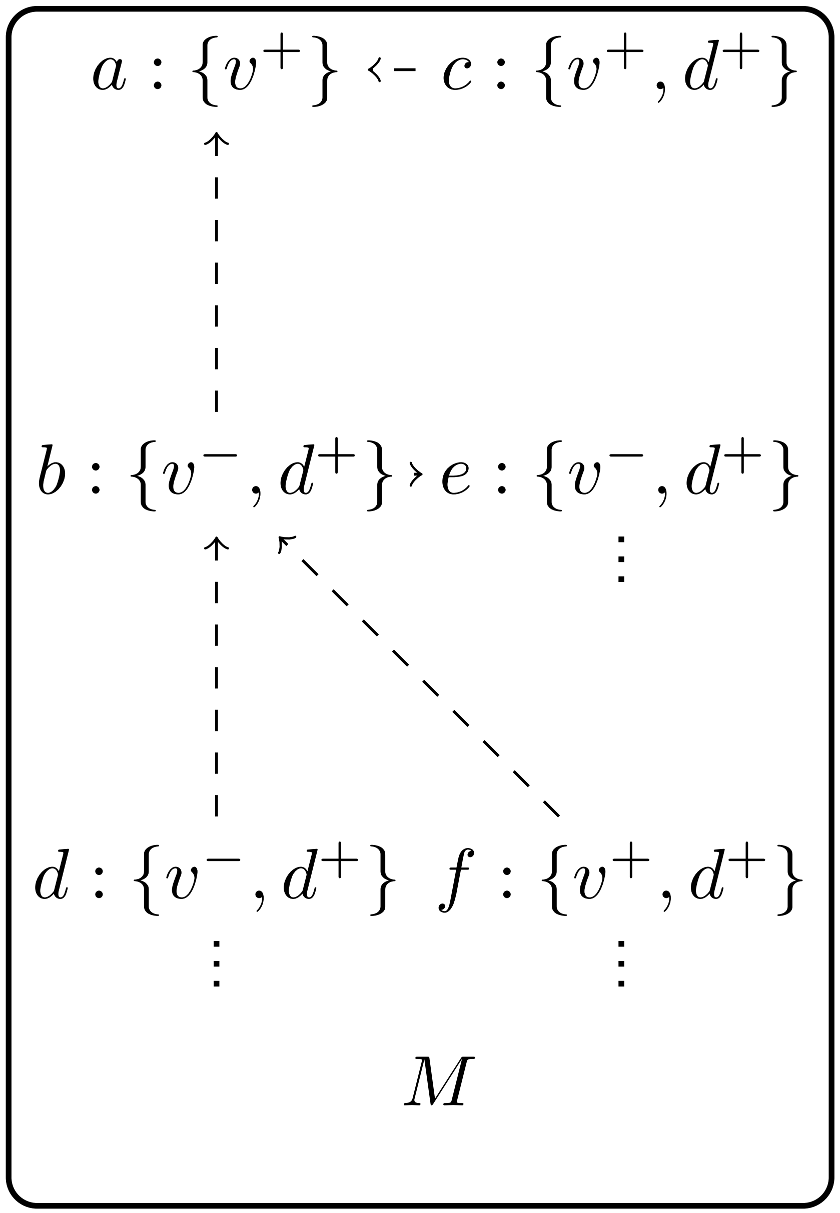
$i \wedge \blacklozenge^{-1}(j \wedge \Diamond_{i:p}^{-1} i \wedge \Diamond_{i:p}^{-1}(\neg i \wedge \neg j))$

Soundness, Completeness and Model Checking

- **SVL** is sound and complete with respect to visibility models
- Model checking **SVL** is in *P*

Example

How can an agent exploit these networks?

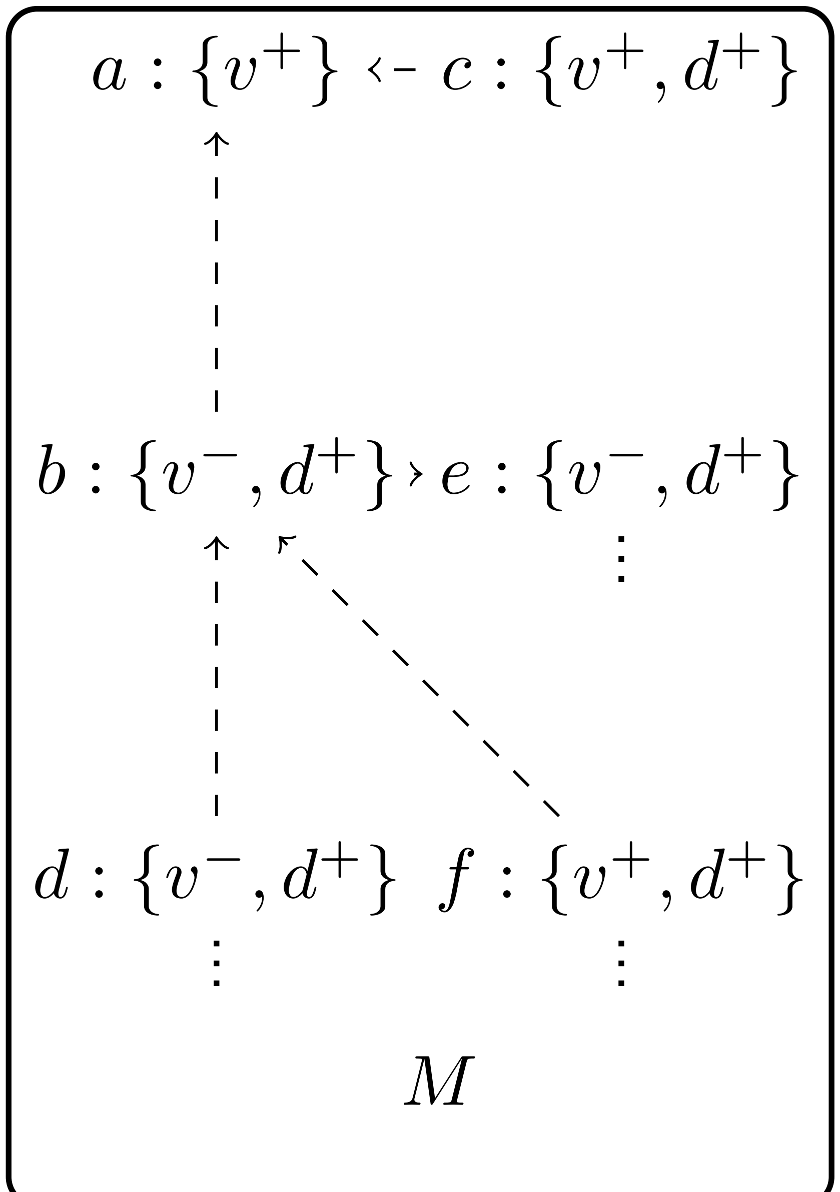


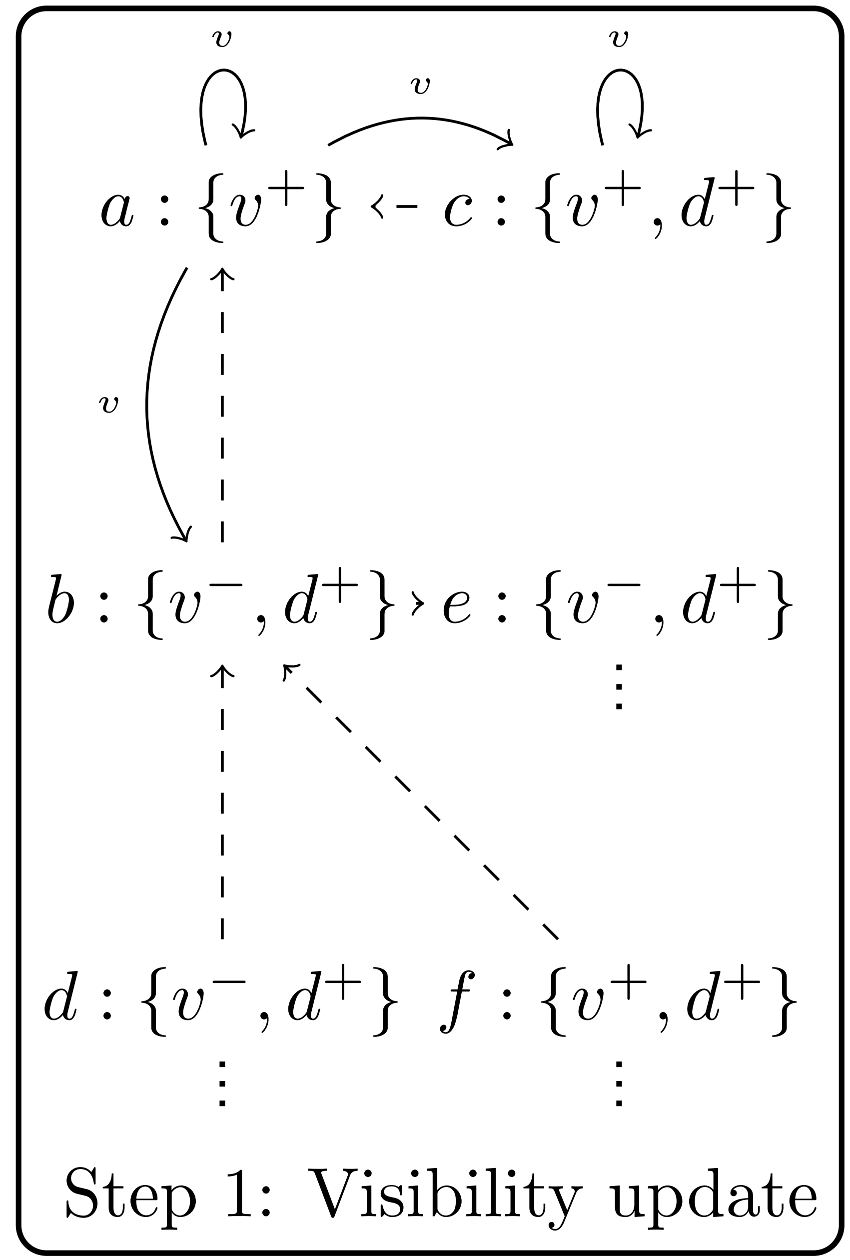
v: vaccination

d: dogs

Example

How can an agent exploit these networks?

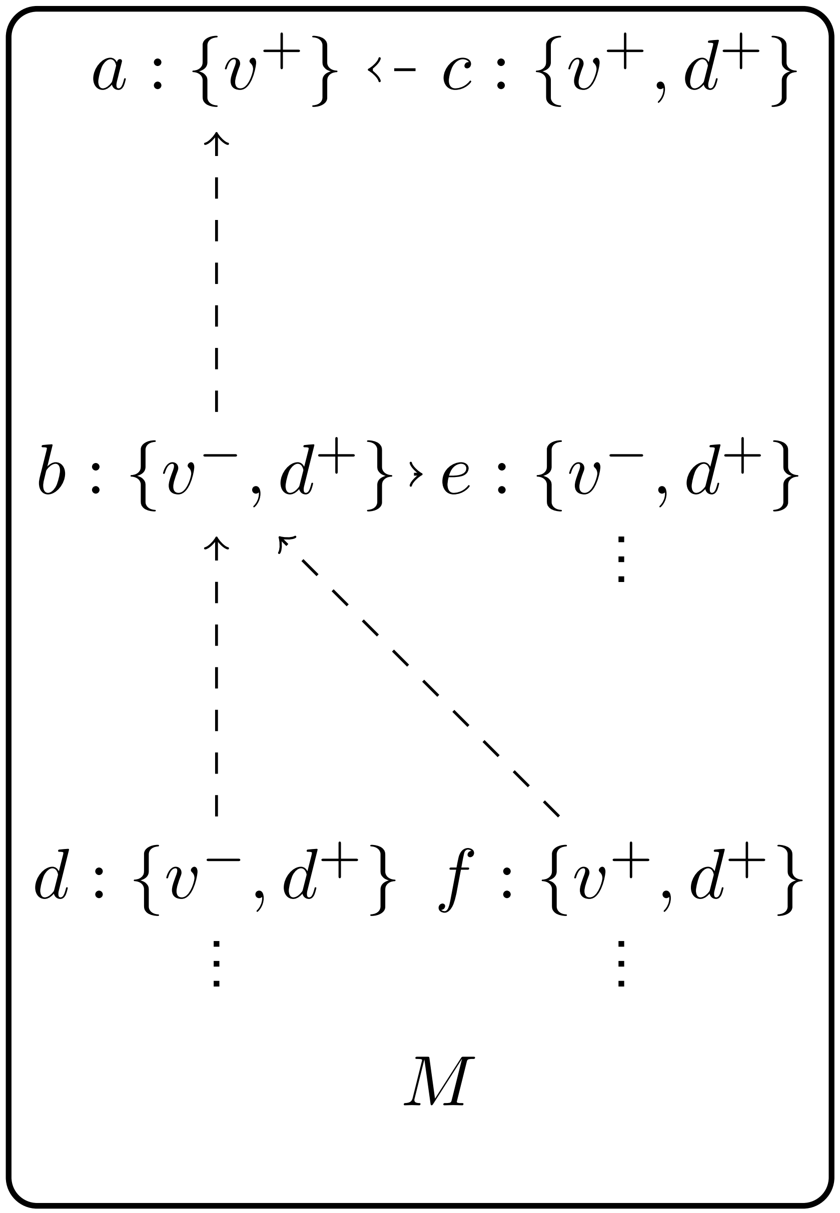


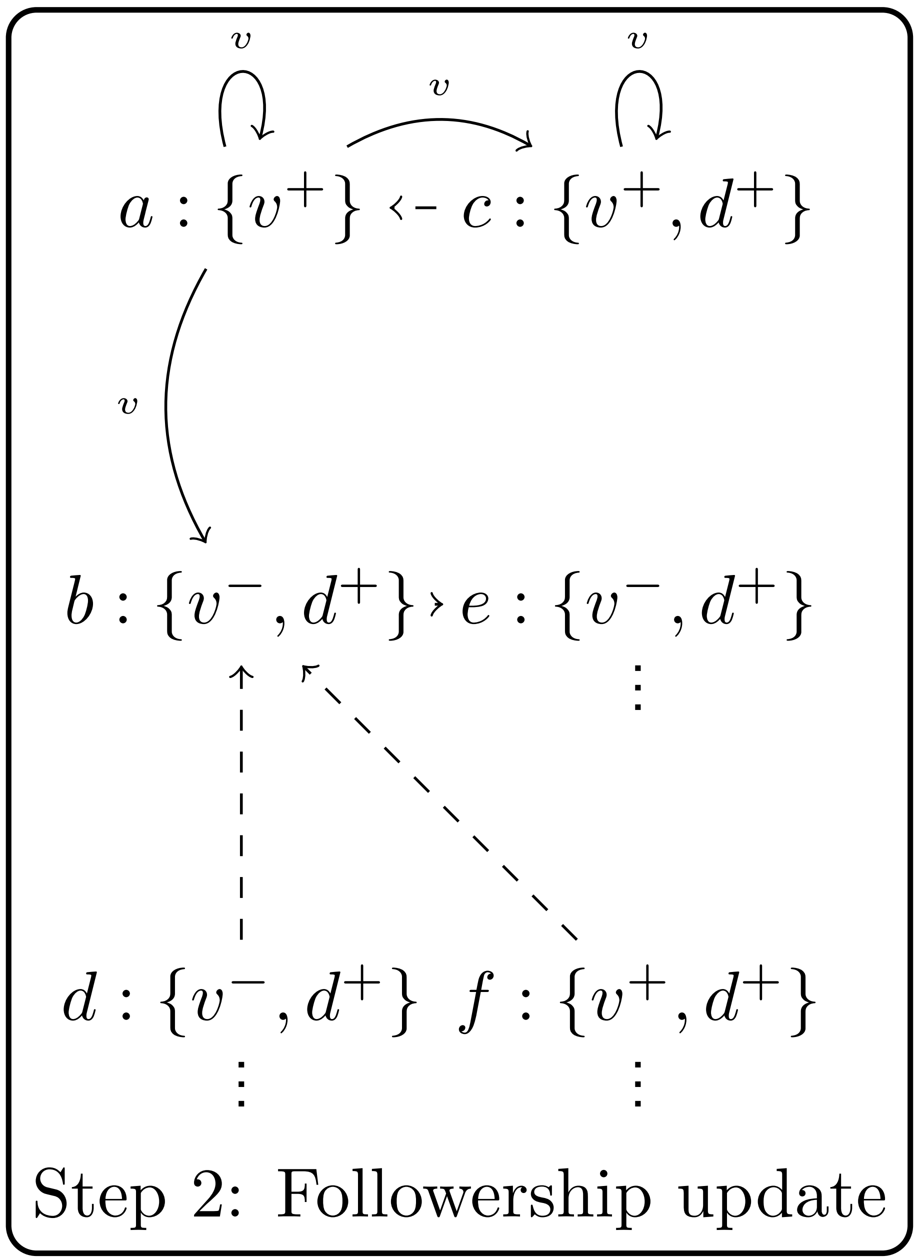


$M^{a:v}$

Example

How can an agent exploit these networks?

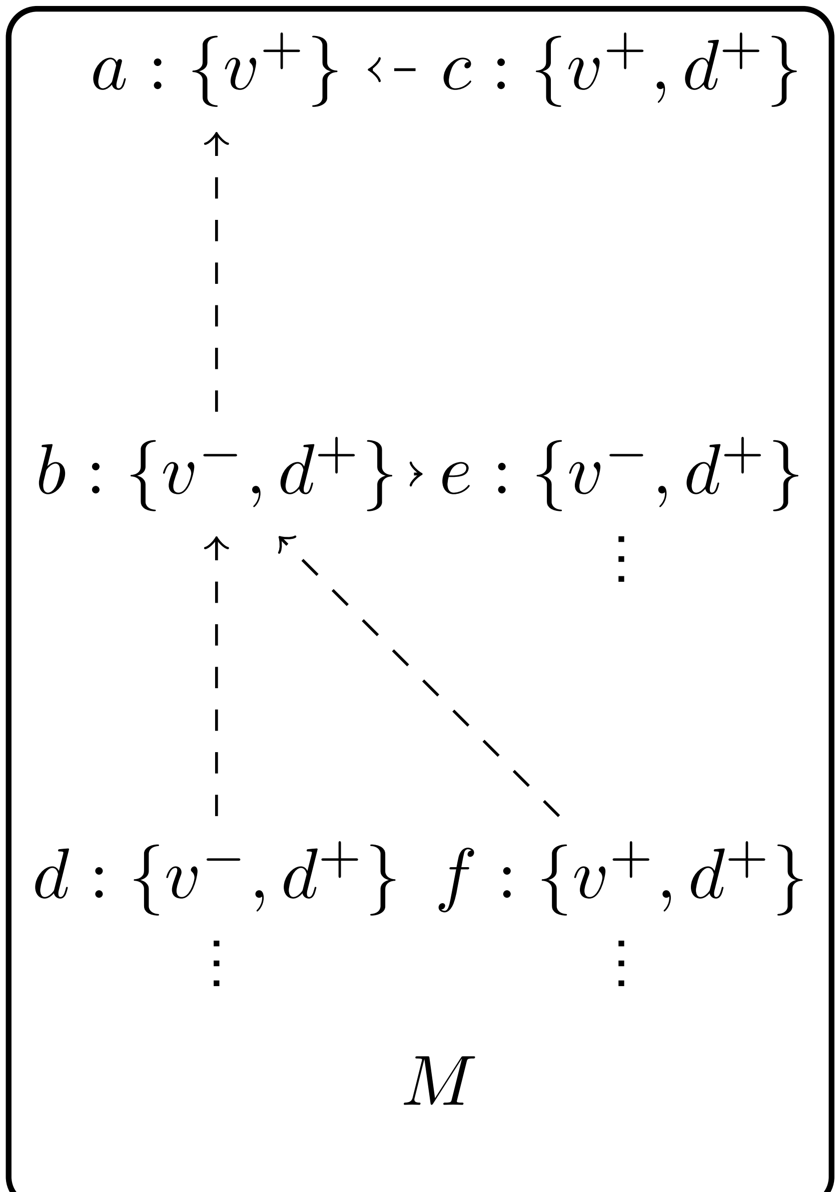


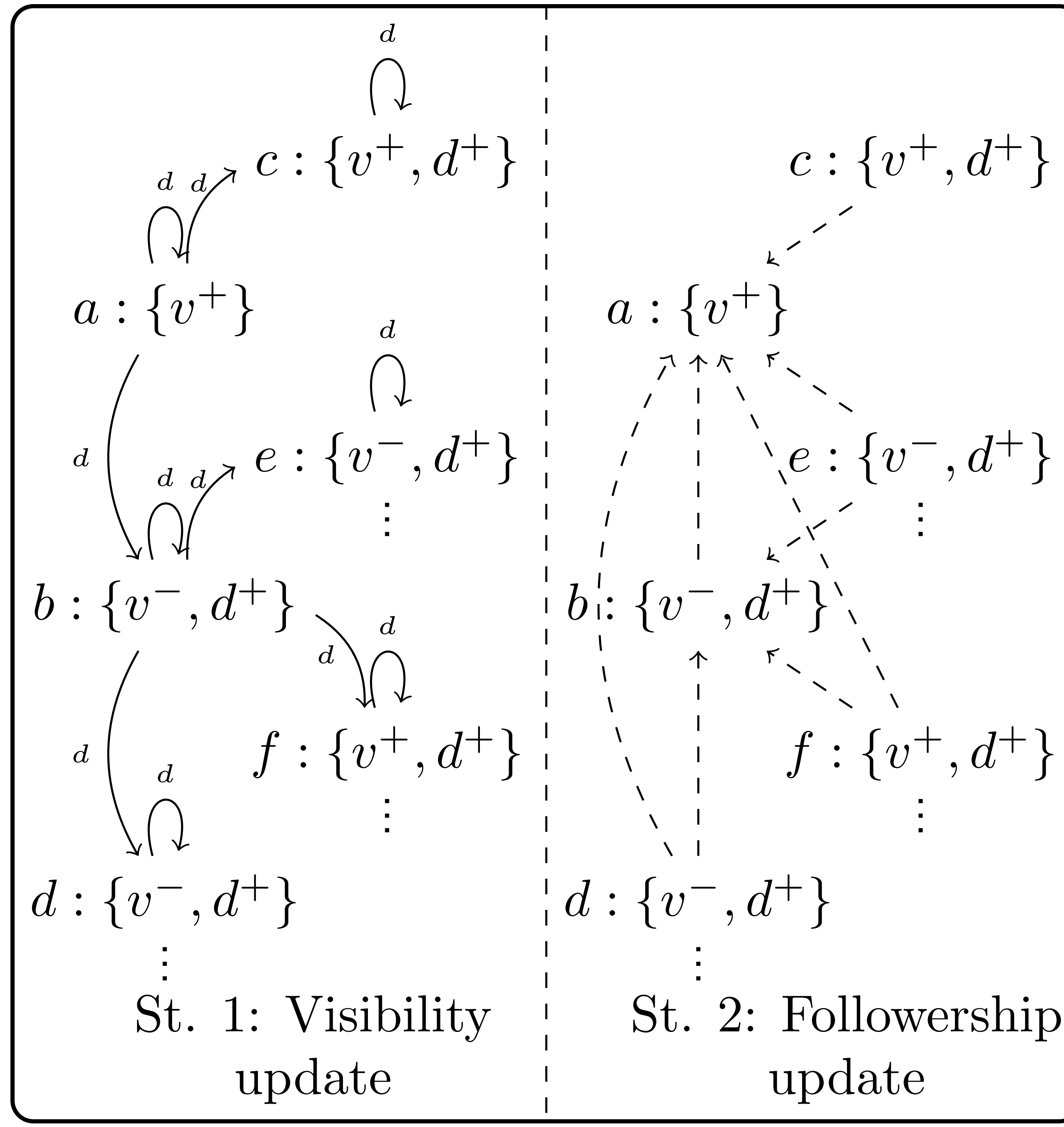


$M^{a:v}$

Example

How can an agent exploit these networks?

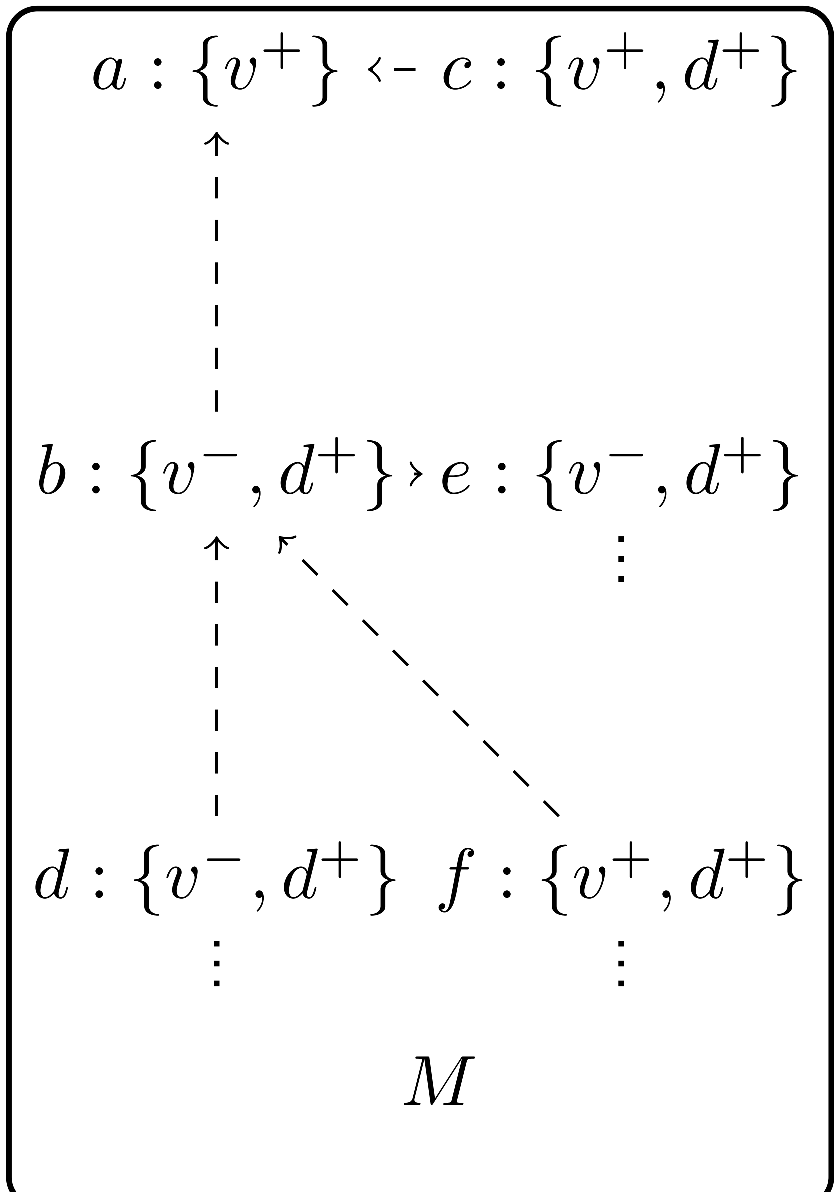


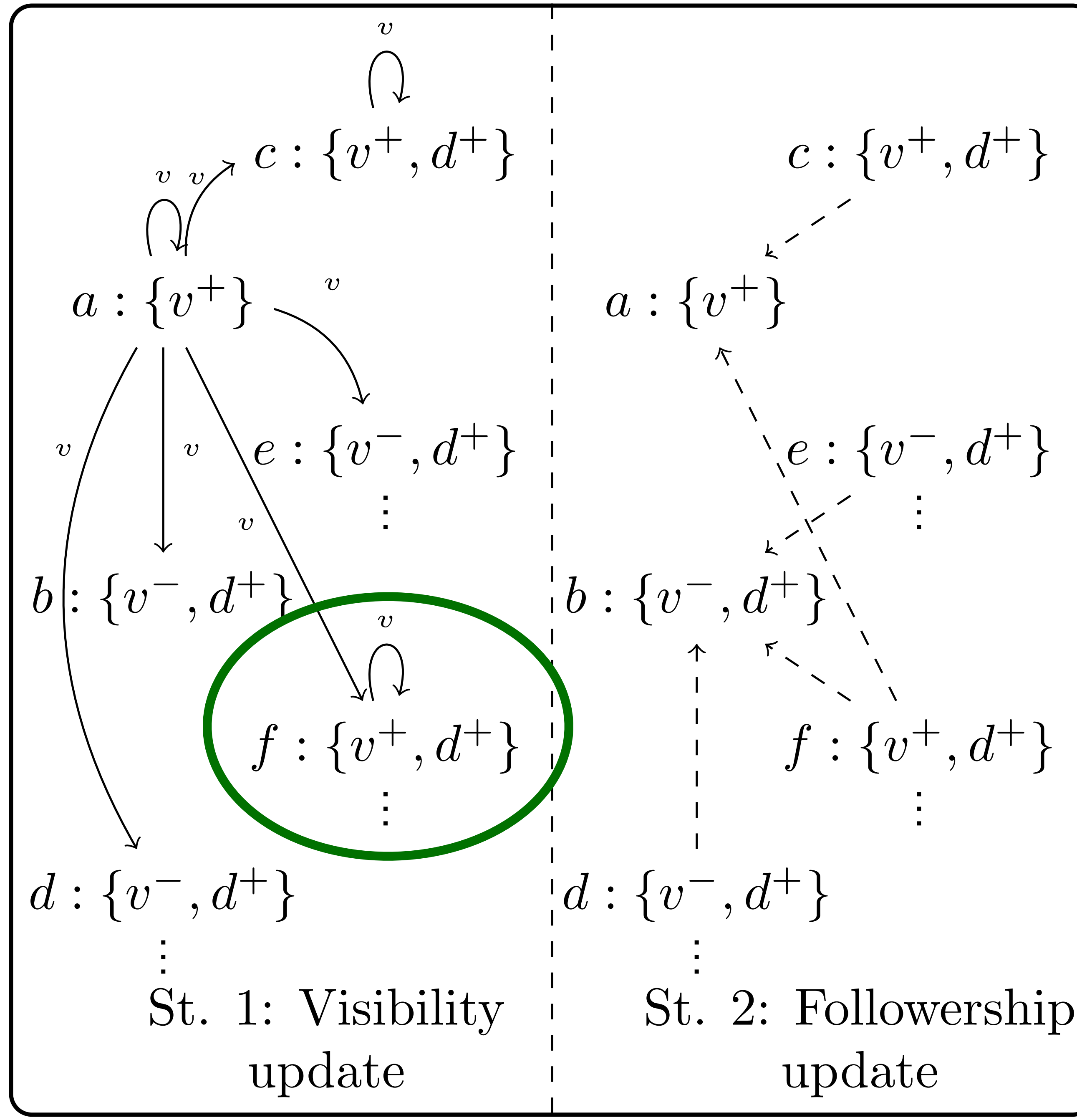


$M^{a:d}$

Example

How can an agent exploit these networks?





$M^{a:d,a:v}$

Example

How can an agent exploit these networks?

- Posting on dogs before vaccines makes more agents exposed to the post on vaccines
- Order of posting is important
- Agents' interests matter
- Exploit an underlying notion of trust

Dynamic Operator: Visibility Logic

Syntax

$\mathcal{SVL}$ +

$$\begin{array}{lcl} \varphi & ::= & [\pi]\varphi \\ \pi & ::= & p \mid (\pi \cup \pi) \end{array}$$

$[\pi]\varphi$: "after the current agent executes action π, φ holds"

$[p \cup q]\varphi$: "whichever topic the current agent posts on, p or q, φ will be true (in both cases)"

Dynamic Operator: Visibility Logic

Semantics

$$M_a \models [p]\varphi \quad \text{iff} \quad M_a^{a:p} \models \varphi$$
$$M_a \models [\pi \cup \tau]\varphi \quad \text{iff} \quad M_a \models [\pi]\varphi \text{ and } M_a \models [\tau]\varphi$$

$M_a^{a:p}$ is defined in two steps:

Visibility update:

$$M* = (A, F, +, -, V, R*)$$

$R^*(a,p)$ is the least fixed point of $f : 2^A \to 2^A$:

$$f(X) = X \cup \{(a,a)\} \cup \{(b,c) \mid (b,b) \in X \text{ and } c \in F(b)\} \cup$$
$$\cup \{(c,c) \mid p \in +(c) \text{ and } \exists b : (b,c) \in X\}.$$

Dynamic Operator: Visibility Logic

Followership update:

1. $F^{a:p}(a) = F(a) \cup \{b\}$, if $a \neq b$, $p \in +(b)$, and $\exists c : (c, b) \in R^*(p, a)$

2. $F^{a:p}(b) = F(b) \setminus \{c\}$, if $p \in -(b)$ and $(c, b) \in R^*(p, a)$

Expressivity and Model Checking

$$\mathcal{SVL} < \mathcal{VL}$$

- VL is more expressive than SVL
    - No reduction axioms for VL are possible
- The model checking problem for VL is *PSPACE*-complete

Future Directions

- Sound and complete axiomatization of VL
- Triggering? Posting *pro* or *contra* a topic
- Discriminate between different posts on the same topic

Thank you!