Intentional Anonymous Public Announcements

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Thomas Ågotnes
University of Bergen, Norway
Shanxi University, China

Rustam Galimullin, University of Bergen, Norway
Ken Satoh, Center for Juris-informatics, Japan
Satoshi Tojo, Asia University, Japan

Anonymous Public Announcements

Example:
We come back to this room after a break, and we see that someone has written “it was raining in Bergen yesterday” on the whiteboard. Only members of our group have access to this room.

What do you learn?

Elevator pitch

- We introduce a new public announcement operator modeling *anonymous public announcements*
- “In-between” an announcement from the “outside” and an announcement from the “inside”
- Assume (common knowledge) that the announcer *intended* to stay anonymous => more revealing!
- It all boils down to the notion of a *safe* announcement

Background: Public Announcement Logic

Background: Epistemic Logic

Given: a set P of atomic propositions and a finite set N of agents

$$\phi ::= \ p \mid \neg\phi \mid (\phi \wedge \phi) \mid K_a\phi$$

Models: $M = (S, \sim, V)$, where

- S is a non-empty set of worlds,
- $\sim$: $N \longrightarrow \wp(S \times S)$ assigns a reflexive, transitive and symmetric accessibility relation, $\sim_a$, to each agent a, and
- $V : P \longrightarrow \wp(S)$ maps each proposition to the set of worlds where it is true.

Background: Epistemic Logic

Interpretation:

$$
\begin{array}{rcl}
M, s \models p & \text{iff} & s \in V(p) \\
M, s \models \neg\phi & \text{iff} & M, s \not\models \phi \\
M, s \models \phi_1 \wedge \phi_2 & \text{iff} & M, s \models \phi_1 \text{ and } M, s \models \phi_2 \\
M, s \models K_a\phi & \text{iff} & \forall t \in S \text{ where } s \sim_a t,\ M, t \models \phi
\end{array}
$$

Background: Public Announcement Logic

$$\phi ::= p \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid K_i\phi \mid [\phi!]\psi$$

Interpretation:

$$M, s \models [\psi!]\phi \quad \text{iff} \quad M, s \models \psi \Longrightarrow M^{\psi!}, s \models \phi$$

$M^{\psi!} = (S', \sim', V')$ is such that:

- $S' = \{s \in S \mid M_s \models \psi\}$;
- for all $a \in N$, $\sim'_a = \sim_a \cap (S' \times S')$;
- for all $p \in P$, $V'(p) = V(p) \cap S'$.

Background: Public Announcement Logic

$[\phi!]\psi$: after ϕ is truthfully announced "from the outside", ψ is true

$[K_a\phi!]\psi$: after ϕ is truthfully announced by agent a, ψ is true

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More information!

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More information!

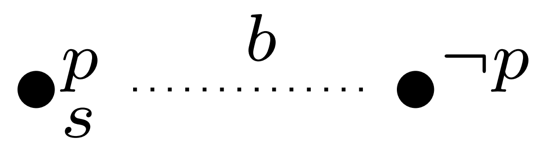
We are looking for something in-between.

Intentionally Anonymous Public Announcements

Assuming common knowledge of the intention to stay anonymous

- We learn that the anonymous agent knew phi
- .. and that she knew that it was *safe* to announce phi
- What does safety mean?

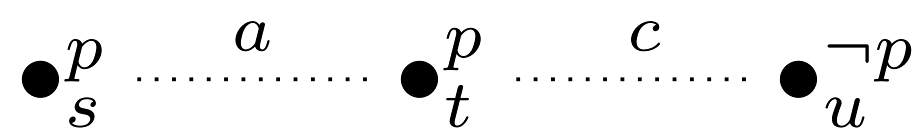
Safety: 2-anonymity is not enough (3 agents)



Safety: 2-anonymity is not enough (3 agents)

$$\bullet^{p}_{s} \;\overset{b}{\cdots\cdots\cdots\cdots}\; \bullet^{\neg p}$$

If p is announced by a or c, the other agent knows who it was


p
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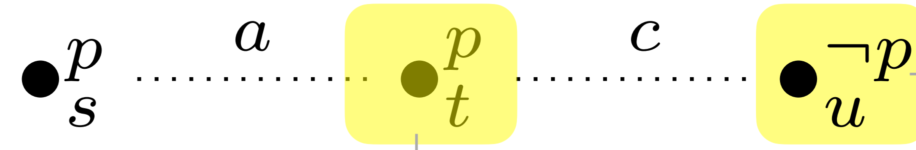
$$\bullet_{s}^{p} \overset{a}{\cdots\cdots\cdots} \bullet_{t}^{p} \overset{c}{\cdots\cdots\cdots} \boxed{\bullet_{u}^{\neg p}}$$

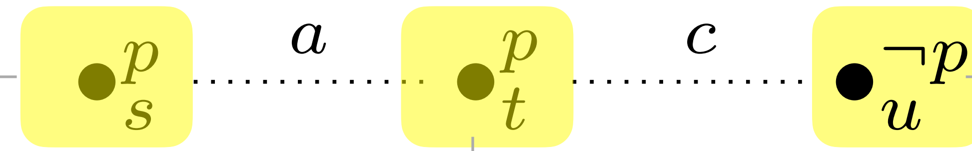
p cannot be announced by anyone



p cannot be announced by anyone

only two agents know p, so p cannot be safely announced

Three agents (a,b,c) know p. But a does not know that p is safe, so p is not safe for a.



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$$\bullet^{p}_{s} \overset{a}{\cdots\cdots} \bullet^{p}_{t} \overset{c}{\cdots\cdots} \bullet^{\neg p}_{u}$$

p cannot be announced by anyone

only two agents know p, so p cannot be safely announced

b knows that p is not safe for a and that c knows that, so p is not safe for b

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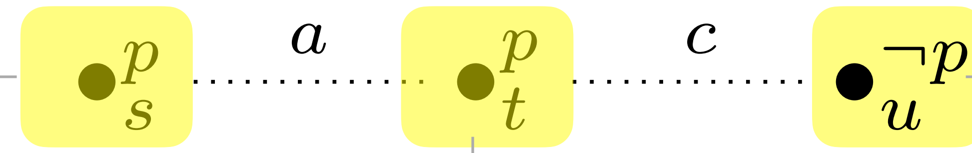

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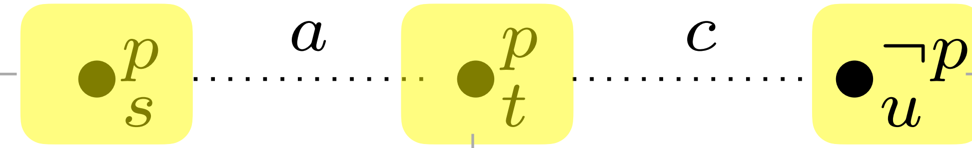

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similarly, p is not safe for c

p cannot safely be announced by anyone!

Safety: thus 3-anonymity is also not enough

Not enough that three agents know ϕ.

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Not enough that three agents know ϕ.

They must also know that three agents know ϕ.

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What about common knowledge? $\bigvee_{a \neq b \neq c} C_{\{a,b,c\}} \phi$

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Sufficient but not *necessary*.

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.. and so on..

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Sufficient but not *necessary*.

We don't need it to be *the same* three agents.

Safety: what we need

$$\text{Let } N^3 = \{\{a, b, c\} : a, b, c \in N, a \neq b \neq c\}$$

$$E_G\phi \equiv \bigwedge_{i \in G} K_i\phi$$

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$$M, s \models \bigvee_{G_1 \in N^3} E_{G_1}\phi$$

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$\ldots$

What we want is the greatest fixed-point of:

$$f(S) = || \bigvee_{G \in N^3} E_G(\phi \wedge x)||^M_{V[x=S]}$$

(where x is not in ϕ)

Safety

$\blacktriangle\phi$: ϕ can safely be announced

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In modal μ-calculus: $\qquad \blacktriangle\phi \leftrightarrow \nu x. \bigvee_{G \in N^3} E_G(\phi \wedge x)$

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In modal μ-calculus: $\quad \blacktriangle\phi \leftrightarrow \nu x. \bigvee_{G \in N^3} E_G(\phi \wedge x)$

$$M, s \models \blacktriangle\phi \Leftrightarrow s \in \bigcup \left\{ S : S \subseteq || \bigvee_{G \in N^3} E_G(\phi \wedge x) ||^M_{[x=S]} \right\}$$

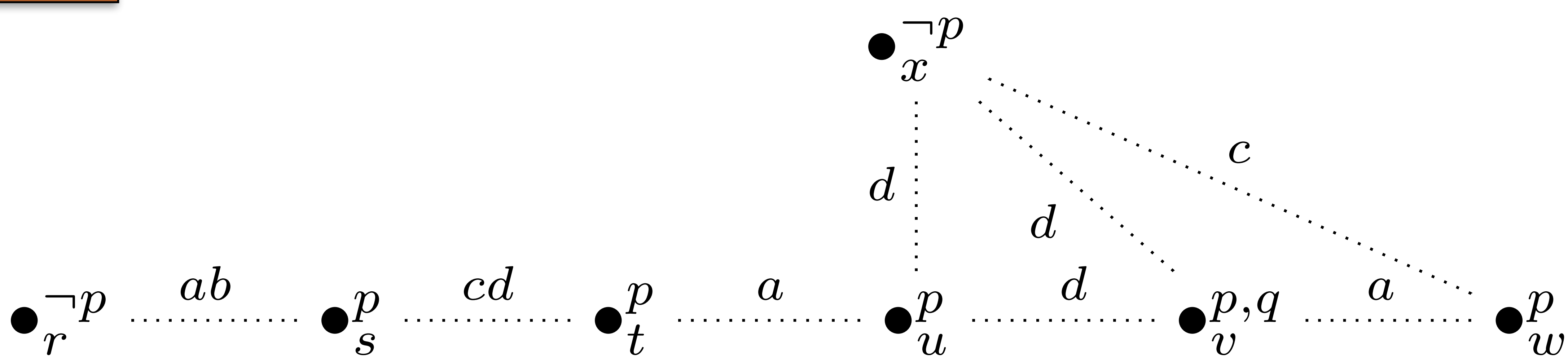
Knowledge of safety

$K_a \blacktriangle \phi$: ϕ can safely be announced by a

(a is a member of a group of three such that ...)

Safety

p safe?



Safety

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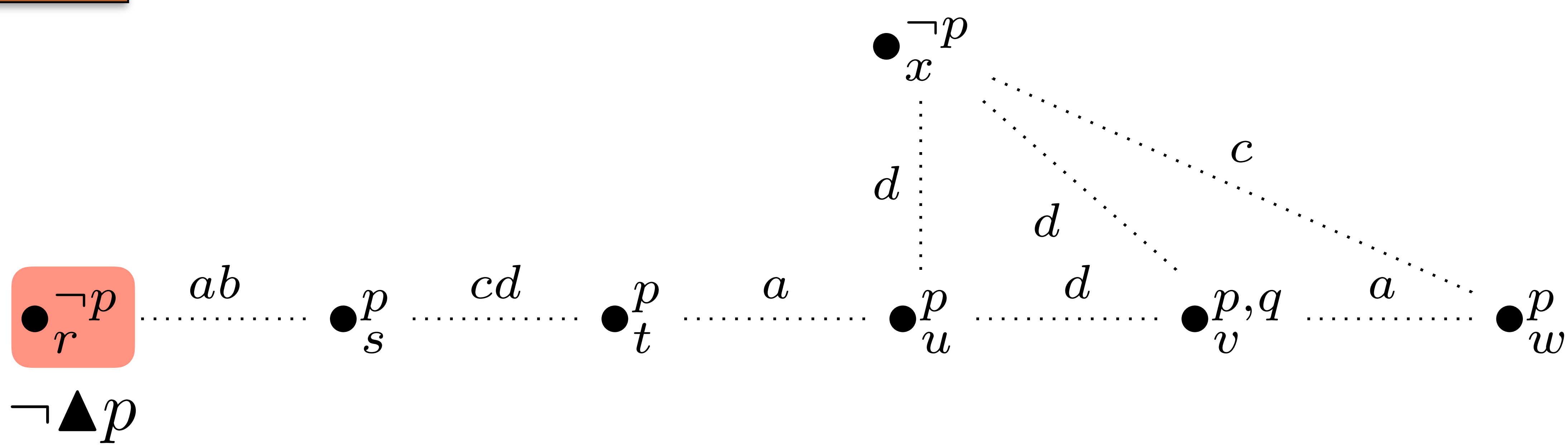

Safety

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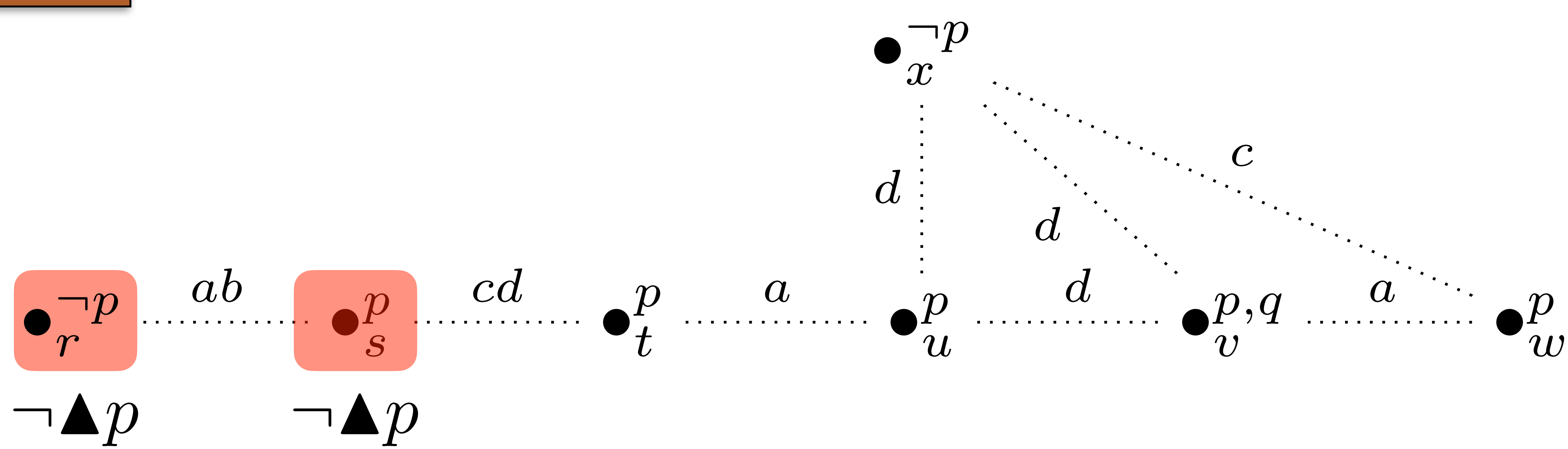

Safety

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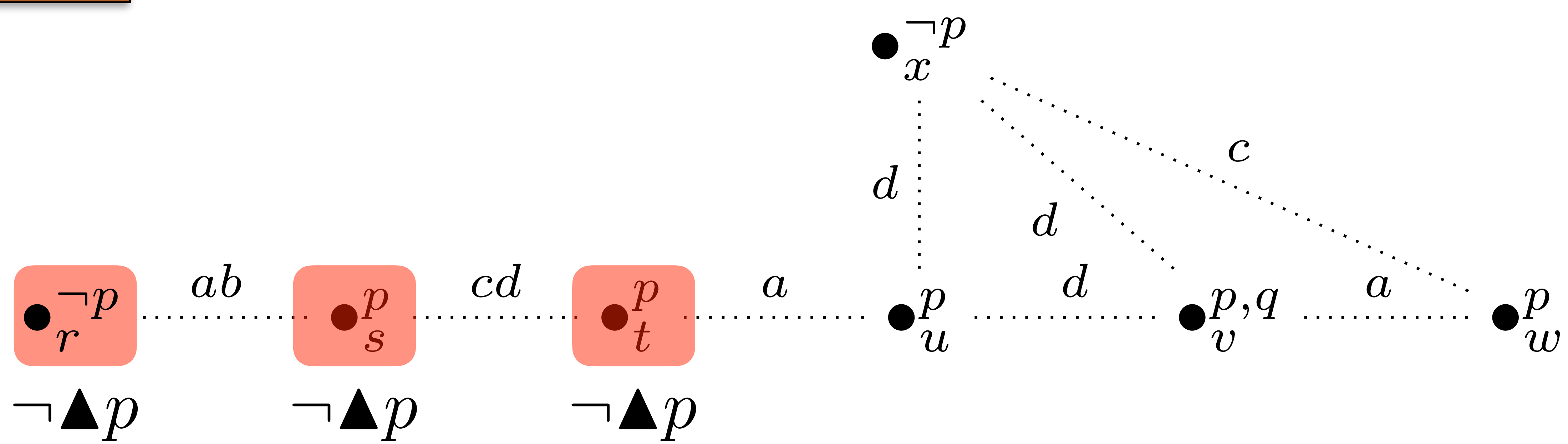

Safety

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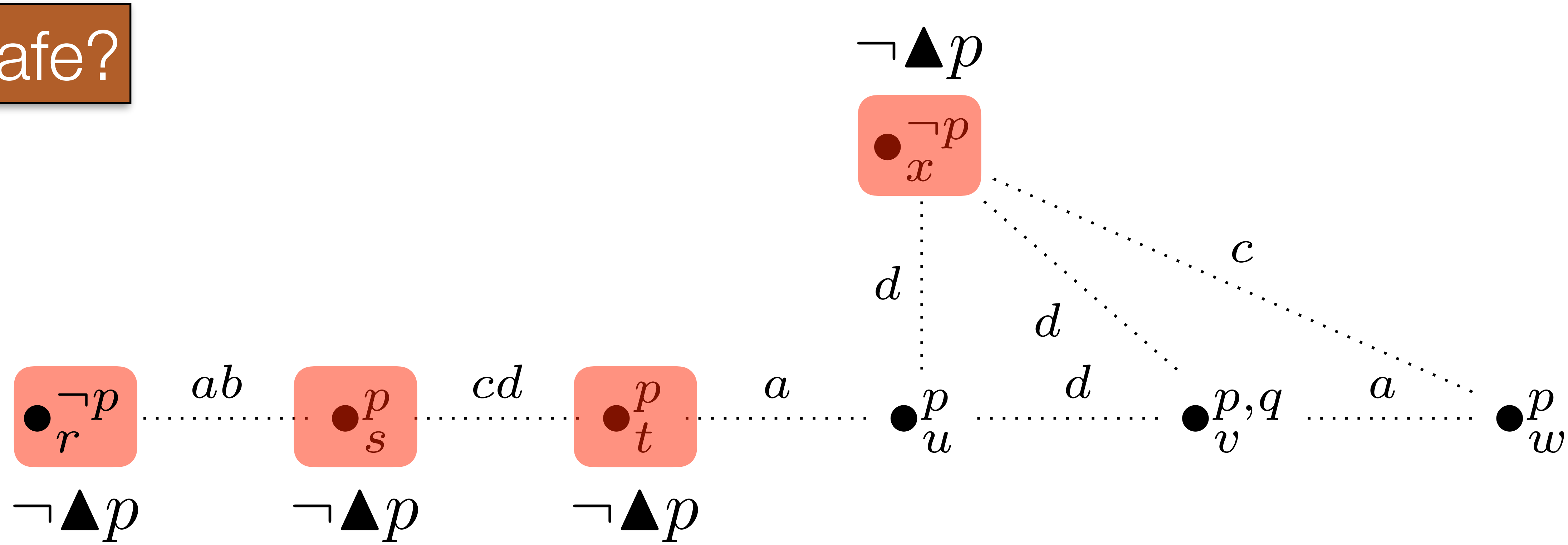

Safety

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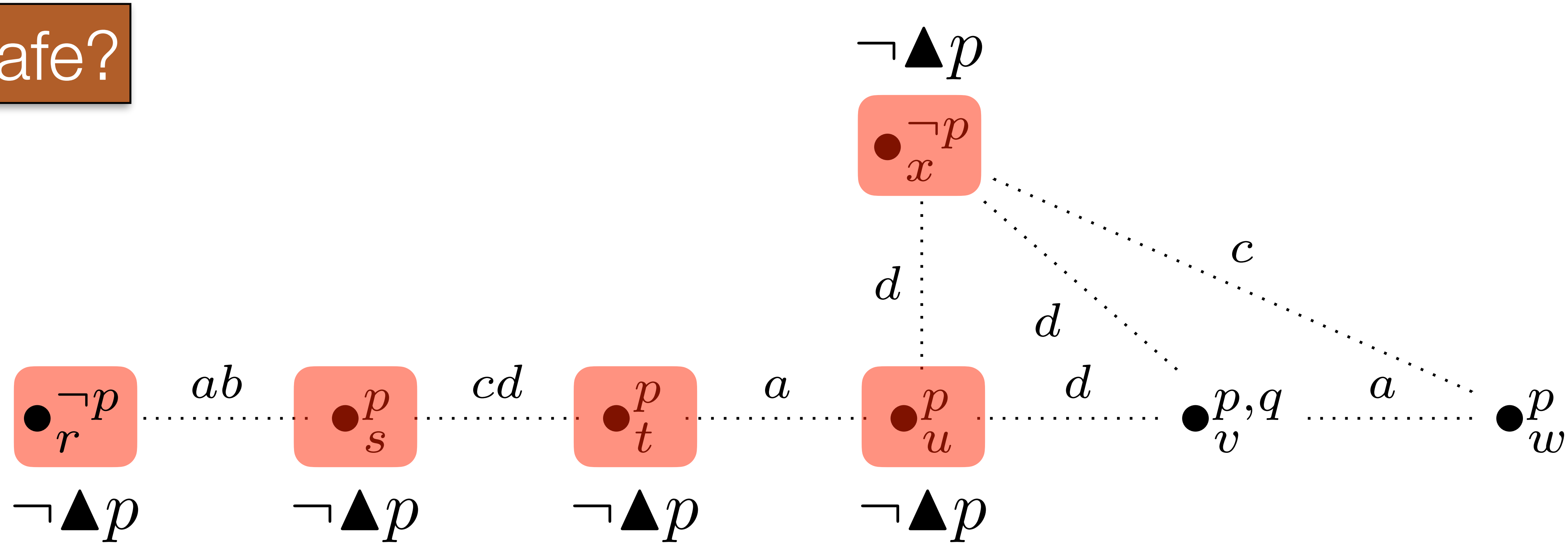

Safety

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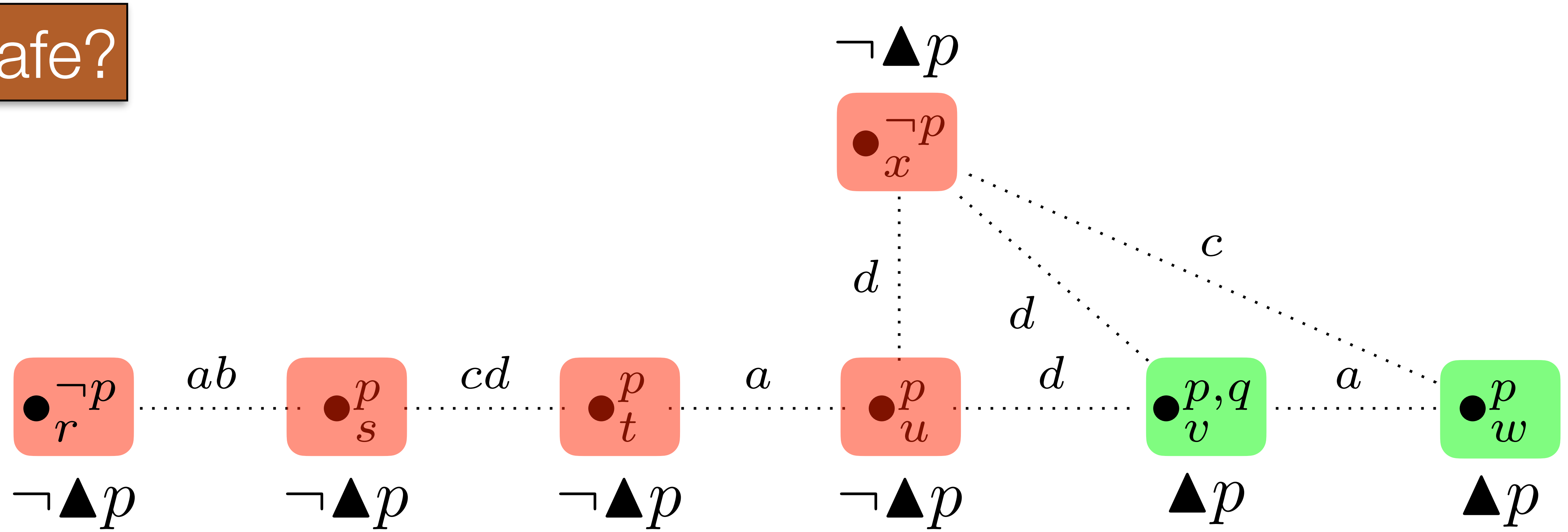
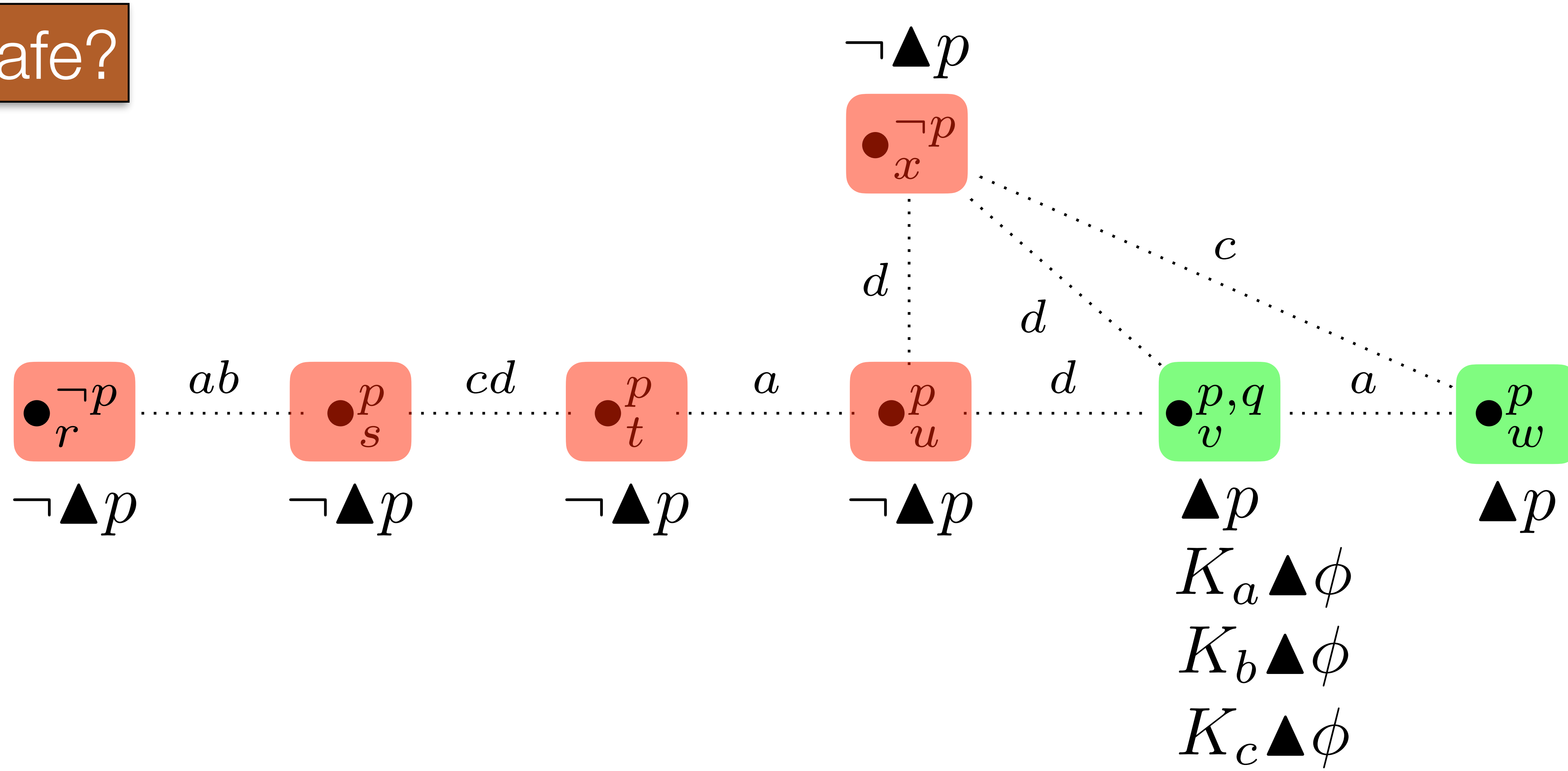


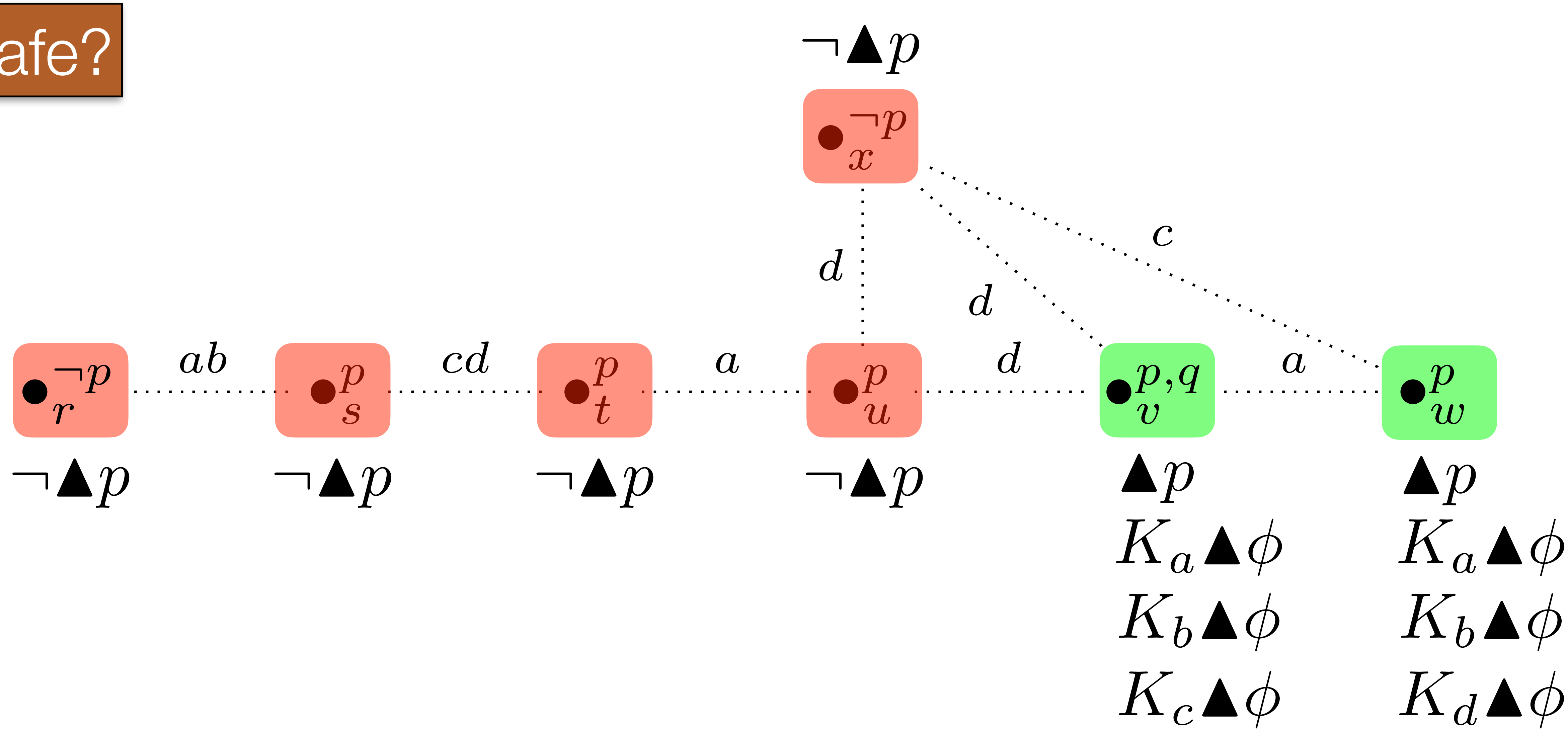
Safety

p safe?



Safety

p safe?



Anonymous Public Announcement Logic with Intentions

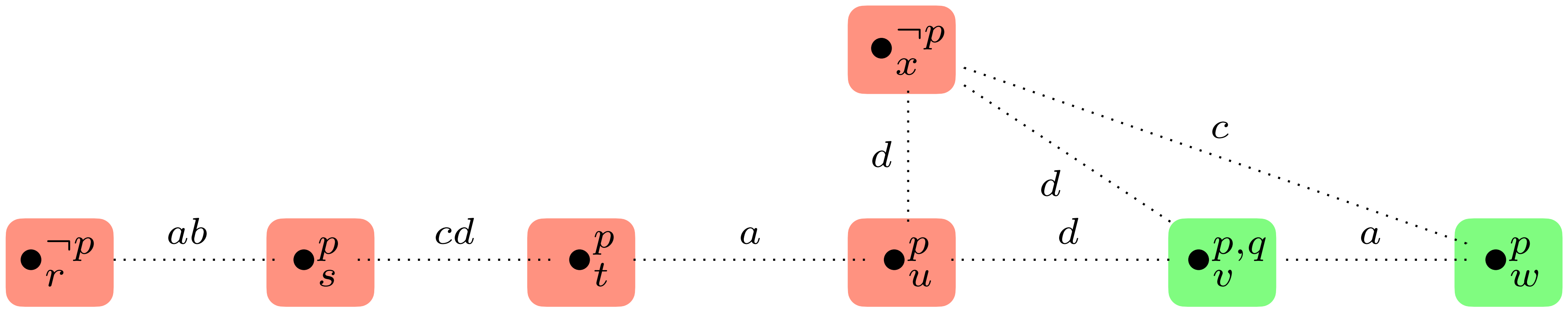
$[\phi\ddagger]\psi$: after any safe announcement of ϕ, ψ is true

Definition 1 *The update of epistemic model $M = (S, \sim, V)$ by the safe pseudo-anonymous announcement of ϕ is the epistemic model $M^{\phi\ddagger} = (S', \sim', V')$ where:*

- $S' = \{(s, a) : s \in S,\ M, s \models K_a \blacktriangle \phi\}$
- $(s, a) \sim'_c (t, b)$ *iff* $s \sim_c b$ *and* $a = c$ *iff* $b = c$
- $V'(s, a) = V(s)$

We then let:

$$M, s \models [\phi\ddagger]\psi \Leftrightarrow \forall a \in N, \left(M, s \models K_a \blacktriangle \phi \Rightarrow M^{\phi\ddagger}, (s, a) \models \psi\right).$$



¬p
x
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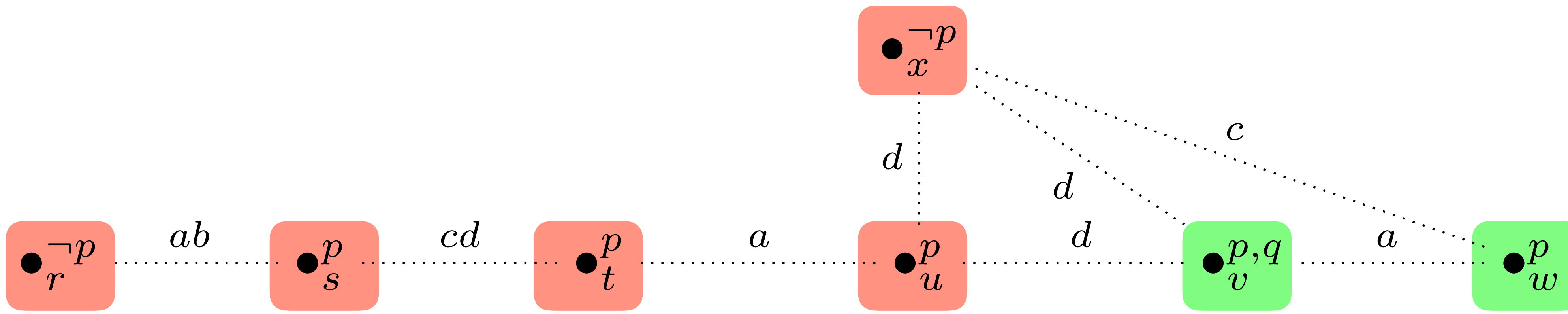
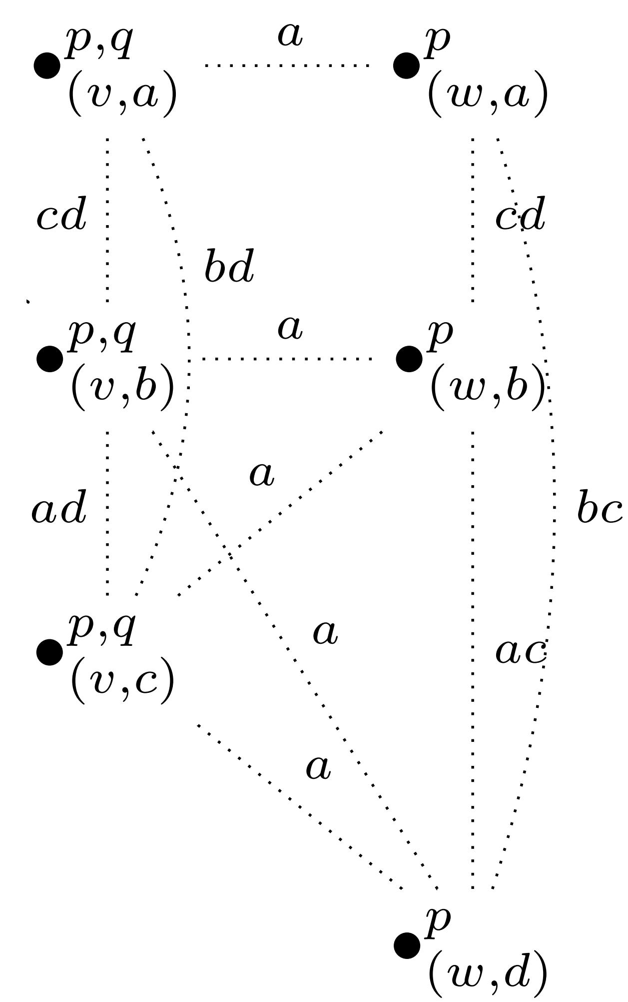
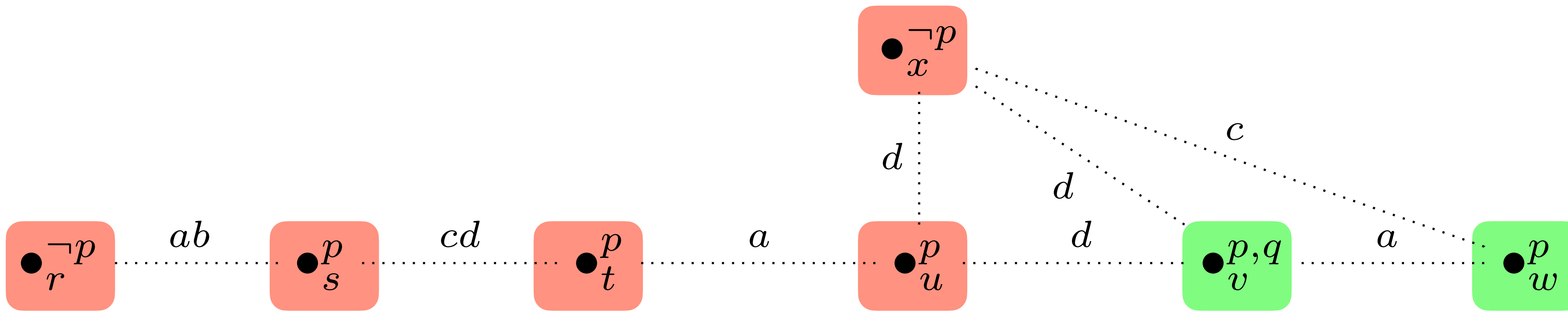


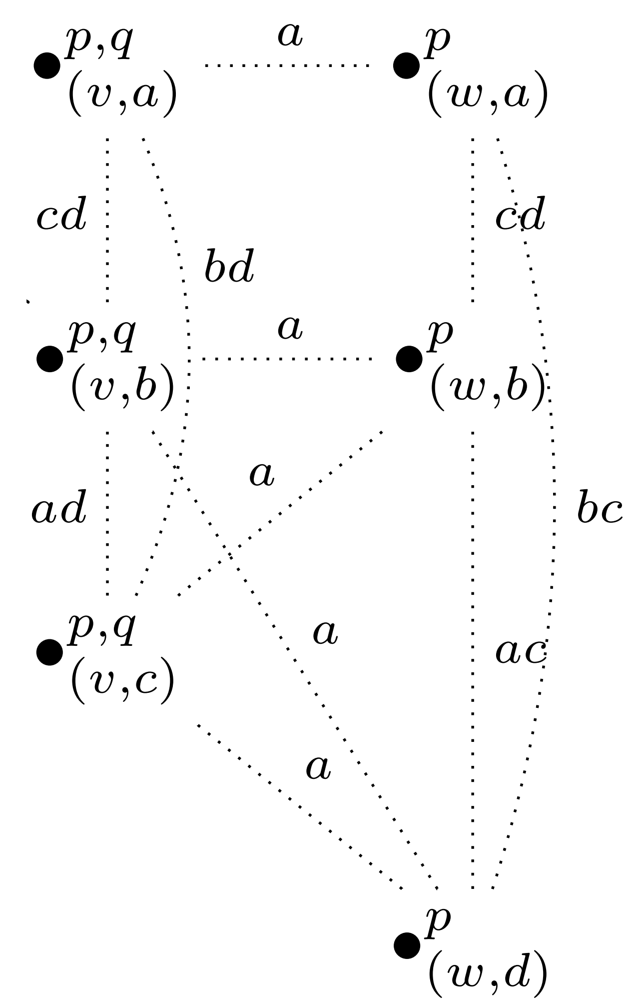
$\Downarrow \quad p\ddagger$

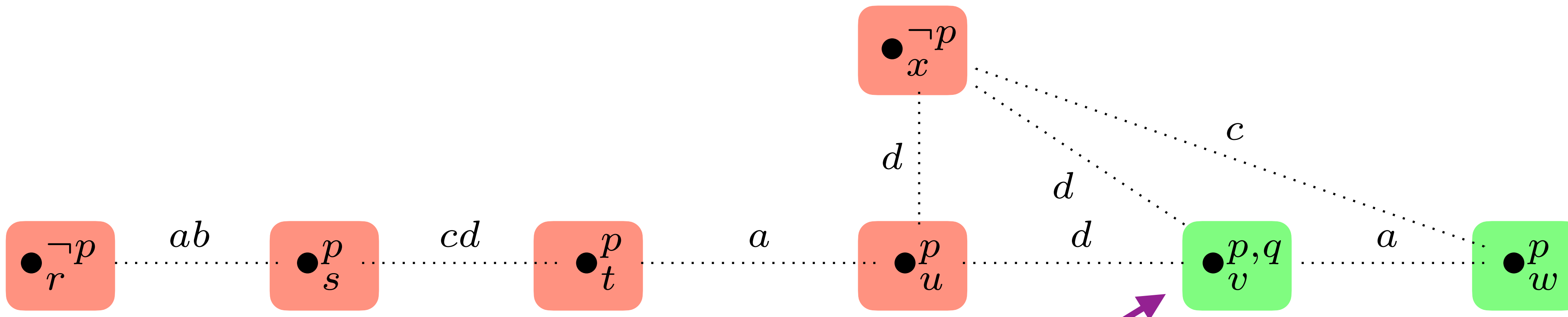




$\Downarrow$ $p‡$

$$M, v \models [p‡]K_cK_dq$$

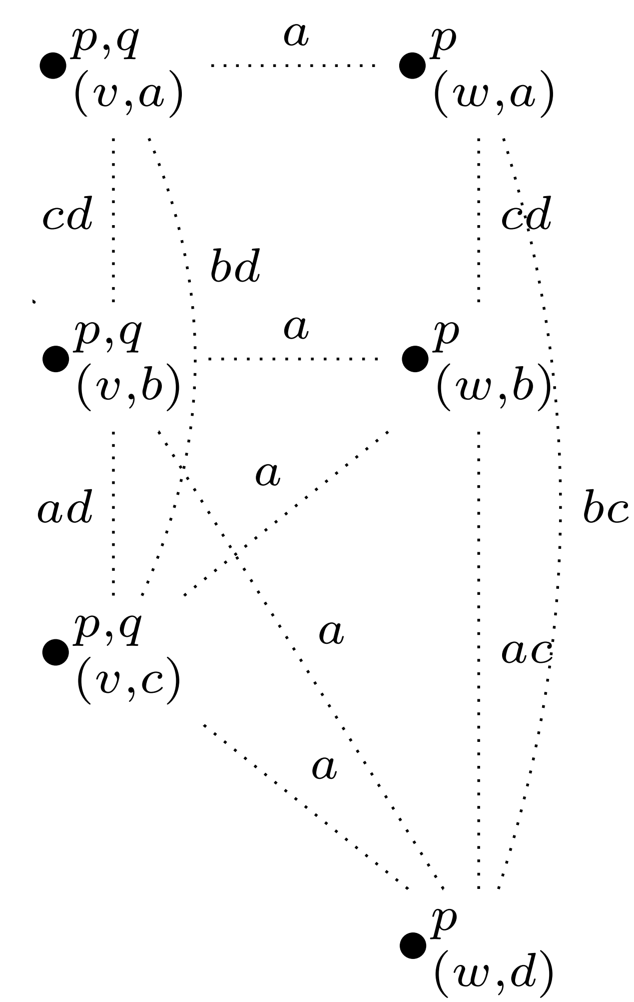


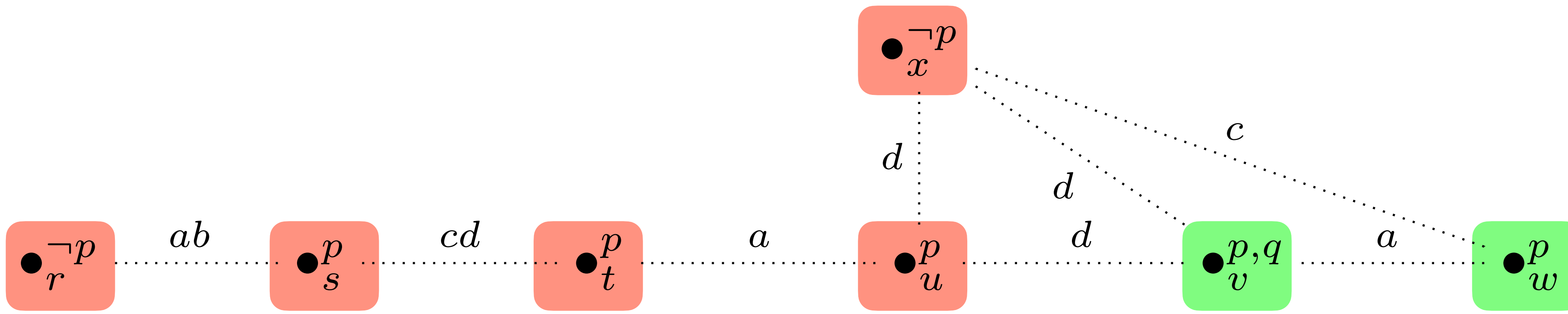


⇓ $p\ddagger$

$$M, v \models [p\ddagger]K_cK_dq$$

a knows *de dicto* that p is safe, but not *de re*





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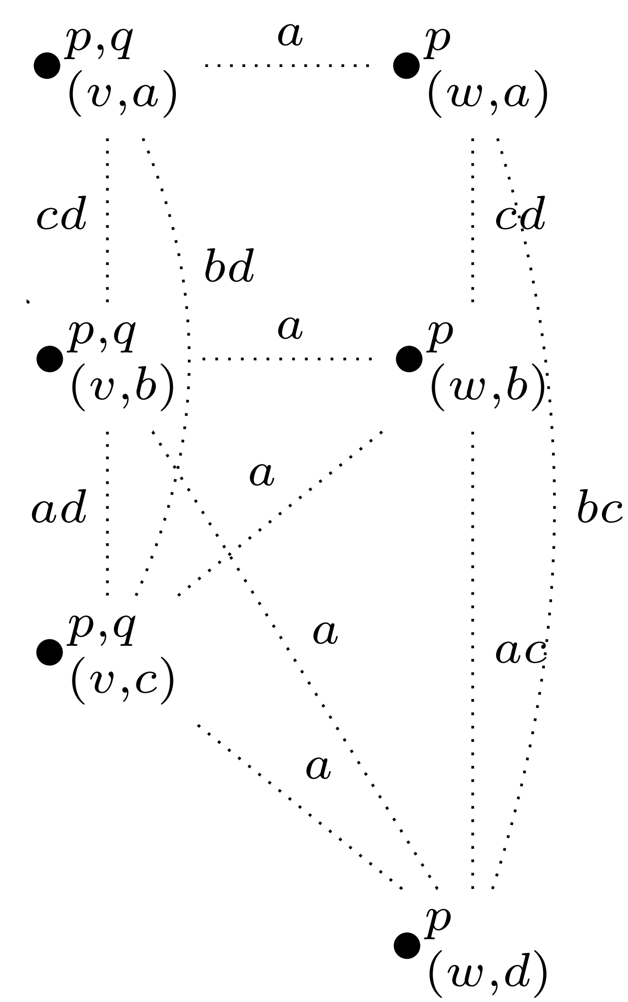
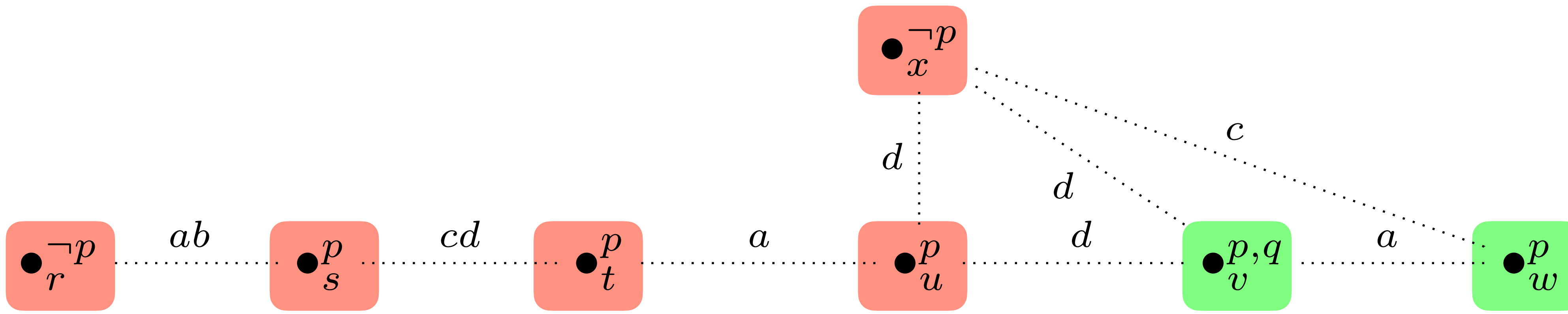
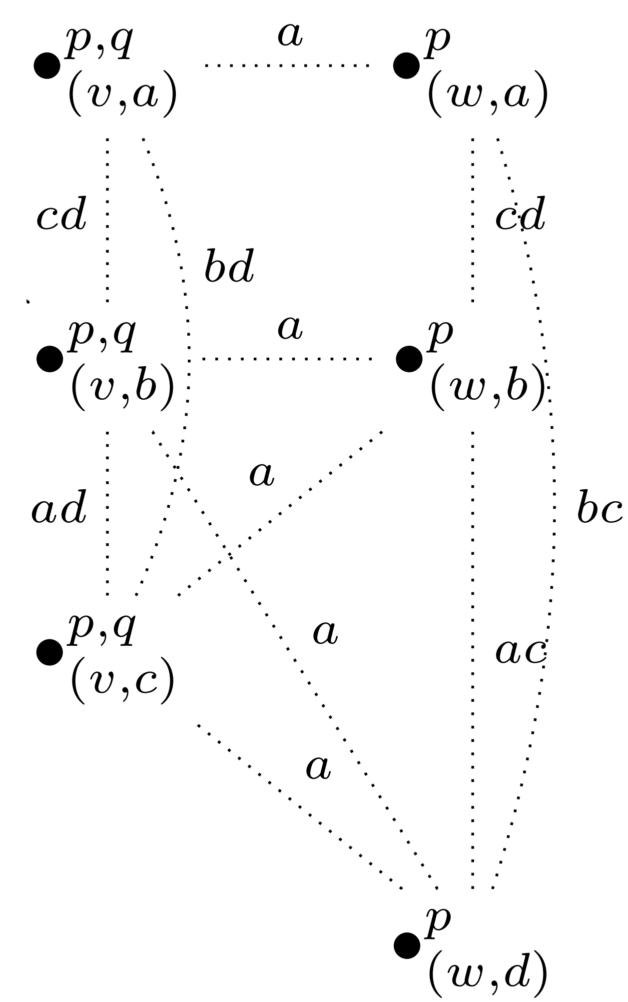




$\Downarrow$ $p\ddagger$

$$M, v \models [p\ddagger]K_cK_dq$$

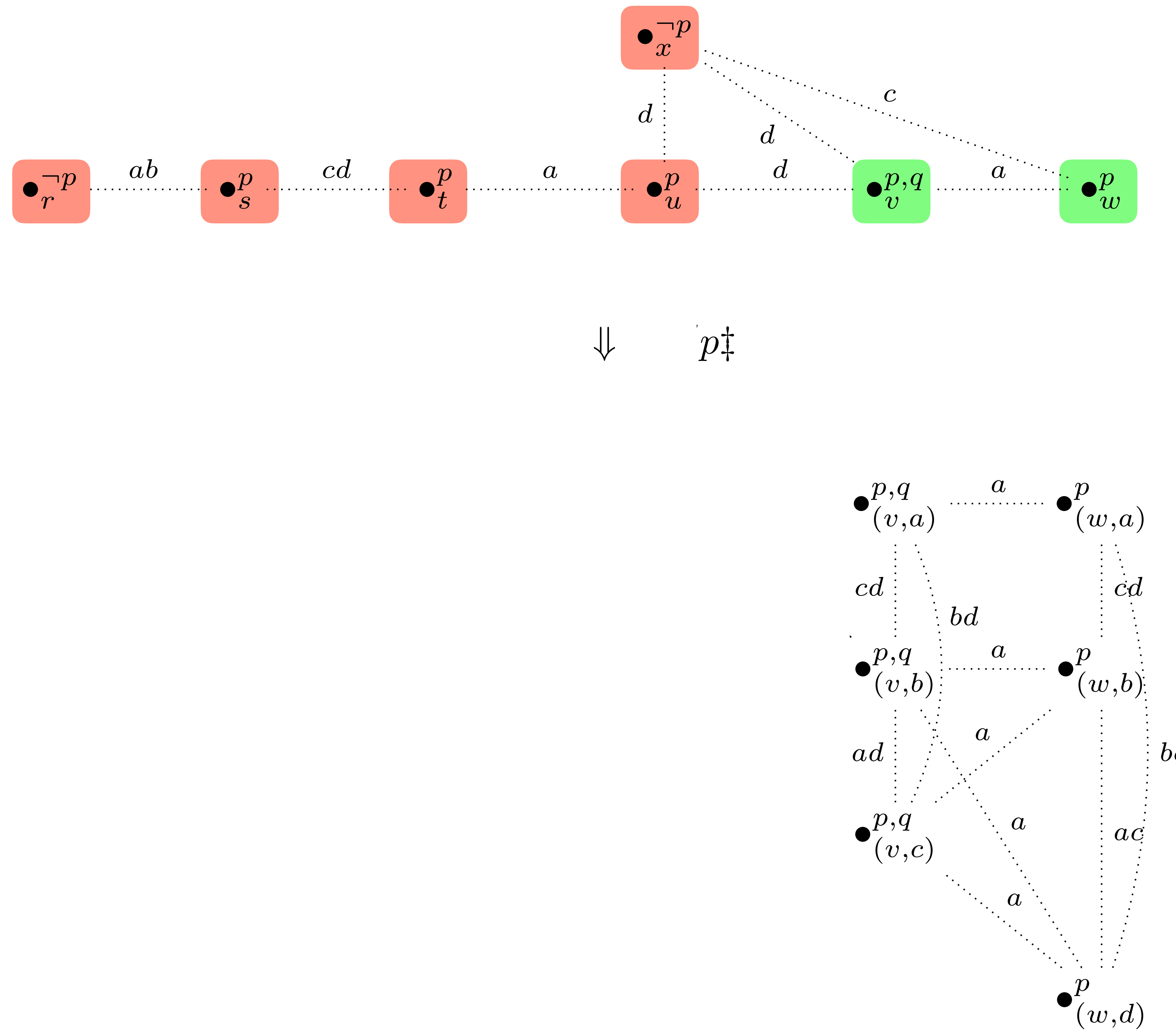
Only the announcer knows who the announcer was



Safety is safe

Lemma 1 *In any update by a safe pseudo-anonymous announcement, it is common knowledge that no-one except the announcer knows who the announcer is (more technically: in any state* (s, a)*, for any agent* $i \neq a$ *there is a state* (s', a') *such that* $(s, a) \sim_i (s', a')$ *and* $a \neq a'$*).*

$K_a \blacktriangle \phi$ is thus both sufficient and necessary for a to safely announce ϕ



¬p x
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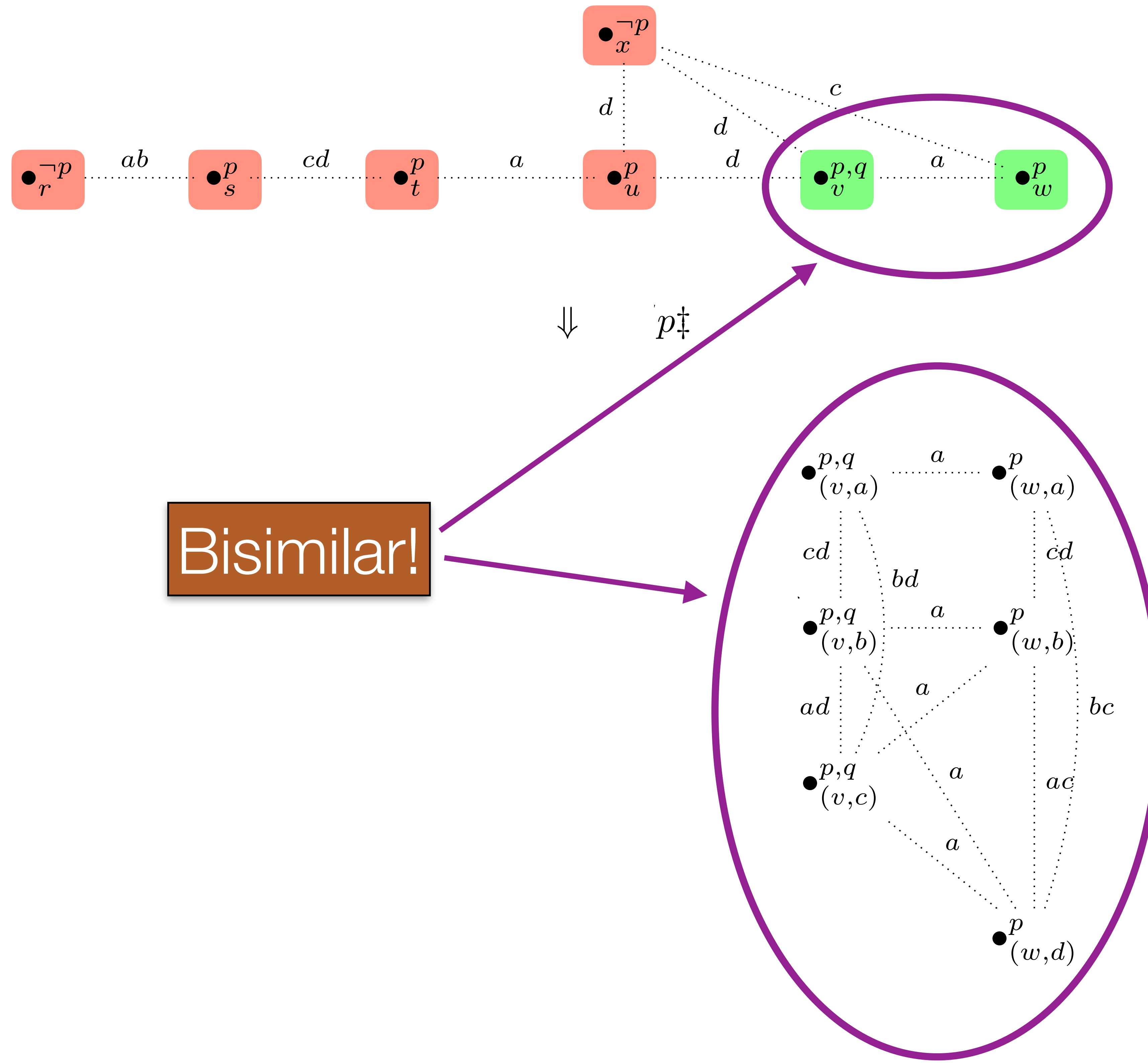

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p t
p u
p,q v
p w
ab
cd
a
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d
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c
a
p‡
Bisimilar!
p,q (v,a)
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p (w,b)
p,q (v,c)
p (w,d)
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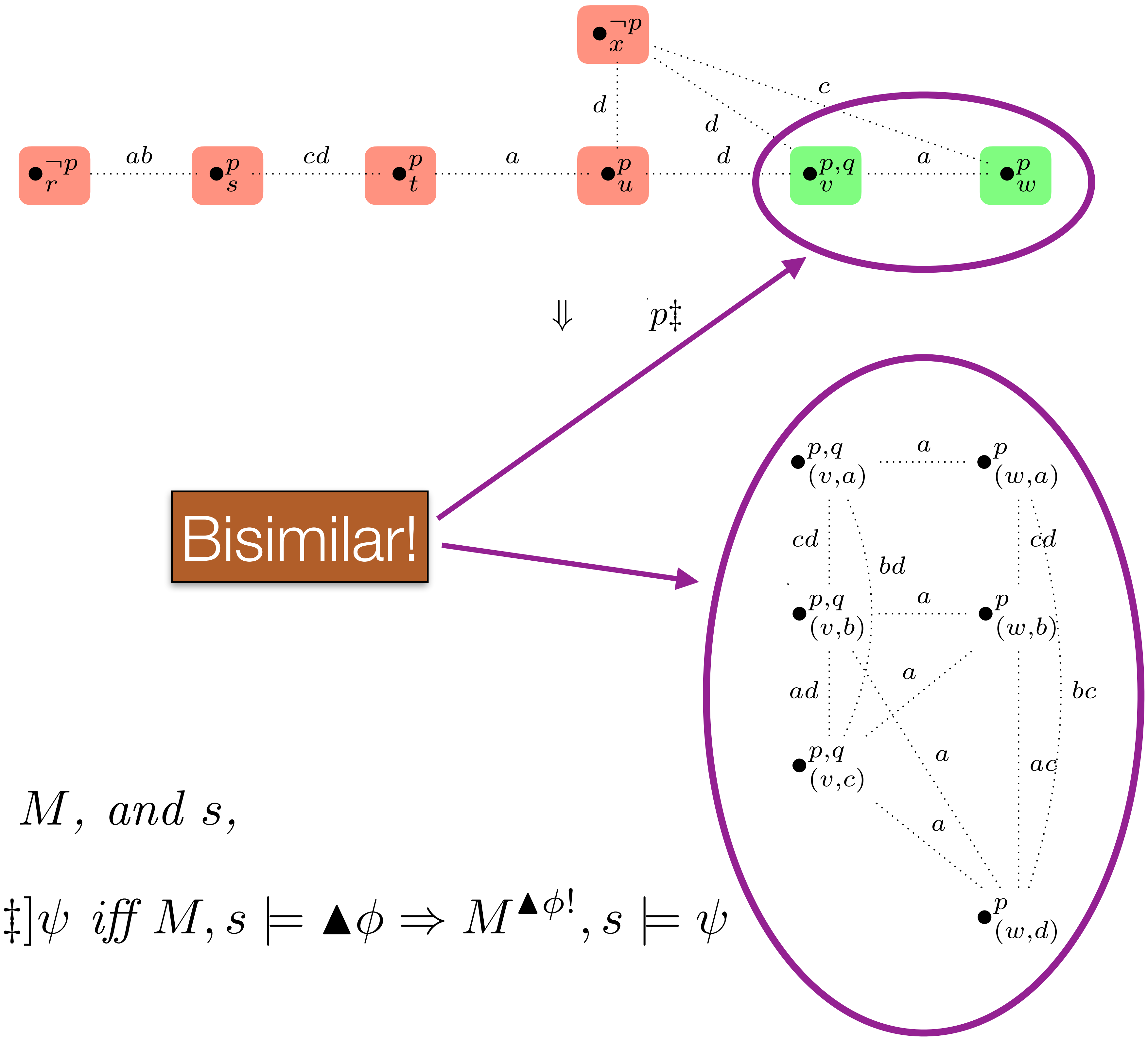
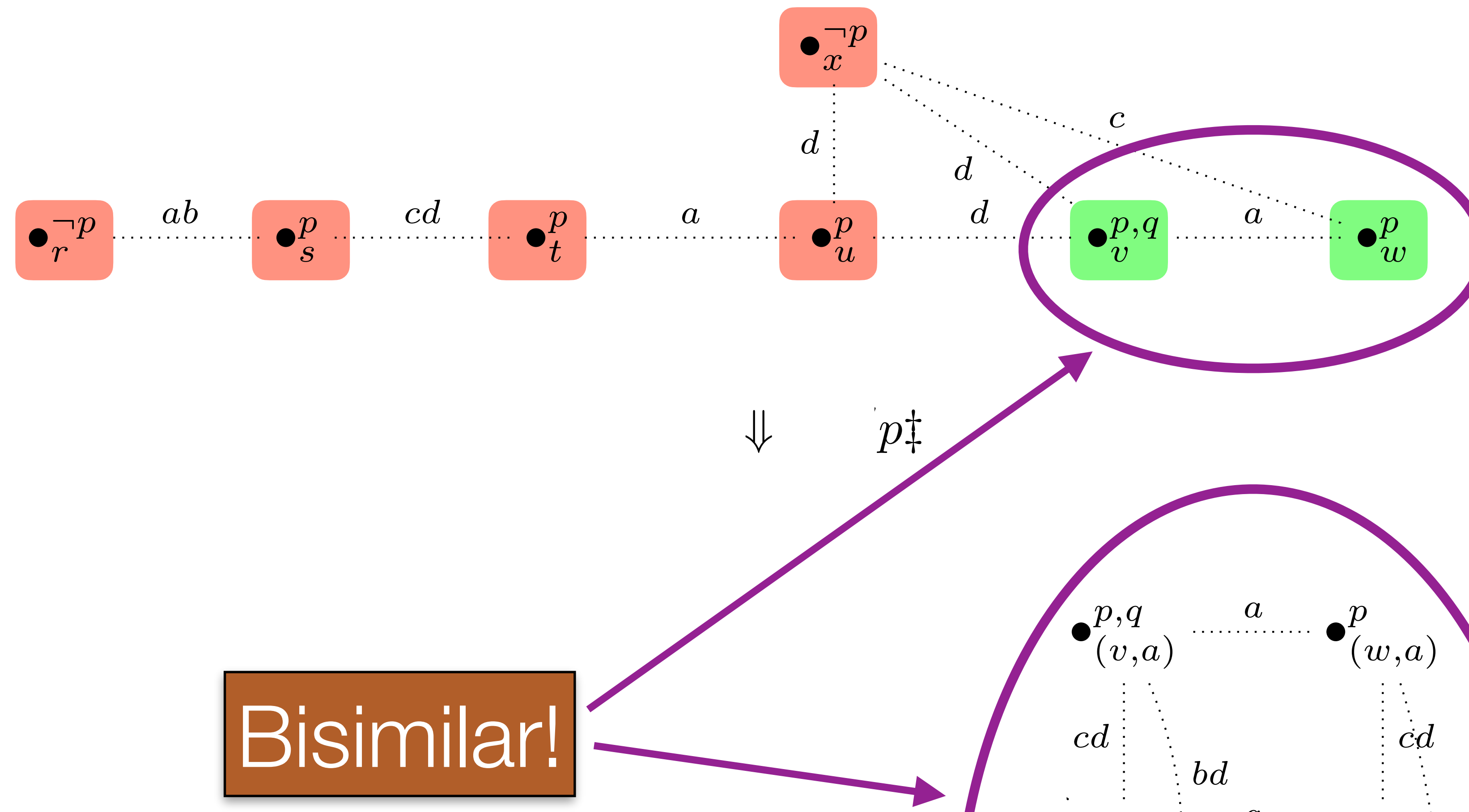
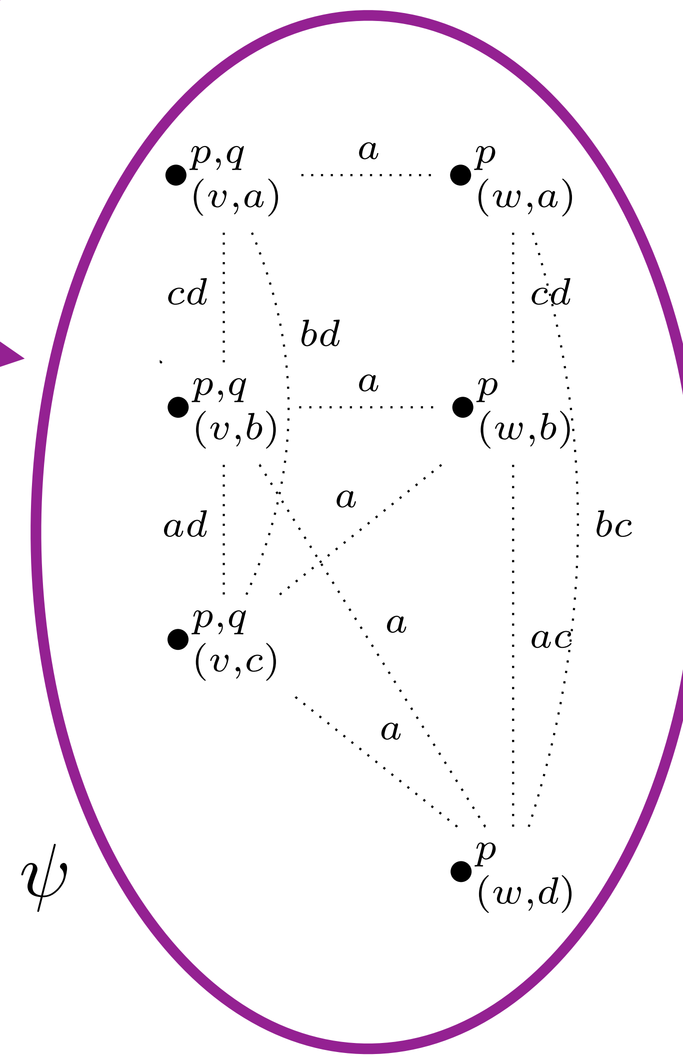


Theorem 1 *For any ϕ, ψ, M, and s,*

$$M, s \models [\phi\ddagger]\psi \text{ iff } M, s \models \blacktriangle\phi \Rightarrow M^{\blacktriangle\phi!}, s \models \psi$$





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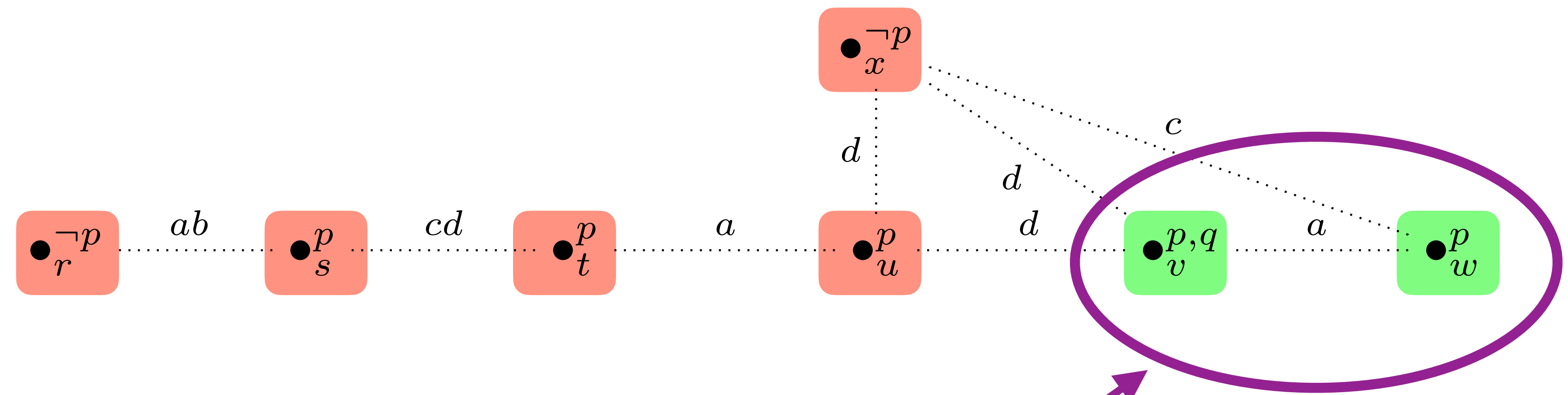
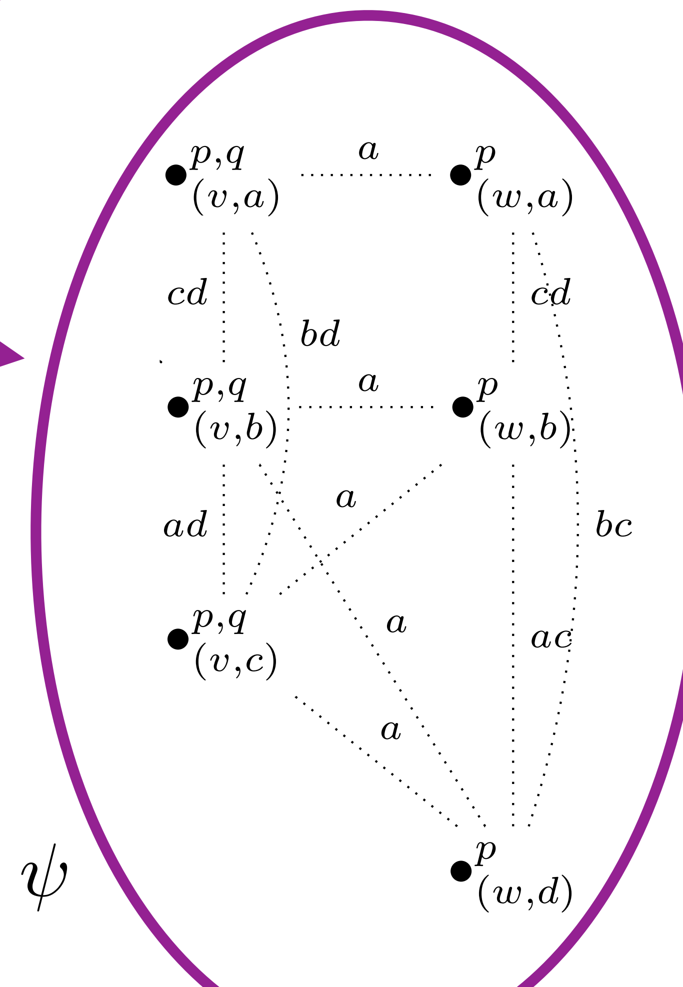
$$\models [\phi\ddagger]\psi \leftrightarrow [\blacktriangle\phi!]\psi$$



$\Downarrow \quad p\ddagger$

Bisimilar!



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$$\models [\phi\ddagger]\psi \leftrightarrow [\blacktriangle\phi!]\psi$$

Safe anonymous announcements are public announcements of safety!

Relationship to action model logic

Definition 1 *The anonymous event model for N agents and formula ϕ is the action model* $\mathsf{M}_\phi^{\mathsf{N}} = (\mathsf{S}, \sim, \mathsf{pre})$ *where*

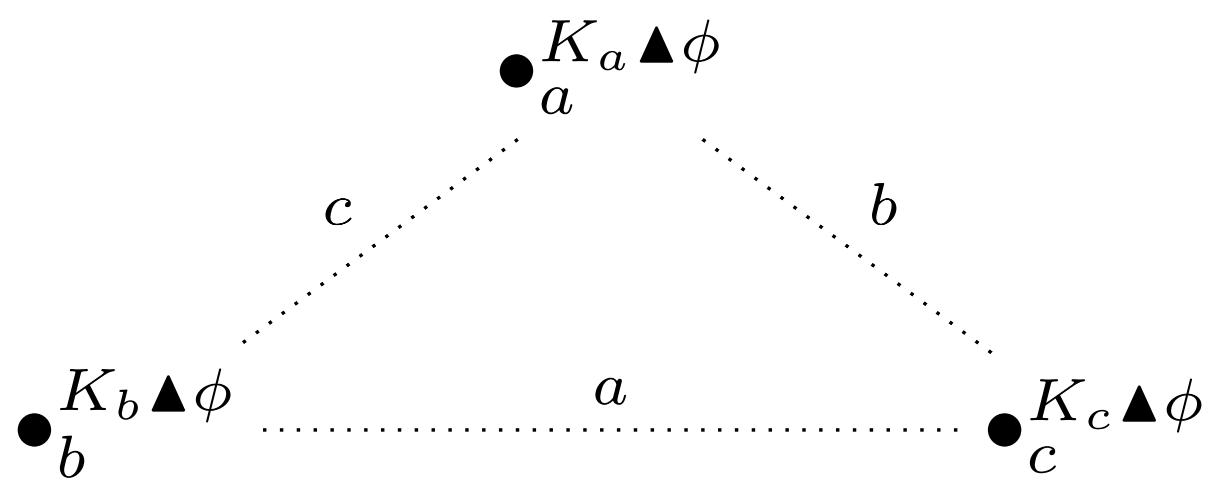
- $\mathsf{S} = N$
- $\mathsf{a} \sim_{\mathsf{b}} \mathsf{c}$ *iff* $(\mathsf{a} = \mathsf{c} \Leftrightarrow \mathsf{b} = \mathsf{c})$
- $\mathsf{pre}(\mathsf{a}) = K_a \blacktriangle \phi$

$$M, s \models [\phi\ddagger]\psi \text{ iff } M, s \models \left[\bigcup_{\mathsf{i} \in \mathsf{N}} (\mathsf{M}_\phi^{\mathsf{N}}, \mathsf{i})\right] \psi$$

Relationship to action model logic

Definition 1 *The anonymous event model for N agents and formula ϕ is the action model* $\mathsf{M}_\phi^\mathsf{N} = (\mathsf{S}, \sim, \mathsf{pre})$ *where*

- $\mathsf{S} = N$
- $\mathsf{a} \sim_\mathsf{b} \mathsf{c}$ *iff* $(\mathsf{a} = \mathsf{c} \Leftrightarrow \mathsf{b} = \mathsf{c})$
- $\mathsf{pre}(\mathsf{a}) = K_a \blacktriangle \phi$



$$M, s \models [\phi\ddagger]\psi \text{ iff } M, s \models \left[\bigcup_{\mathsf{i} \in \mathsf{N}} (\mathsf{M}_\phi^\mathsf{N}, \mathsf{i})\right] \psi$$

Languages

$\mathcal{L}_{\ddagger}$ $\quad \phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid [\phi\ddagger]\phi$

$\mathcal{L}_{\blacktriangle}$ $\quad \phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid \blacktriangle\phi$

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It is all about safety!

Axiomatising safety: no normality

$\mathcal{L}_{\blacktriangle}$:

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid \blacktriangle\phi$$

$$M, s \models \blacktriangle\phi \Leftrightarrow s \in \bigcup \left\{ S : S \subseteq || \bigvee_{G \in N^3} E_G(\phi \wedge x) ||^M_{[x=S]} \right\}$$

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Not *normal*!

Axiomatising safety: no induction axiom

$$\not\models \blacktriangle(\phi \to \bigvee_{G \in N^3} E_G \phi) \to (\phi \to \blacktriangle\phi)$$

But we do have an induction *rule*:
If $\models \phi \to \bigvee_{G \in N^3} E_G \phi$, then $\models \phi \to \blacktriangle\phi$

Axiomatising safety: no compactness

$$\{\blacktriangle_n \phi : n \geq 0\} \cup \{\neg \blacktriangle \phi\}$$

$$\blacktriangle_n = \phi \wedge \bigvee_{G_1 \in N^3} E_{G_1}(\phi \wedge \bigvee_{G_2 \in N^3} E_{G_2}(\phi \wedge \bigvee_{G_3 \in N^3} E_{G_3}(\phi \wedge \cdots \wedge \bigvee_{G_n \in N^3} E_{G_n} \phi)))$$

Axiomatising safety

all instances of propositional tautologies	Prop
From $\phi \to \psi$ and ϕ, derive ψ	Modus ponens
$K_a(\phi \to \psi) \to (K_a\phi \to K_a\psi)$	Distribution
$K_a\phi \to \phi$	Truth
$\neg K_a\phi \to K_a\neg K_a\phi$	Negative introspection
From ϕ, derive $K_a\phi$	Necessitation
$\blacktriangle\phi \to \bigvee_{G\in N^3} E_G(\phi \wedge \blacktriangle\phi)$	Mix
From $\phi \to \bigvee_{G\in N^3} E_G\phi$, derive $\phi \to \blacktriangle\phi$	Induction
From $\phi \to \psi$, derive $\blacktriangle\phi \to \blacktriangle\psi$	Monotonicity

Axiomatising safety

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Theorem 1 *The system is sound and weakly complete.*

Completeness: complications

- Non-normality
  - No standard (relational) path semantics!
- Non-compactness
- Fixed-points

Conclusions

- The logic of intentional anonymous public announcements
- Key idea: *safety*
- Fixed-point operator
- Intentions => more revealing! Similar to in Russian cards.
- Safe anonymous announcements = public announcements of safety
- Expressive power: all boils down to epistemic logic + safety
  - which we axiomatised
- Future work: group knowledge (stronger safety), quantification a la APAL/GAL, self-reference.