Knowledge, Time, and Change

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UNITED KINGDOM · CHINA · MALAYSIA

Introduction

What do we expect from agents?

- Perform epistemic actions (learn, cheat, suspect, etc.)[1]
- Have varying beliefs about time (be mistaken about time passed, knowing that something was true 'yesterday,' etc.)[2]
- Change basic facts of a world (flip coins, change cards, etc.)[3]

How about common knowledge in such a setting?

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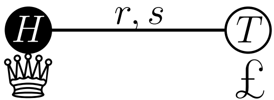

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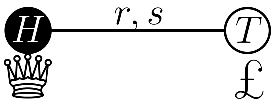

Example

$(M, H) \models K_r(♕ \vee £), \neg K_s ♕ \wedge \neg K_s £, K_r K_s K_r(♕ \vee £)$

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The coin is under a cup, and as Sophie leaves the room, Russell decides to cheat. He looks under the cup, sees that the coin is heads and flips it. Sophie saw this on a hidden camera, and hastened back to the room. During this, Russell, feeling guilty, decides to flip the coin back. Now, Sophie knows that Russell did something, but she does not know whether he just looked underneath the cup, or did something else. Moreover, Sophie is not sure how many actions Russell has performed.

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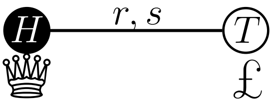

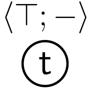

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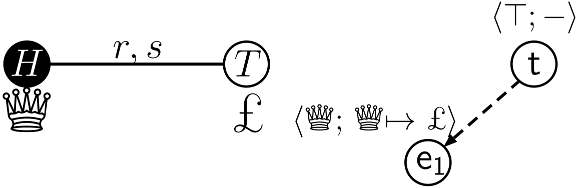

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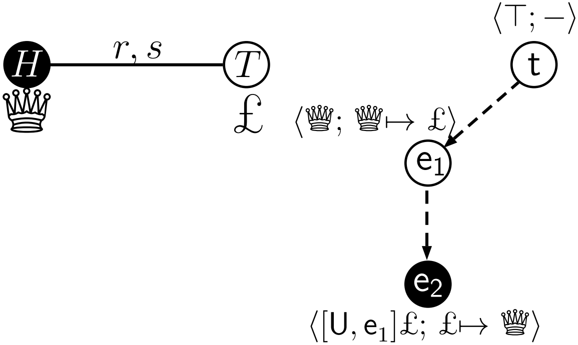

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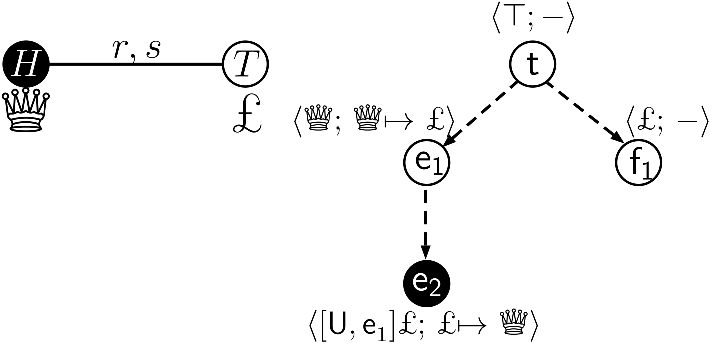

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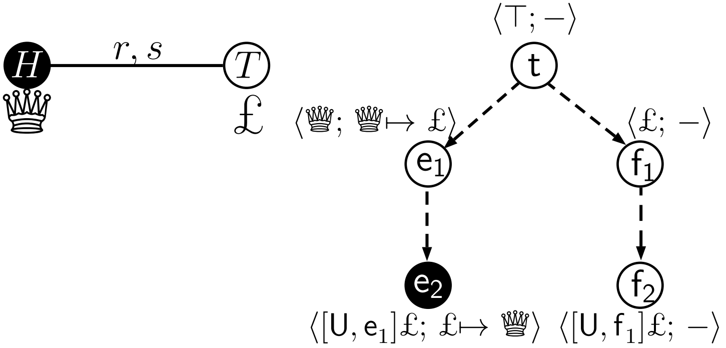

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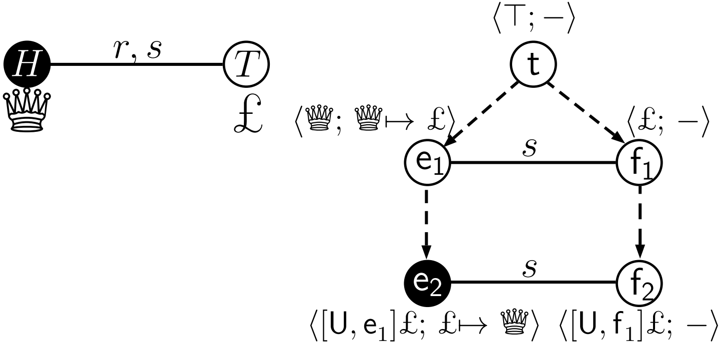

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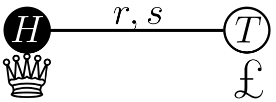

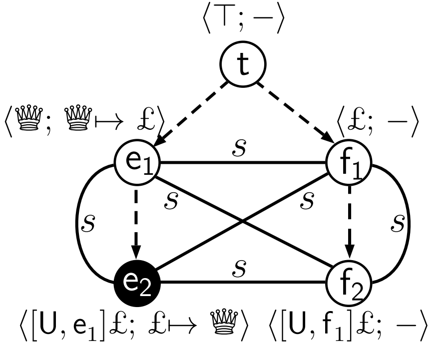

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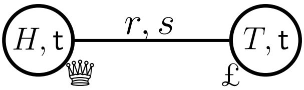

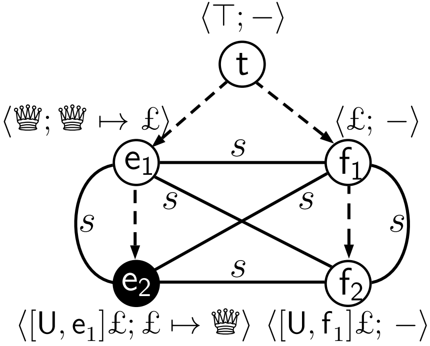

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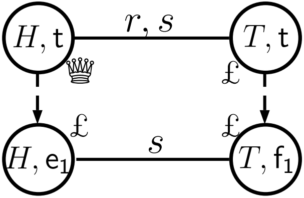

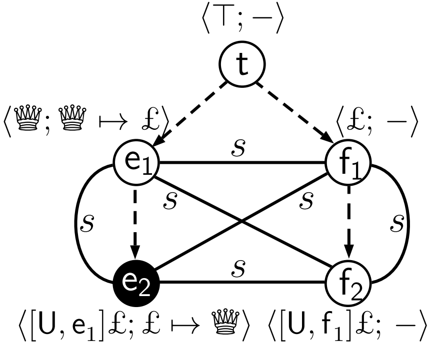

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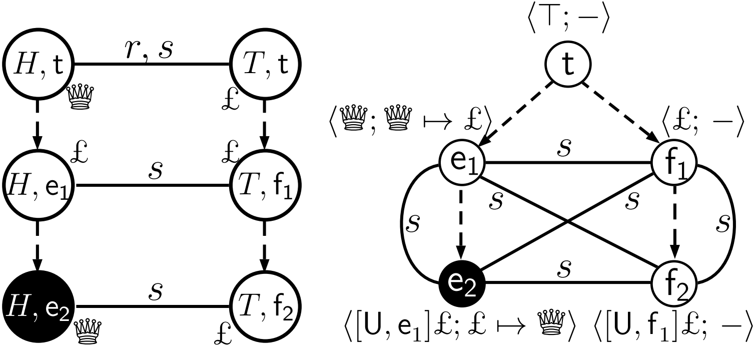

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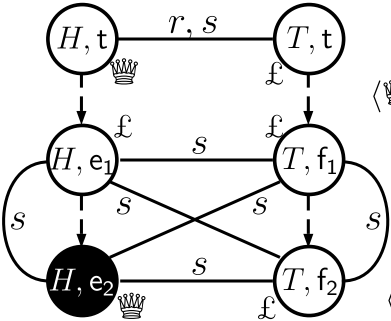

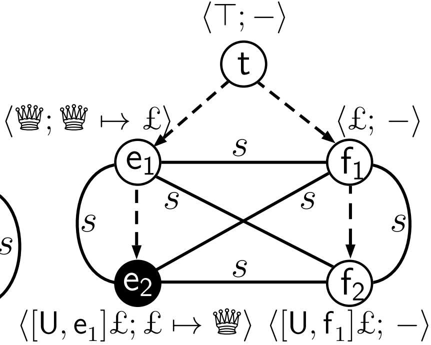

Reduction Axioms

Reduction axioms 'push through' an update operator, and allow to get rid of it altogether

- $[\mathsf{U},\mathsf{e}]p \leftrightarrow (\mathsf{pre}(\mathsf{e}) \rightarrow \mathsf{post}(\mathsf{e})(p))$[4] (we add here ontic changes)
- $[\mathsf{U},\mathsf{e}]\neg\varphi \leftrightarrow (\mathsf{pre}(\mathsf{e}) \rightarrow \neg[\mathsf{U},\mathsf{e}]\varphi)$
- $[\mathsf{U},\mathsf{e}](\varphi \land \psi) \leftrightarrow ([\mathsf{U},\mathsf{e}]\varphi \land [\mathsf{U},\mathsf{e}]\psi)$
- $[\mathsf{U},\mathsf{e}][a]\varphi \leftrightarrow (\mathsf{pre}(\mathsf{e}) \rightarrow \bigwedge_{(\mathsf{e},\mathsf{f})\in \mathsf{R}(\mathsf{a})}[a][\mathsf{U},\mathsf{f}]\varphi)$
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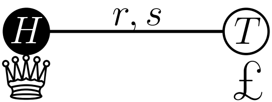

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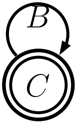

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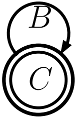

s
s, r, s
s, r, s, r, r, s, s, s
$\ldots$

Updates and Automata

$$[\mathsf{U}, \mathsf{e}][\mathfrak{A}]\varphi \leftrightarrow \bigwedge_{\mathsf{f}\in\mathsf{E}}[\mathfrak{U}_{(\mathsf{U},\mathsf{e},\mathsf{f})} \otimes \mathfrak{A}][\mathsf{U}, \mathsf{f}]\varphi$$ [5]

[5] Barteld Kooi and Johan van Benthem. "Reduction axioms for epistemic actions". In: *AiML-2004: Advances in Modal Logic, Department of Computer Science, University of Manchester, Technical report series, UMCS-04-9-1* (2004), pp. 197–211.

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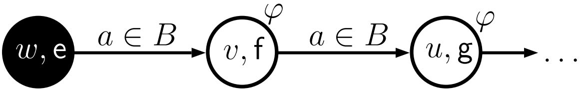

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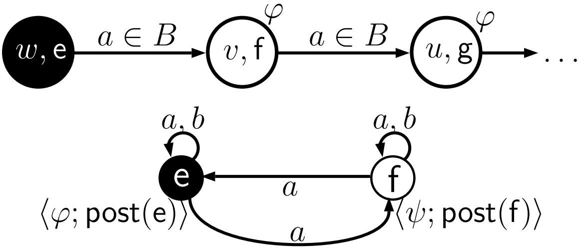

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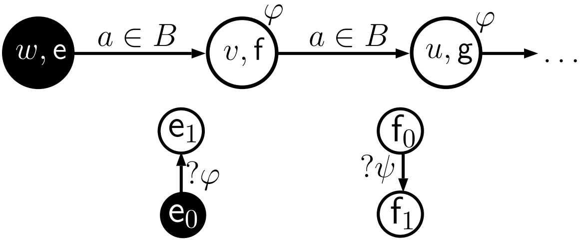

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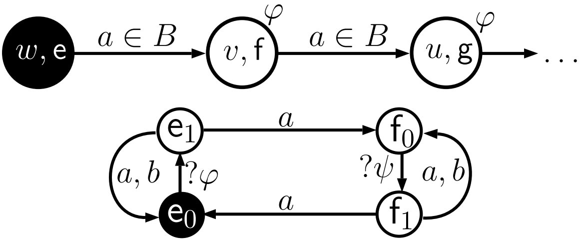

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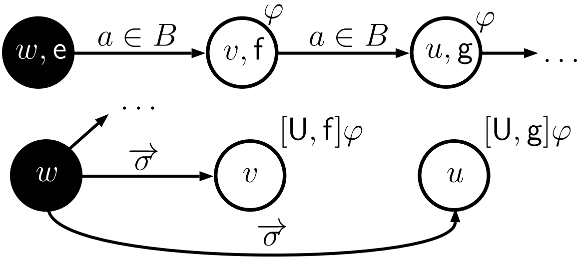

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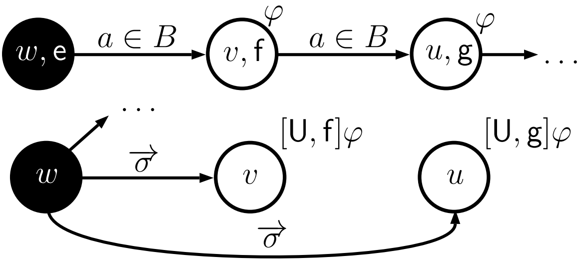

With 'update and automata' axiom DETL with ontic changes and <u>common knowledge is complete</u>.

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Summary

- Added ontic changes to DETL[6]
- Added common knowledge to DETL and employed the existing technique to show completeness of the resulting system
- Future work:
  - Add distributed knowledge to the system
  - Consider more temporal operators

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Thank you for attention!

Language

Definition (Language of $\mathbf{APDL}^{+\ltimes}$)

The *language* $\mathcal{L}_{\mathbf{APDL}^{+\ltimes}}$ *of* $\mathbf{APDL}^{+\ltimes}$ is as follows:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid [a]\varphi \mid [?\varphi]\varphi \mid [Y]\varphi \mid [\mathfrak{A}]\varphi \mid [\mathsf{U}, \mathsf{e}]\varphi,$$

where $p \in P$, $a \in A$, $\mathfrak{A}$ is an automaton over $A \cup \{?\varphi \mid \varphi \in \mathcal{L}\}$, and U is an update model. All the usual abbreviations of propositional logic and conventions for deleting parentheses hold, and $\langle\alpha\rangle\varphi$ is equivalent to $\neg[\alpha]\neg\varphi$.

Epistemic Models

Definition (Epistemic model)

An *epistemic model* (with yesterday) is a quadruple $M = (W, R, \rightsquigarrow^M, V)$, where

- W is a non-empty set of states,
- $R : A \to \mathcal{P}(W \times W)$ is an accessibility relation for each agent $a \in A$,
- $\rightsquigarrow^M: W \to W$ is a temporal, 'yesterday', relation between states,
- $V : P \to \mathcal{P}(W)$ is a valuation of propositional variables $p \in P$.

Update Models

Definition (Update model)

An (epistemic-temporal) *update model* is a tuple $\mathsf{U} = (\mathsf{E}, \mathsf{R}, \rightsquigarrow^{\mathsf{U}}, \mathsf{pre}, \mathsf{post})$, where

- E is a finite non-empty set of events,
- $\mathsf{R} : A \to \mathcal{P}(\mathsf{E} \times \mathsf{E})$ is an accessibility relation for each agent $a \in A$,
- $\rightsquigarrow^{\mathsf{U}}: \mathsf{E} \to \mathsf{E}$ is a temporal, 'yesterday', relation between events,
- $\mathsf{pre} : \mathsf{E} \to \mathcal{L}$ assigns to each event a precondition,
- $\mathsf{post} : \mathsf{E} \to (P \to \mathcal{L})$ assigns to each event a postcondition for each propositional variable. Each post(e) is either identity id or finitely different from it. In the latter case, the finite difference is called a domain $dom(\mathsf{post}(\mathsf{e}))$.

Semantics

Definition (Semantics of $\mathbf{APDL}^{+\ltimes}$)

$$
\begin{array}{lll}
(M, w) \models p & \text{iff} & w \in V(p) \\
(M, w) \models \neg\varphi & \text{iff} & (M, w) \not\models \varphi \\
(M, w) \models \varphi \wedge \psi & \text{iff} & (M, w) \models \varphi \text{ and } (M, w) \models \psi \\
(M, w) \models [a]\varphi & \text{iff} & \text{for all } v \in W : (w, v) \in R(a) \\
 & & \text{implies } (M, v) \models \varphi \\
(M, w) \models [Y]\varphi & \text{iff} & \text{for all } v \in W : v \rightsquigarrow^{M} w \\
 & & \text{implies } (M, v) \models \varphi \\
(M, w) \models C_B\varphi & \text{iff} & \text{for all } v \in W : (w, v) \in (\bigcup_{a \in B} R(a))^{*} \\
 & & \text{implies } (M, v) \models \varphi \\
(M, w) \models [\mathsf{U}, \mathsf{e}]\varphi & \text{iff} & (M, w) \models \mathsf{pre}(\mathsf{e}) \\
 & & \text{implies } (M \cdot \mathsf{U}, (w, \mathsf{e})) \models \varphi
\end{array}
$$

Execution

Definition (Execution of an update model)

The result of *executing* (U, e) in (M, w) with $(M, w) \models \mathsf{pre}(\mathsf{e})$ is the epistemic model $(M \cdot \mathsf{U}, (w, \mathsf{e})) = ((W^{M \cdot \mathsf{U}}, R^{M \cdot \mathsf{U}}, \rightsquigarrow^{M \cdot \mathsf{U}}, V^{M \cdot \mathsf{U}}), (w, \mathsf{e}))$, where

- $W^{M \cdot \mathsf{U}} = \{(v, \mathsf{f}) \mid (M, v) \models \mathsf{pre}(\mathsf{f})\}$,
- $R^{M \cdot \mathsf{U}}(a) = \{((v, \mathsf{f}), (u, \mathsf{g})) \mid (v, u) \in R(a) \text{ and } (\mathsf{u}, \mathsf{g}) \in \mathsf{R}(a)\}$,
- $\rightsquigarrow^{M \cdot \mathsf{U}} = \{((v, \mathsf{f}), (u, \mathsf{g})) \mid (v, u) \in \rightsquigarrow^{M}, \mathsf{f} = \mathsf{g}$ and f is a past state; or $v = u$ and $(\mathsf{f}, \mathsf{g}) \in \rightsquigarrow^{\mathsf{U}}\}$,
- $V^{M \cdot \mathsf{U}}(p) = \{$(a) f is a past state and $(M, v) \models \mathsf{post}(\mathsf{f})(p)$; or (b) $p \in dom(\mathsf{post}(\mathsf{f}))$ and $(M, v) \models \mathsf{post}(\mathsf{f})(p)$; or (c) for some state $(u, \mathsf{g}) \in W^{M \cdot \mathsf{U}}$ with $d(u, \mathsf{g}) < d(v, \mathsf{f})$ such that for all $(u', \mathsf{g}') \in W^{M \cdot \mathsf{U}}$ with $d(u, \mathsf{g}) < d(u', \mathsf{g}') < d(v, \mathsf{f})$ and $p \notin dom(\mathsf{g}')$, either (a) holds for (u, g), or $p \in dom(\mathsf{g})$ and $(M, u) \models \mathsf{post}(\mathsf{g})(p)\}$.