Coalition and Group Announcement Logic

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Introduction

In this poster, we consider a combination of logics for cooperative announcements – Coalition and Group Announcement Logic and provide its complete axiomatisation. Moreover, we partially answer the question of how group and coalition announcement operators interact, and settle some other open problems.

Preliminaries

Formulas of the logic are interpreted in **epistemic models**.

An *epistemic model* is a triple $M = (W, \sim, V)$, where

- W is a non-empty set of states;
- $\sim: A \to \mathcal{P}(W \times W)$ assigns an equivalence relation to each agent;
- $V : P \to \mathcal{P}(W)$ assigns a set of states to each propositional variable.

Public announcement of φ restricts a given model to the states where φ holds.

$$(M, w) \models [\varphi]\psi \text{ iff } (M, w) \models \varphi \text{ implies } (M^\varphi, w) \models \psi$$

And the corresponding diamond version of the operator.

$$(M, w) \models \langle\varphi\rangle\psi \text{ iff } (M, w) \models \varphi \text{ and } (M^\varphi, w) \models \psi$$

Coalition and Group Announcements

Let $\mathcal{L}^G_{EL}$ denote the set of formulas of the type $\bigwedge_{i\in G} K_i\varphi_i$, where for every $i \in G$ it holds that $\varphi_i \in \mathcal{L}_{EL}$. We are primarily interested in the following types of announcements:

Group Announcements

'whatever agents from G announce, φ holds'

$$(M, w) \models [G]\varphi \text{ iff } \forall\psi\in\mathcal{L}^G_{EL} : (M, w) \models [\psi]\varphi$$

'agents from G have an announcements, such that φ holds'

$$(M, w) \models \langle G\rangle\varphi \text{ iff } \exists\psi\in\mathcal{L}^G_{EL} : (M, w) \models \langle\psi\rangle\varphi$$

Coalition Announcements

'whatever agents from G announce, there is an announcement by $A \setminus G$, such that φ holds'

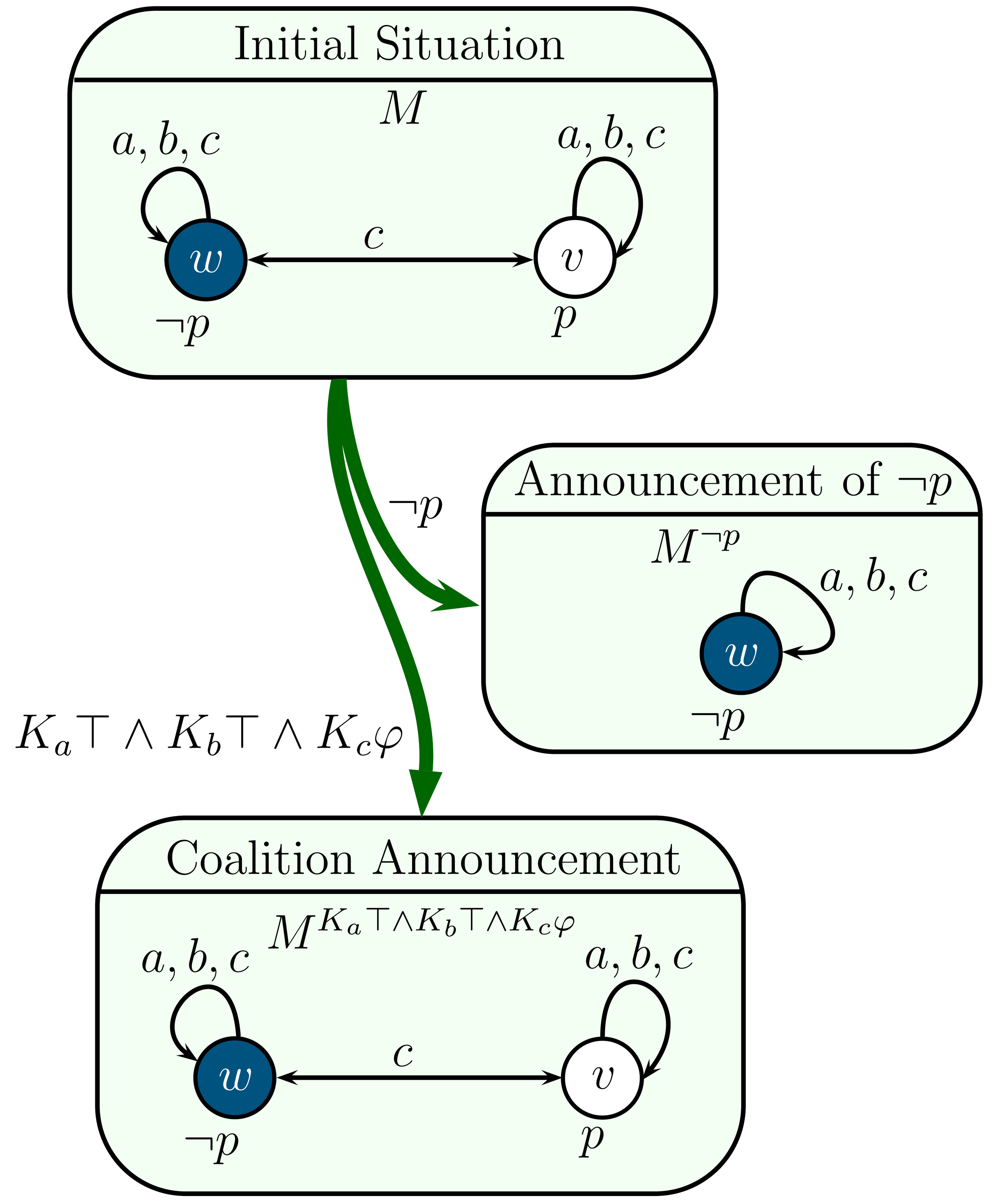
$$(M, w) \models [\![G]\!]\varphi \text{ iff } \forall\psi\in\mathcal{L}^G_{EL}\ \exists\chi\in\mathcal{L}^{A\setminus G}_{EL} : (M, w) \models \psi \to \langle\psi \wedge \chi\rangle\varphi$$

'whatever agents from G announce, there is an announcement by $A \setminus G$, such that φ holds'

$$(M, w) \models [\![G]\!]\varphi \text{ iff } \forall\psi\in\mathcal{L}^G_{EL}\ \exists\chi\in\mathcal{L}^{A\setminus G}_{EL} : (M, w) \models \psi \to \langle\psi \wedge \chi\rangle\varphi$$

Example

Ann, Bob, and Cath are travelling by train from Nottingham to Liverpool through Manchester. Cath was sound asleep all the way, and she has just woken up. In the following figure, p denotes proposition 'the agents have already passed Manchester', and $\neg p$ – 'the agents have not passed Manchester yet.'



Interaction between coalition and group announcements

Let us examine the axiom: $\langle\!\langle G\rangle\!\rangle\varphi \to \langle G\rangle[A \setminus G]\varphi$. Whatever formula φ agents from $A \setminus G$ announce after an announcement ψ by G, they could have announced it in the beginning: $K_{A\setminus G}[K_G\psi]\varphi$, or 'if you announce ψ, we will know φ'.

Language and Axiomatisation

The *language of coalition and group announcement logic* $\mathcal{L}_{CoGAL}$ is as follows:

$$\varphi, \psi ::= p \mid \neg\varphi \mid (\varphi \wedge \psi) \mid K_a\varphi \mid [\varphi]\psi \mid [G]\varphi \mid [\![G]\!]\varphi,$$

where $p \in P$, $a \in A$, $G \subseteq A$, and all the usual abbreviations of propositional logic (such as $\vee, \to, \leftrightarrow$) and conventions for deleting parentheses hold.

We define $\mathcal{L}_{GAL}$ as the language without the operator $[\![G]\!]$, $\mathcal{L}_{PAL}$ the language without $[G]$ as well, and $\mathcal{L}_{EL}$ the purely epistemic language which in addition does not contain announcement operators $[\varphi]$.

Axiomatisation of **CoGAL** is a union of axiomatisation of group announcement logic, interaction axiom for group and coalition announcements $A11$, rule of inference for coalition announcements $R6$, and necessitation $R4$.

$(A0)$ propositional tautologies,	$(A10)$ $[G]\varphi \to [\psi]\varphi$, where $\psi \in \mathcal{L}^G_{EL}$,
$(A1)$ $K_a(\varphi \to \psi) \to (K_a\varphi \to K_a\psi)$,	$(A11)$ $\langle\!\langle G\rangle\!\rangle\varphi \to \langle G\rangle[A \setminus G]\varphi$,
$(A2)$ $K_a\varphi \to \varphi$,	$(R0)$ $\vdash \varphi, \varphi \to \psi \Rightarrow \vdash \psi$,
$(A3)$ $K_a\varphi \to K_aK_a\varphi$,	$(R1)$ $\vdash \varphi \Rightarrow \vdash K_a\varphi$,
$(A4)$ $\neg K_a\varphi \to K_a\neg K_a\varphi$,	$(R2)$ $\vdash \varphi \Rightarrow \vdash [\psi]\varphi$,
$(A5)$ $[\varphi]p \leftrightarrow (\varphi \to p)$,	$(R3)$ $\vdash \varphi \Rightarrow \vdash [G]\varphi$,
$(A6)$ $[\varphi]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi]\psi)$,	$(R4)$ $\vdash \varphi \Rightarrow \vdash [\![G]\!]\varphi$,
$(A7)$ $[\varphi](\psi \wedge \chi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\chi)$,	$(R5)$ $(\forall\psi\in\mathcal{L}^G_{EL} \vdash \eta([\psi]\varphi)) \Rightarrow \vdash \eta([G]\varphi)$,
$(A8)$ $[\varphi]K_a\psi \leftrightarrow (\varphi \to K_a[\varphi]\psi)$,	$(R6)$ $(\forall\psi\in\mathcal{L}^G_{EL}\ \exists\chi\in\mathcal{L}^{A\setminus G}_{EL}$
$(A9)$ $[\varphi][\psi]\chi \leftrightarrow [\varphi \wedge [\varphi]\psi]\chi$,	$\vdash \eta(\psi \to \langle\psi \wedge \chi\rangle\varphi)) \Rightarrow \vdash \eta([\![G]\!]\varphi)$.

Coalition and group announcement logic is **sound** and **complete**.

Future Work

- Establish whether the other direction of axiom $A11$, $\langle G\rangle[A \setminus G]\varphi \to \langle\!\langle G\rangle\!\rangle\varphi$, is valid;
- Logics of quantified announcements were shown to be undecidable. So, we plan to find decidable fragments of **CAL**;
- Provide an axiomatisation of the logic without group announcements;
- Investigate epistemic planning with group actions.

References

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