Group Announcement Logic with Distributed Knowledge

Rustam GALIMULLIN [1]
Thomas Ågotnes [2]
Natasha ALECHINA [3]

[1]CNRS, LSV, ENS Paris-Saclay, France

[2]University of Bergen, Norway & University of Southwest, China

[3]University of Nottingham, UK

What this talk is about

- Public Announcement Logic (PAL)[1] can be extended with quantification over announcements made by agents
- Resulting formalism, Group Announcement Logic (GAL)[2], reasons about the ability of agents to achieve their epistemic goals
- We study how this kind of an ability is related to the implicit knowledge of a group (a.k.a. distributed knowledge D_G)
- We provide a complete axiomatisation of GAL with D_G

[1] Jan Plaza. "Logics of public communications". In: *Synthese* 158.2 (2007), pp. 165–179.

[2] Thomas Ågotnes et al. "Group announcement logic". In: *Journal of Applied Logic* 8.1 (2010), pp. 62–81.

Models

Definition (Epistemic Model)

An epistemic model is a triple $M = (S, \sim, V)$, where

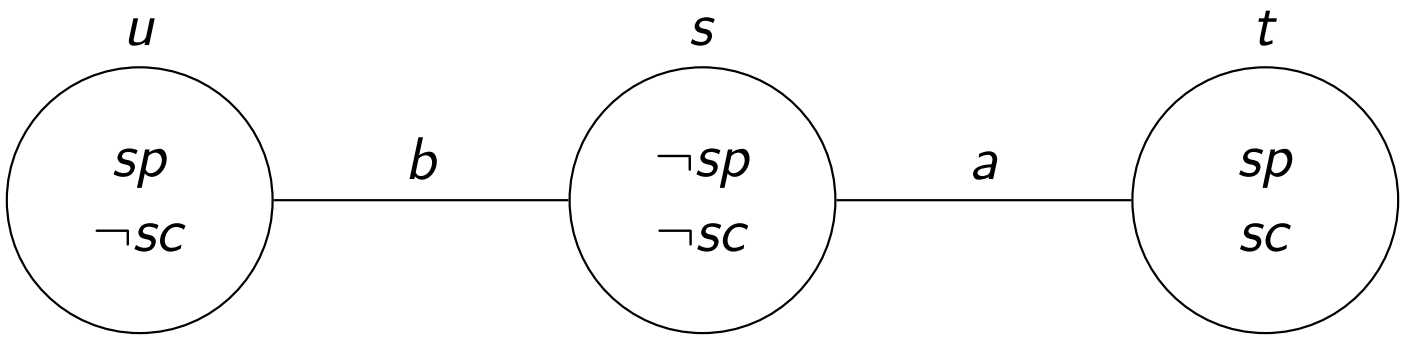
- S is a non-empty set of states,
- $\sim_a : A \to 2^{S \times S}$ is an equivalence relation to each agent a,
- $V : P \to 2^S$ is the valuation function.

A pair M_s with $s \in S$ is called a pointed model.
An announcement of φ in a pointed model M_s results in an updated pointed model M_s^φ containing only φ-states:

- $S^\varphi = [\![\varphi]\!]_M$,
- $\sim_a^\varphi = \sim_a \cap (S^\varphi \times S^\varphi)$,
- $V^\varphi(p) = V(p) \cap S^\varphi$.

Example

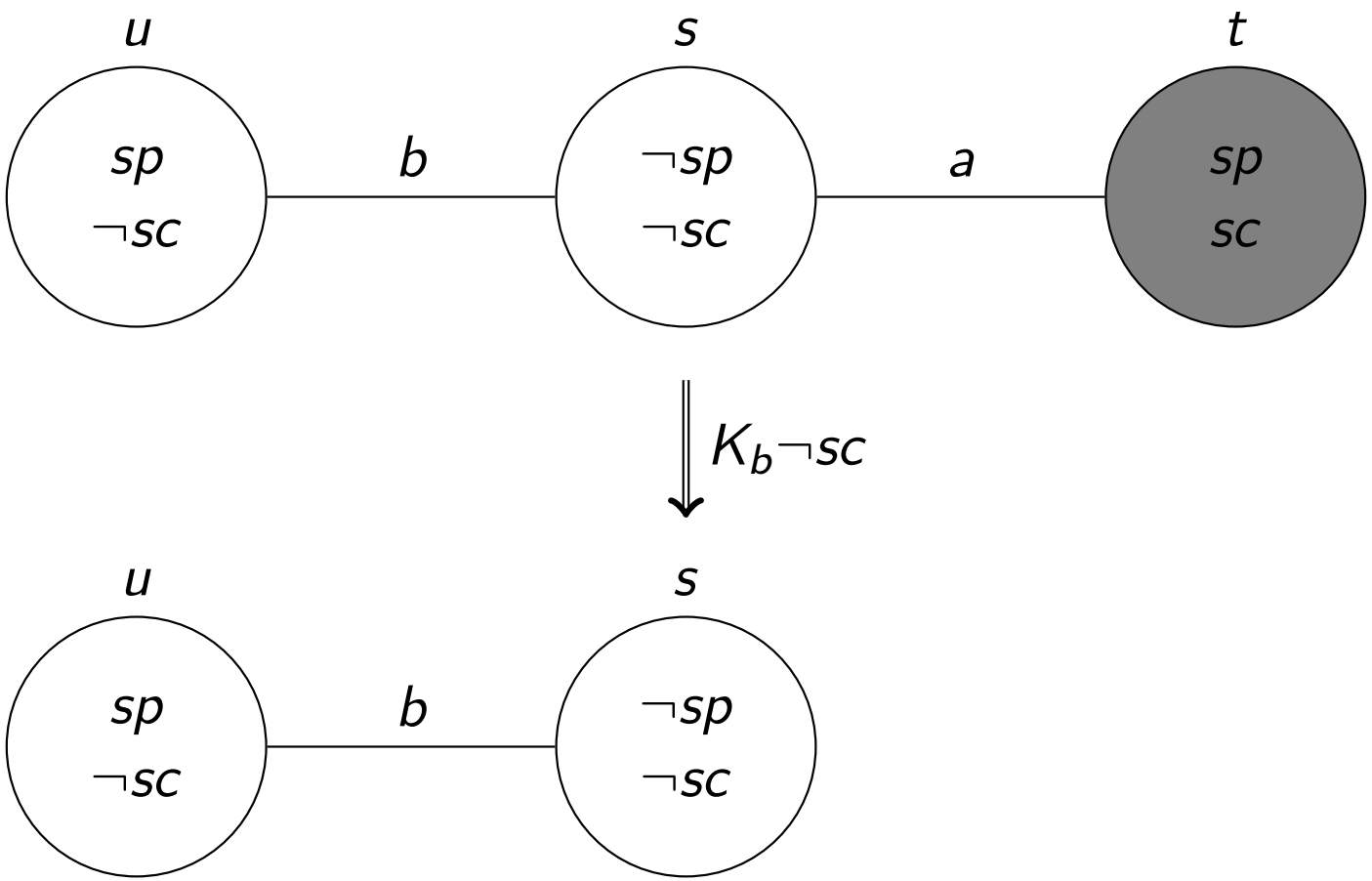
Alice and Bob argue about spiders. Both of them do not know whether spiders are insects (sp). However, Bob knows that scorpions are not insects ($\neg sc$), and Alice knows that if spiders were insects, then scorpions would have been insects as well.



$M_s \models \neg sp \wedge \neg sc$, $M_s \models \neg K_a(\neg sp \wedge \neg sc)$, $M_s \models D_{\{a,b\}}(\neg sp \wedge \neg sc)$

Example

Bob says that he knows that scorpions are not insects ($K_b \neg sc$).



$M_s^{K_b \neg sc} \models K_a(\neg sp \wedge \neg sc)$, $M_s^{K_b \neg sc} \models \neg K_b(\neg sp \wedge \neg sc)$

Semantics of PALD

Definition (Semantics)

$$
\begin{array}{lll}
M_s \models p & \text{iff} & s \in V(p) \\
M_s \models \neg\varphi & \text{iff} & M_s \not\models \varphi \\
M_s \models \varphi \wedge \psi & \text{iff} & M_s \models \varphi \text{ and } M_s \models \psi \\
M_s \models K_a\varphi & \text{iff} & \forall t \in S : s \sim_a t \text{ implies } M_t \models \varphi \\
M_s \models D_G\varphi & \text{iff} & \forall t \in S : s \sim_G t \text{ implies } M_t \models \varphi \\
M_s \models [\varphi]\psi & \text{iff} & M_s \not\models \varphi \text{ or } M_s^{\varphi} \models \psi
\end{array}
$$

$\sim_G$ denotes $\bigcap_{a \in G} \sim_G$

Dual of $[\varphi]\psi$

$$M_s \models \langle\varphi\rangle\psi \quad \text{iff} \quad M_s \models \varphi \text{ and } M_s^{\varphi} \models \psi$$

Announcements by Groups

We are interested in the following restrictions on announcements:

- Announcements are made by agents
- Agents can only announce what they know
- Groups of agents can announce conjunctions of formulas, where each conjunct is a formula known by an agent in the group

Group announcements and Distributed Knowledges

Group Announcement Logic with Distributed Knowledge (GALD)
$=$ PALD $+ \{[G]\varphi, \langle G\rangle\varphi\}$

$\langle G\rangle\varphi$: 'there is an announcement by agents from G such that φ holds in the resulting model'

$[G]\varphi$: 'whatever agents from G announce, φ holds in the resulting model'

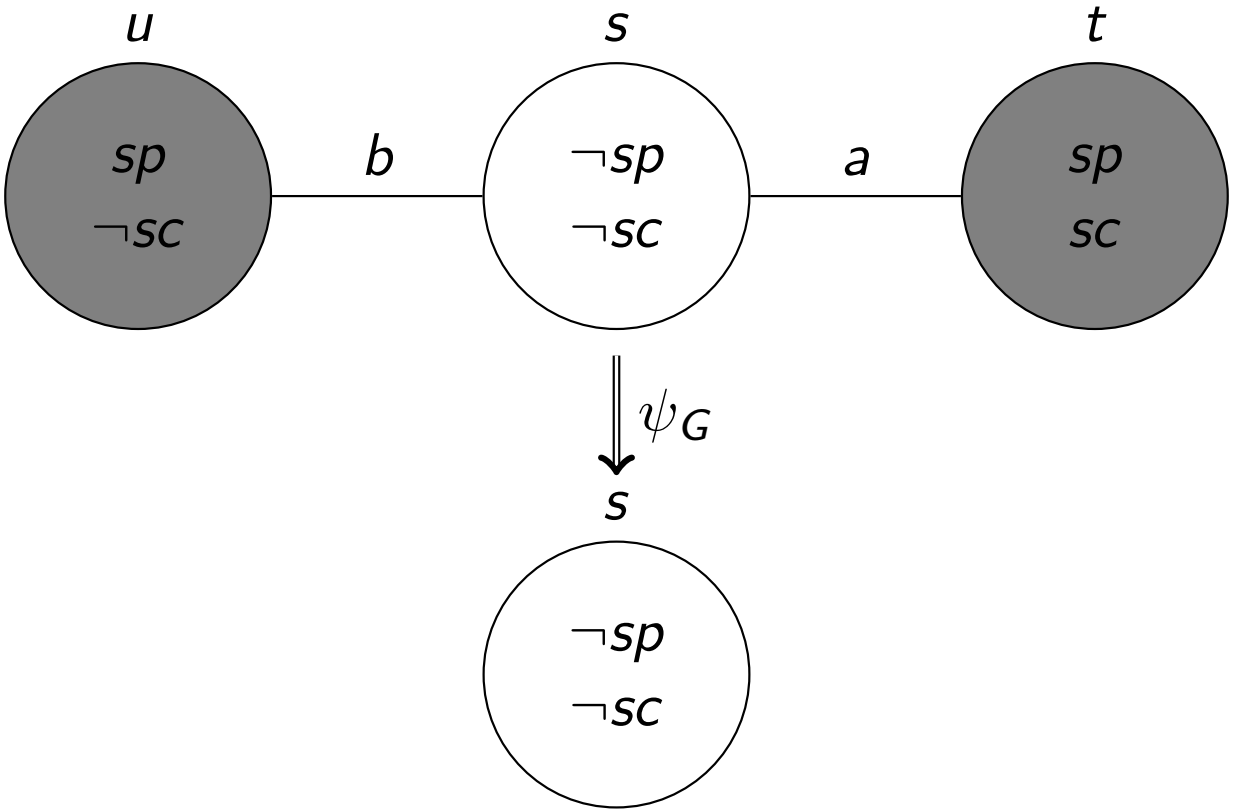
$\psi_G := \bigwedge_{a\in G} K_a\psi_a$, where ψ_a is an epistemic formula.

Definition (Semantics)

$$M_s \models [G]\varphi \quad \text{iff} \quad \forall\psi_G : M_s \models [\psi_G]\varphi$$
$$M_s \models \langle G\rangle\varphi \quad \text{iff} \quad \exists\psi_G : M_s \models \langle\psi_G\rangle\phi$$

Example

Alice and Bob have an announcement after which they both know that spiders are not insects: $\psi_G := K_b \neg sc \wedge K_a(sc \leftrightarrow sp)$. In such a way they make their distributed knowledge explicit.



$M_s \models D_{\{a,b\}}(\neg sp \wedge \neg sc)$, $M_s \models \langle G \rangle K_{\{a,b\}}(\neg sp \wedge \neg sc)$

Possible interactions between D_G and $\langle G \rangle$

Can distributed knowledge of a group become known to its members through pubic communication? Not always.

$$\langle G \rangle D_G \varphi \to D_G \langle G \rangle \varphi$$
$$D_G \langle G \rangle \varphi \to \langle G \rangle \varphi$$

$$\langle G \rangle \varphi \not\to D_G \langle G \rangle \varphi$$
$$D_G \langle G \rangle \varphi \not\to \langle G \rangle D_G \varphi$$

$D_G \not\to \langle G \rangle E_G \varphi$ (the Moore sentence $\varphi := p \wedge \neg K_a p$)
$\langle G \rangle E_G \varphi \not\to D_G \varphi$ (agents may learn φ which was not their D.K.)

$D_G \varphi^+ \to \langle G \rangle E_G \varphi^+$ (on finite bisimulation contracted models, φ^+ is positive)

Group Announcement Logic with Distributed Knowledge

Axiomatisation: $\text{PALD}^a + \text{GAL}^b$

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[a] Yì N. Wáng and Thomas Ågotnes. "Public announcement logic with distributed knowledge: expressivity, completeness and complexity". In: *Synthese* 190.1 (2013), pp. 135–162.

[b] Thomas Ågotnes et al. "Group announcement logic". In: *Journal of Applied Logic* 8.1 (2010), pp. 62–81.

Theorem

GALD is sound and complete

- Infinitary axiomatisation
- Conservative semantics

Resolved Distributed Knowledge

R_G models the situation after all agents in the group have shared their knowledge (publicly observable private communication)[a]

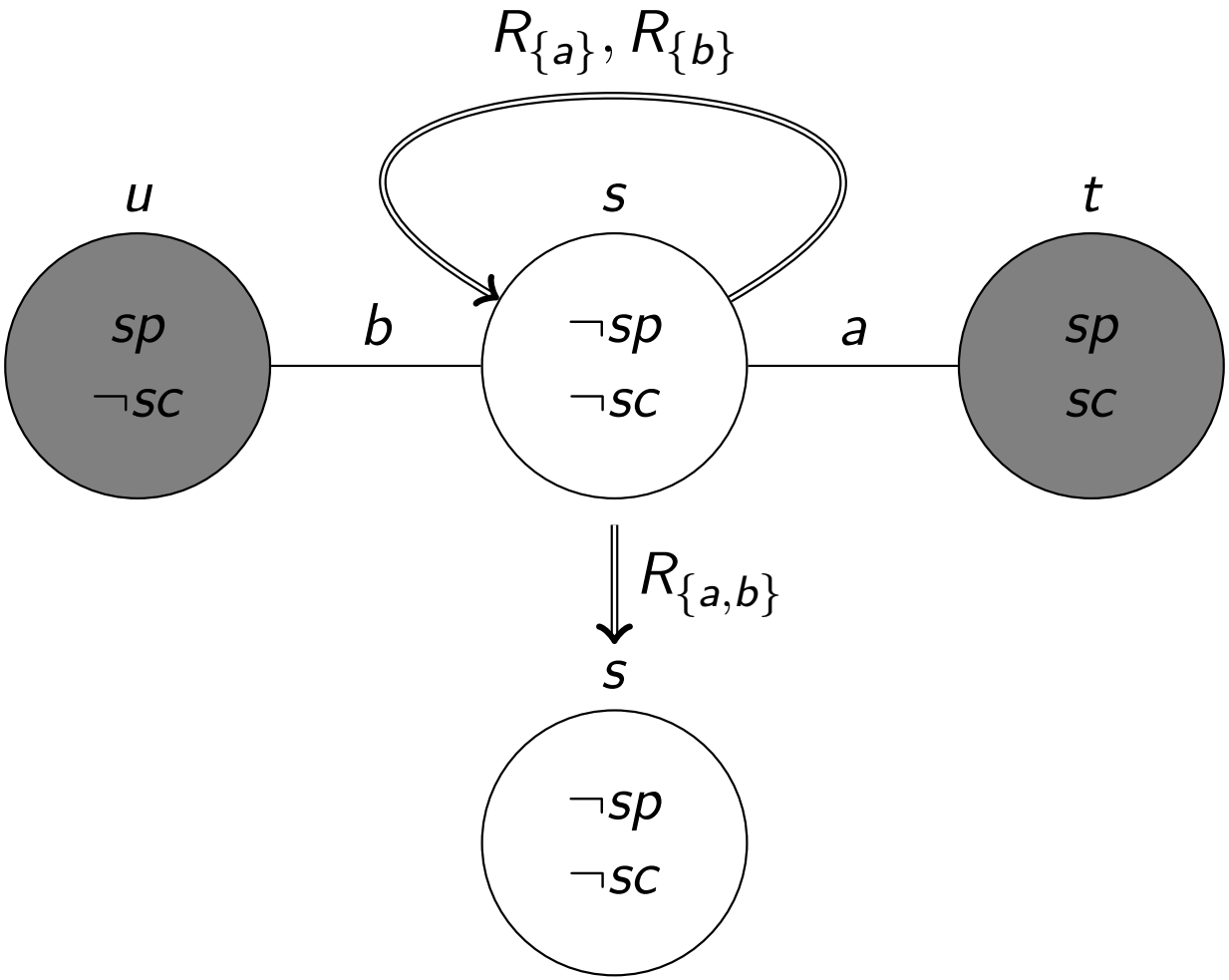
[a] Thomas Ågotnes and Yì N. Wáng. "Resolving distributed knowledge". In: *Artificial Intelligence* 252 (2017), pp. 1–21.

Definition (Semantics)

$$M_s \models R_G\varphi \quad \text{iff} \quad M_G \models \varphi$$

$M^G = (S, \sim^G, V)$, where $\sim_a^G$ denotes $\bigcap_{b \in G} \sim_b$ if $a \in G$, and $\sim_a$ otherwise

Example



Properties of Resolution

Distributed knowledge and resolved distributed knowledge are different

$$D_G\varphi \not\to R_G\varphi,\ R_G\varphi \not\to D_G\varphi$$

The result of the resolution cannot be achieved through public communication

$$\langle G\rangle\varphi \not\to R_G\varphi,\ R_G\varphi \not\to \langle G\rangle\varphi$$

In particular, private communication between all agents is different from public communication between all agents

$$R_A\varphi \not\to \langle A\rangle\varphi$$

Note on Expressivity

Definition (Expressivity)

L_1 is at least as expressive as L_2 ($L_2 \leqslant L_1$) iff for all $\varphi \in L_2$ there is an equivalent $\psi \in L_1$. If $L_1 \leqslant L_2$ and $L_2 \leqslant L_1$, then L_1 and L_2 are equally expressive ($L_1 \equiv L_2$).

- PAL+D $\equiv$ EL+D[3]
- EL + D + R $\equiv$ EL+D[4]
- PAL $\leqslant$ GAL[5], and hence PAL+D $\leqslant$ GAL+D
- GAL+R $\not\leqslant$ GAL+D$^-$, where the latter is the fragment without D in public announcements (shown in the paper)

[3] Yì N. Wáng and Thomas Ågotnes. "Public announcement logic with distributed knowledge: expressivity, completeness and complexity". In: *Synthese* 190.1 (2013), pp. 135–162.

[4] Thomas Ågotnes and Yì N. Wáng. "Resolving distributed knowledge". In: *Artificial Intelligence* 252 (2017), pp. 1–21.

[5] Thomas Ågotnes et al. "Group announcement logic". In: *Journal of Applied Logic* 8.1 (2010), pp. 62–81.

Open Problems

- Axiomatisation of EL+R
- Axiomatisation of GAL+R
- Relative expressivity of GAL+D and GAL+R
- Investigating extensions of other logics of quantified announcements (APAL[6] and CAL[7])
- Finite model property of GAL

[6] Philippe Balbiani et al. "'Knowable' as 'known after an announcement'". In: *Review of Symbolic Logic* 1.3 (2008), pp. 305–334.

[7] Thomas Ågotnes and Hans van Ditmarsch. "Coalitions and Announcements". In: *Proceedings of AAMAS 2008*. Ed. by Lin Padgham et al. IFAAMAS, 2008, pp. 673–680.

Thank you for attention!