Quantifying Over Public Announcements

Recent Results and Open Questions

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First Things First

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Tim French (University of Western Australia)

Plan of the Talk

Part I. Introduction to Epistemic Logic and Public Announcement Logic

Part II. Introduction to Arbitrary Public Announcement Logic

Open Problem I and a partial solution

Part III. Introduction to Group and Coalition Announcement Logics

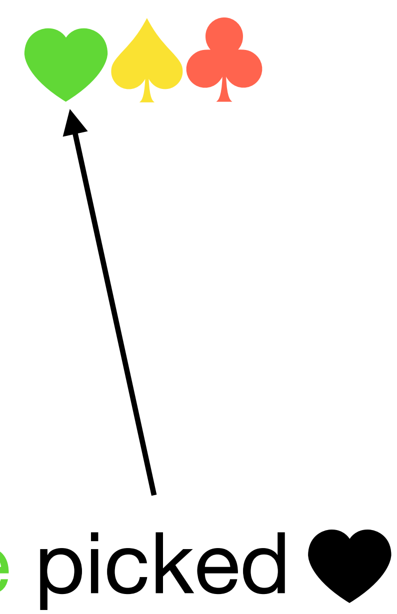
Open Problems II and III and their partial solutions

Part I

Introduction to Epistemic Logic and Public Announcement Logic

Card Example

Three agents, Alice, Bob, and Carol, have each drawn one card from a deck of {♥ ♠ ♣}



Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

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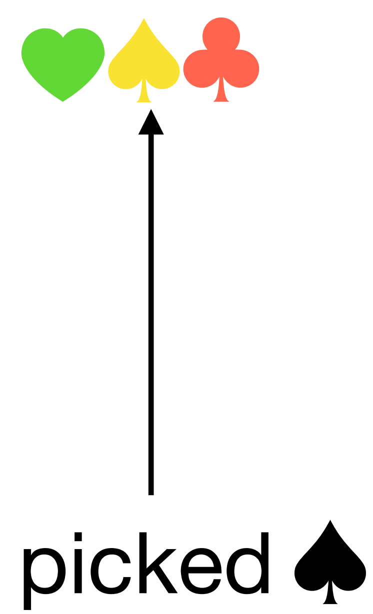

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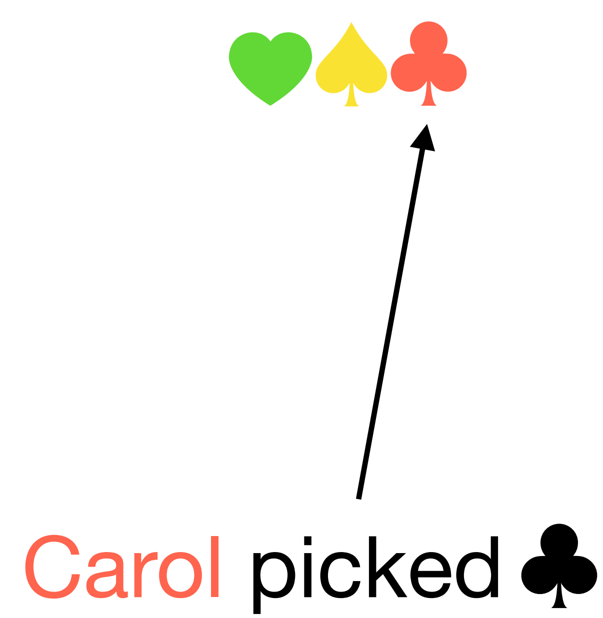

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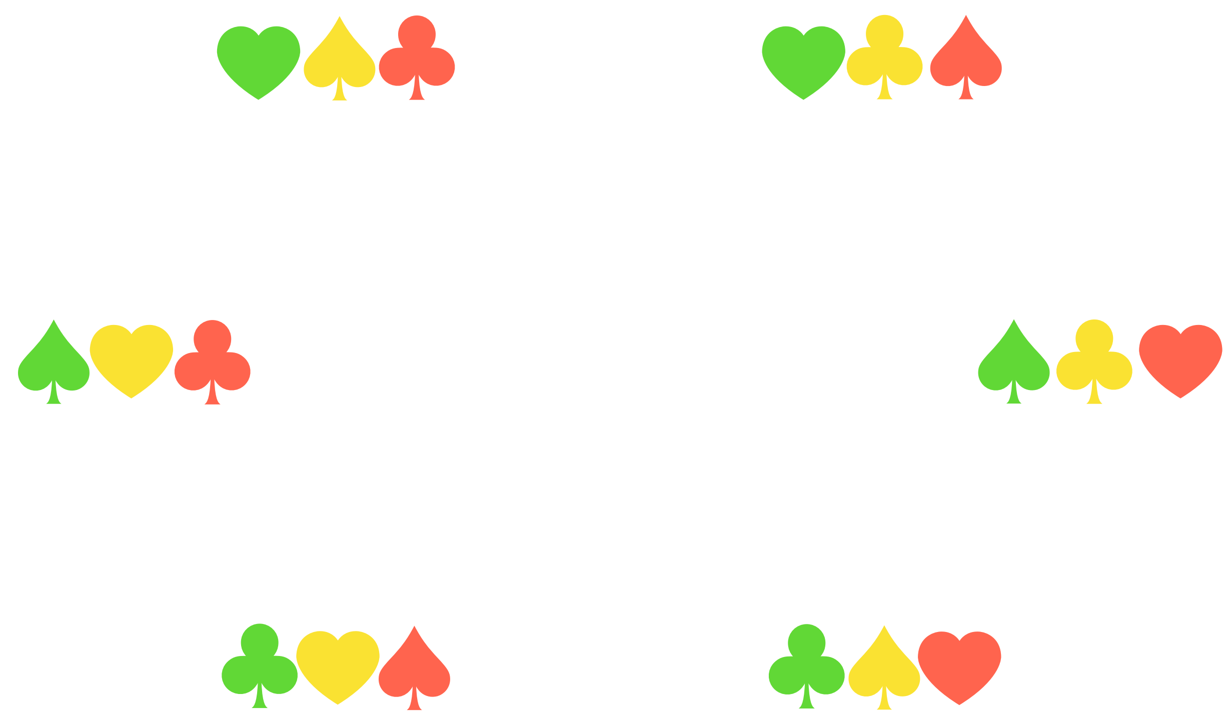

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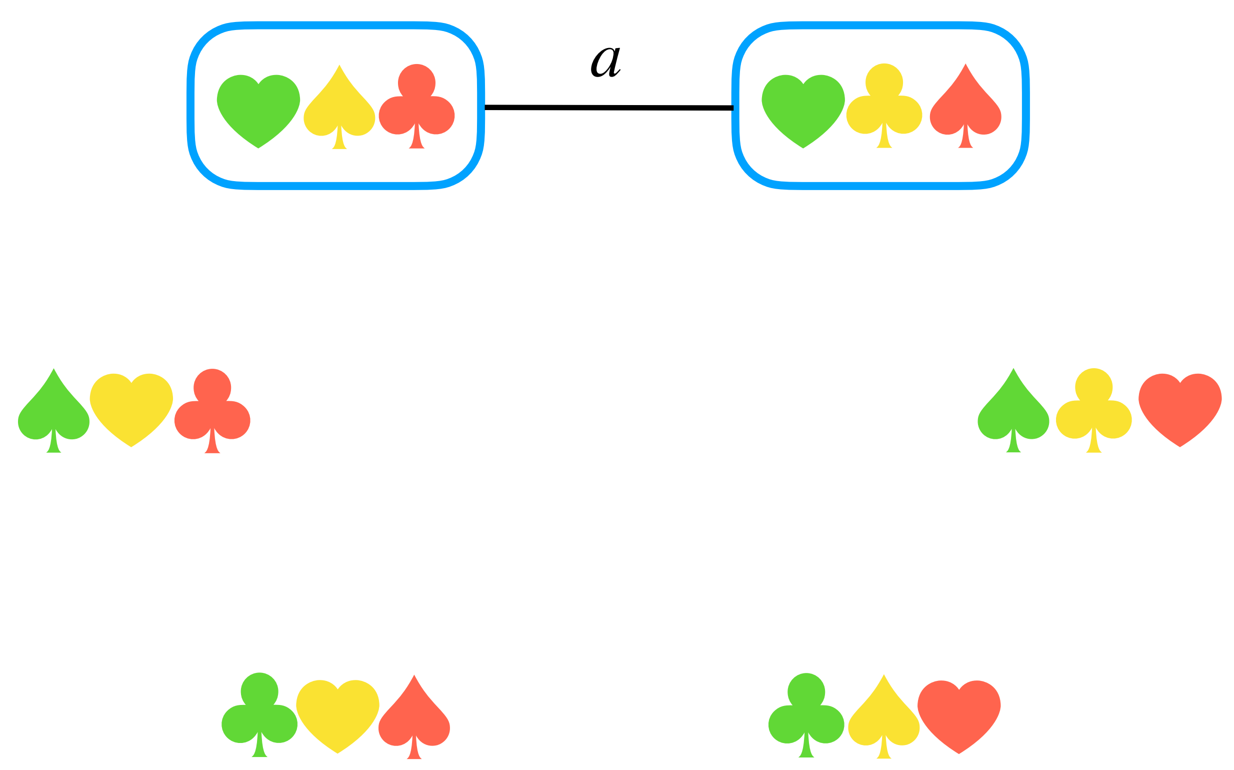

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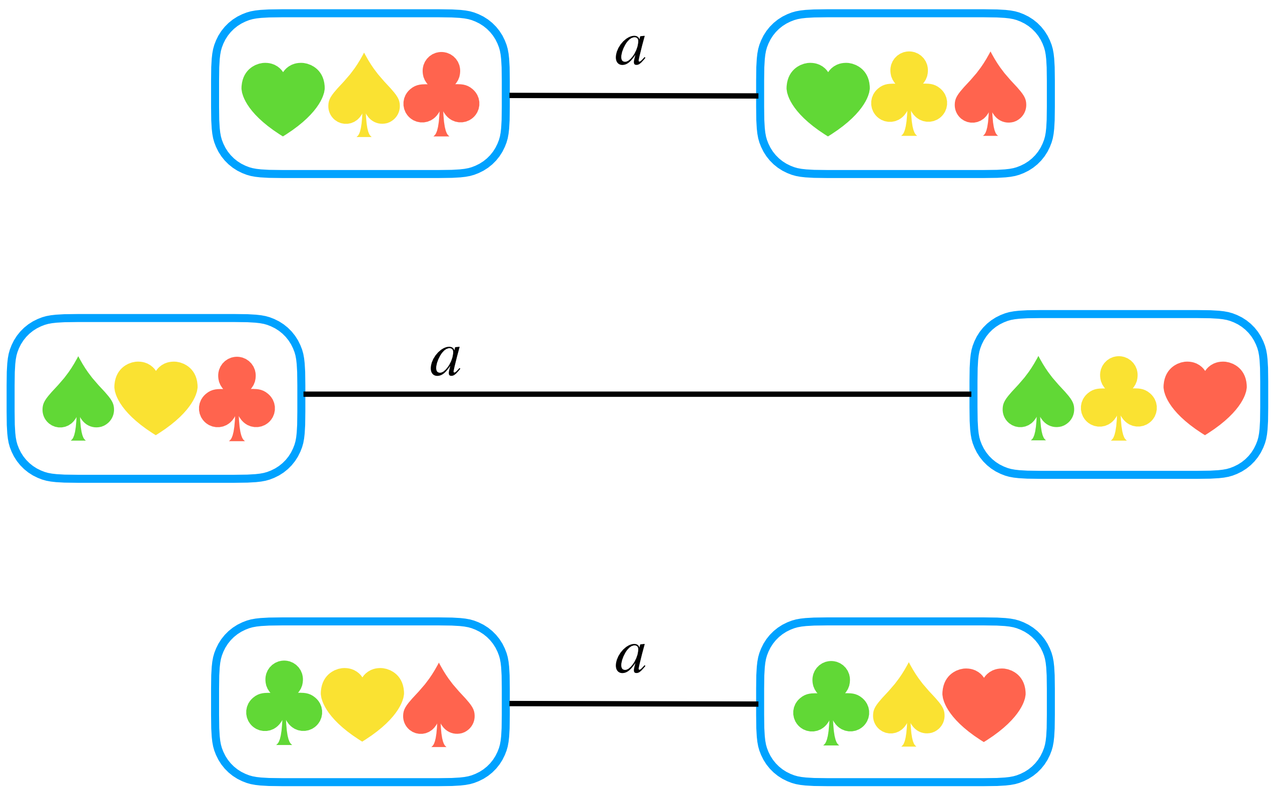

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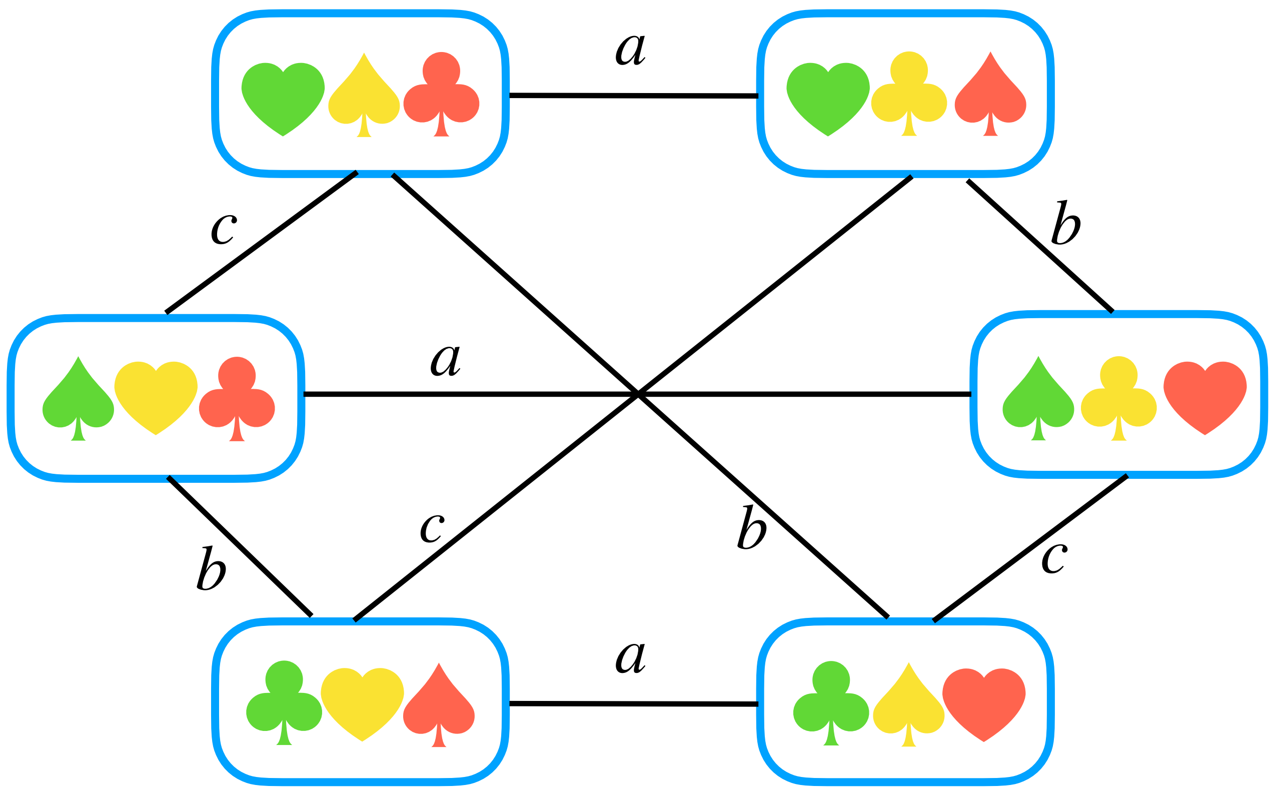

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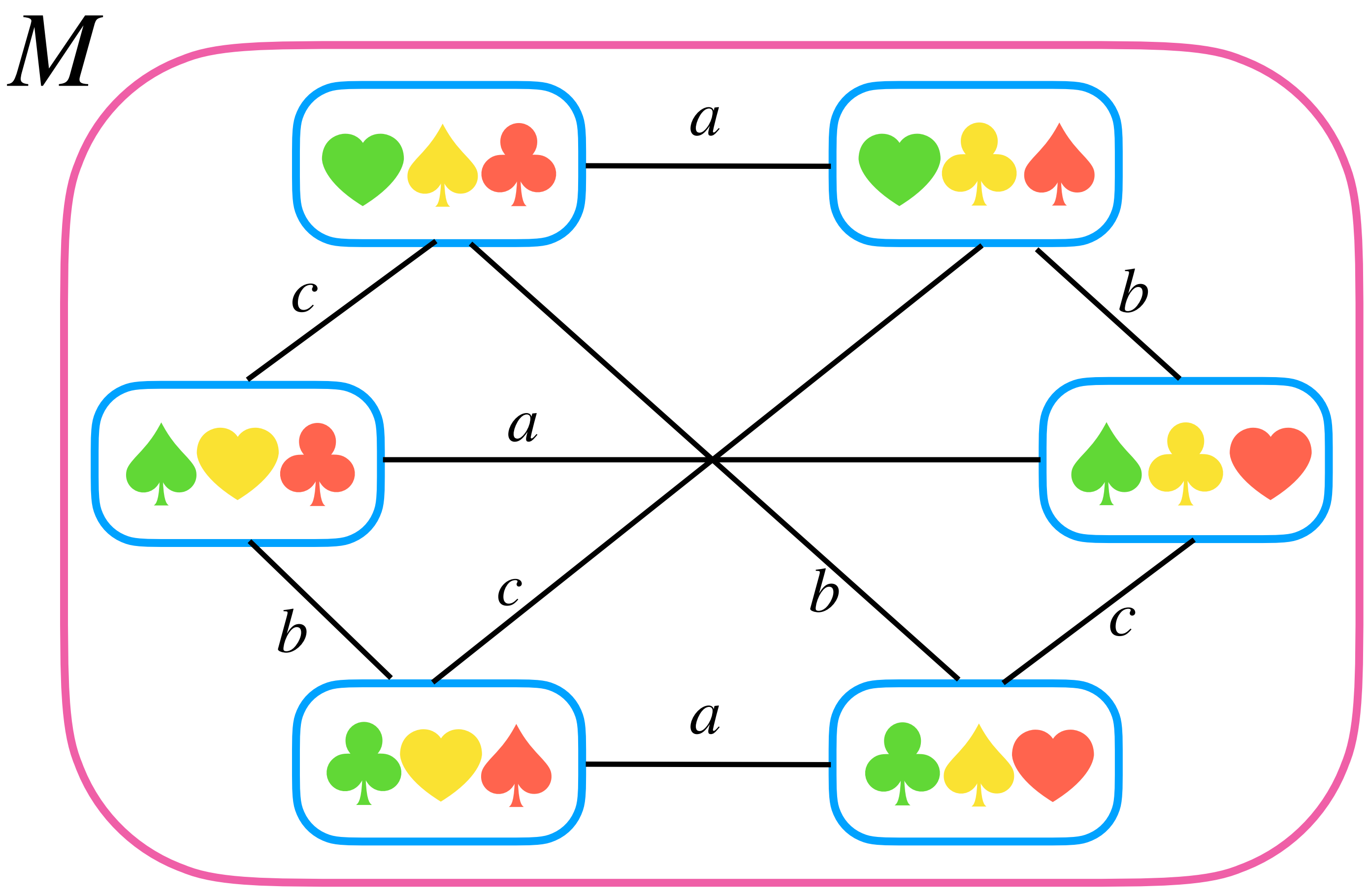

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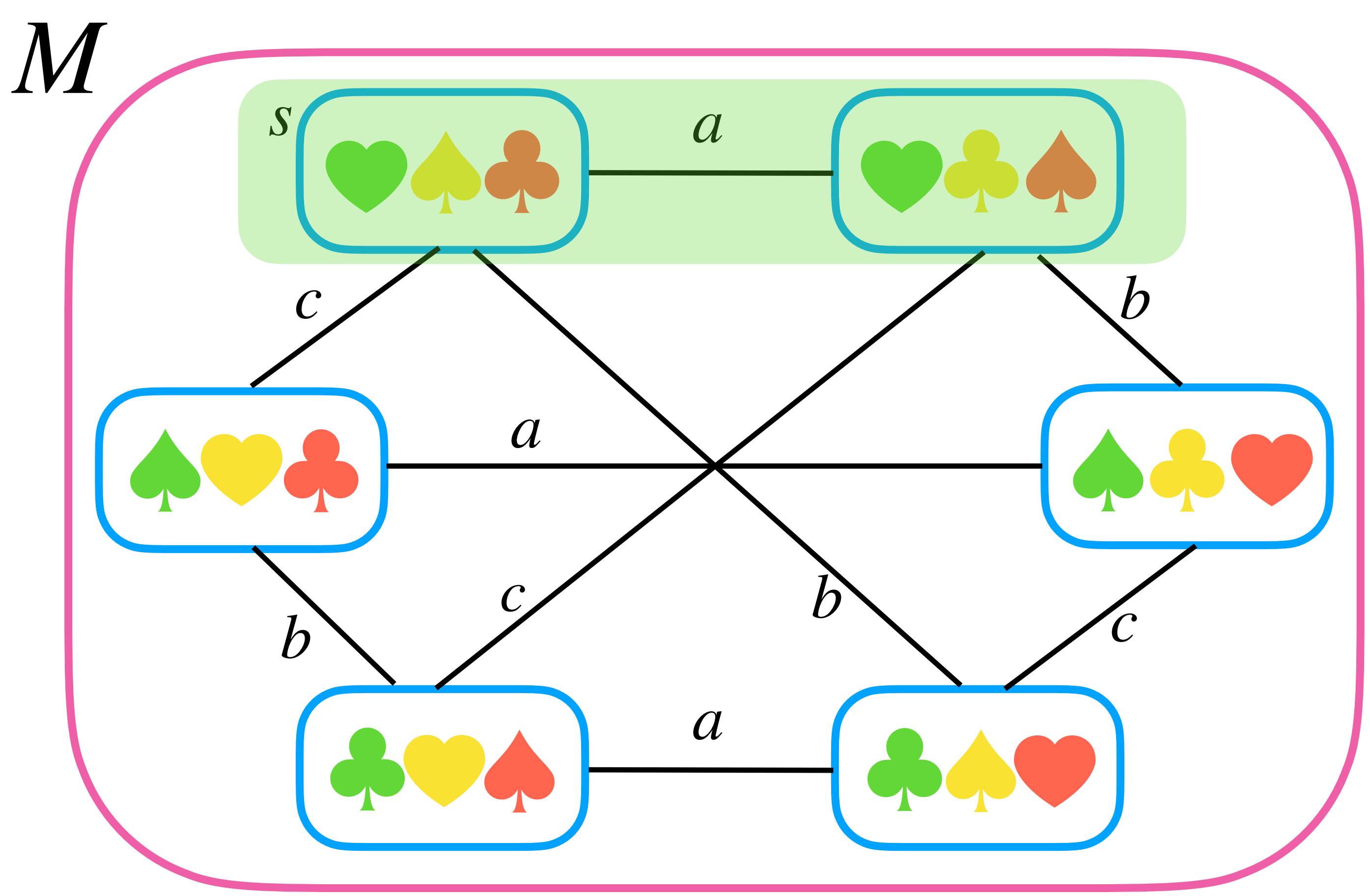

$M_s \vDash ♥_a \wedge ♠_b \wedge ♣_c$

$M_s \vDash \Box_a (♠_b \vee ♣_b)$

$\Box_a \varphi$: An agent a knows φ if φ is true in all a-reachable states

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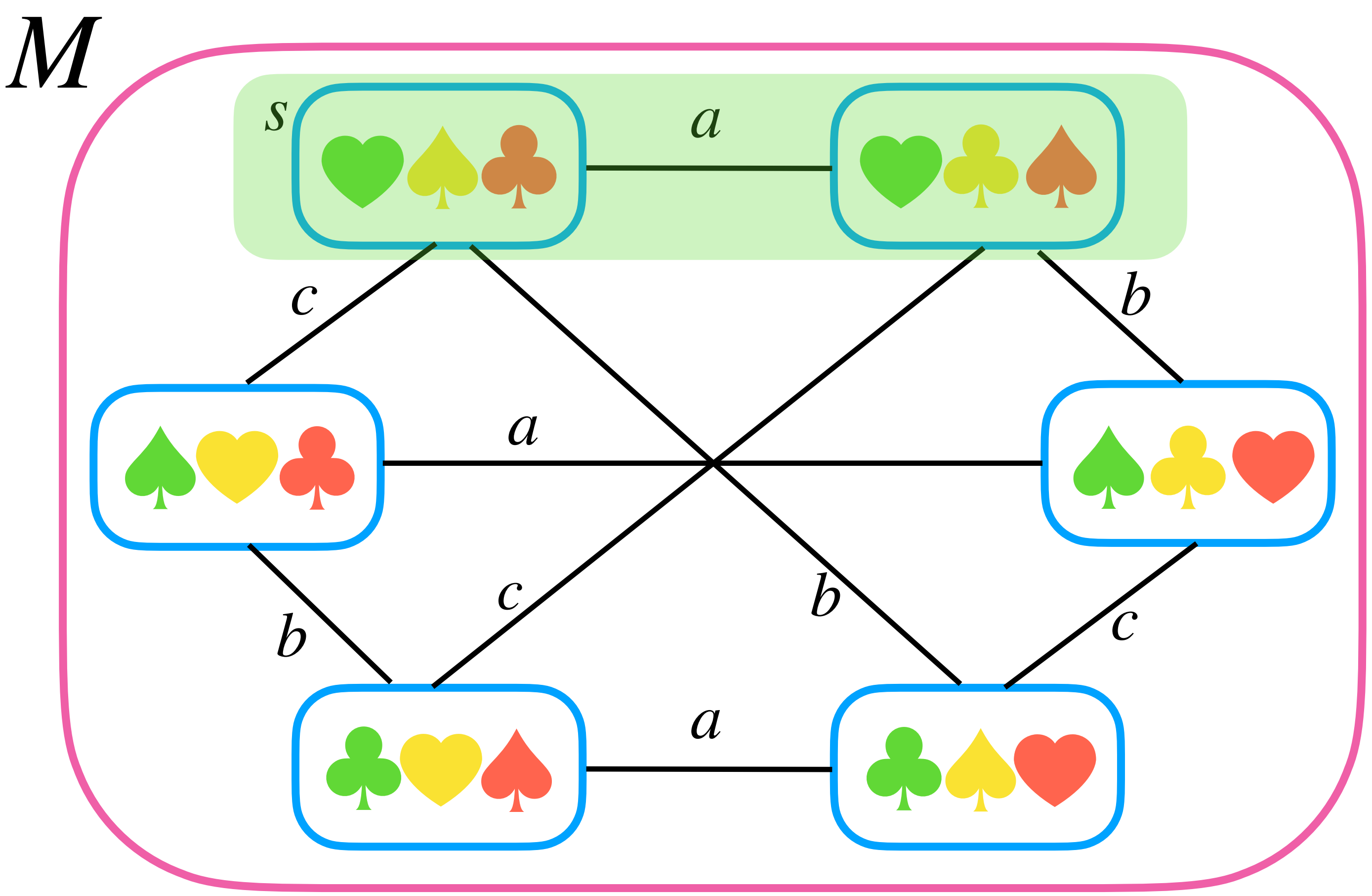

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$M_s \vDash \Diamond_a (♠_b \wedge ♣_c)$

$M_s \vDash \Box_c \Box_b ♣_c$

$\Diamond_a \varphi$: An agent a considers φ possible if φ is true in at least one a-reachable state

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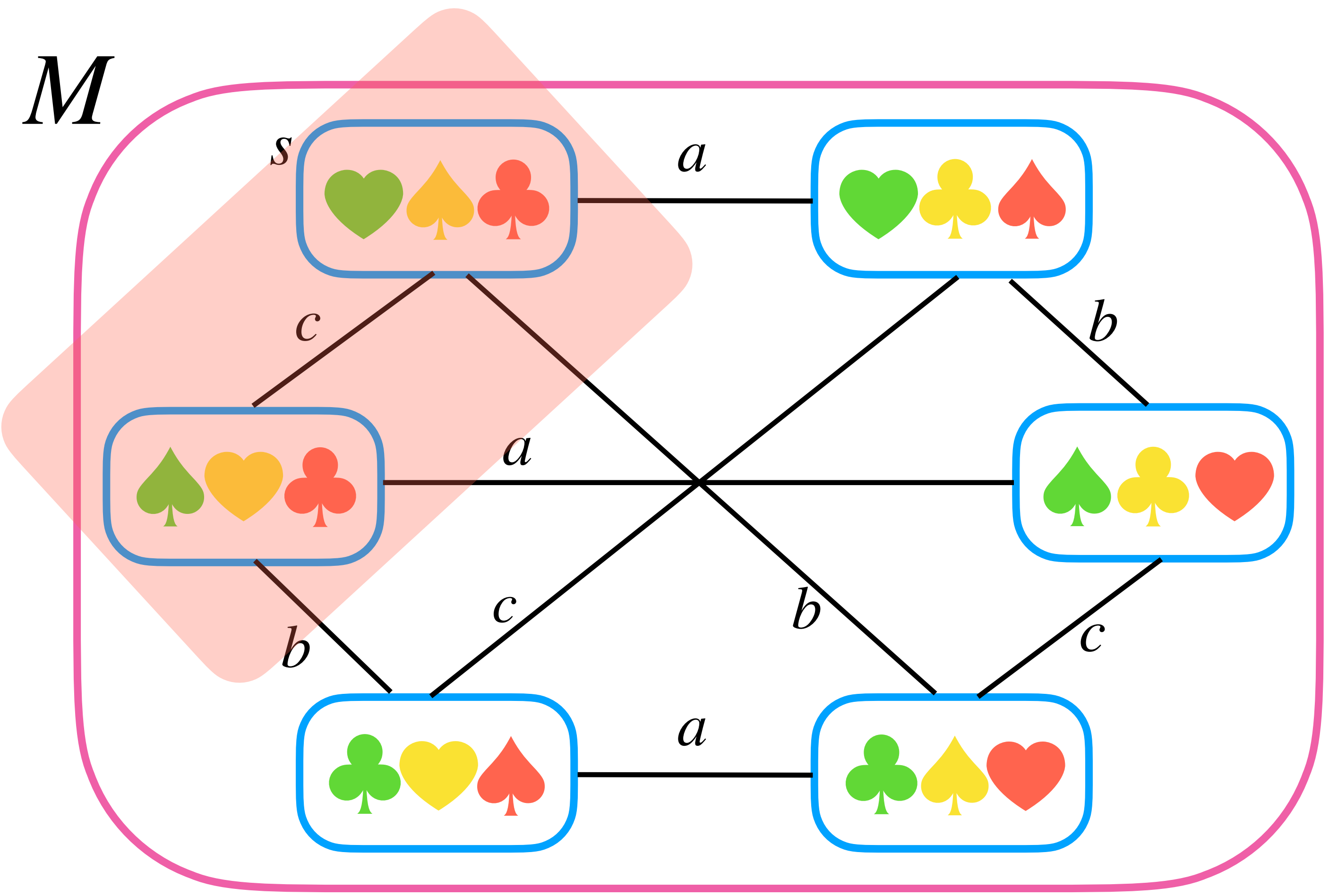

$$M_s \models \heartsuit_a \wedge \spadesuit_b \wedge \clubsuit_c$$

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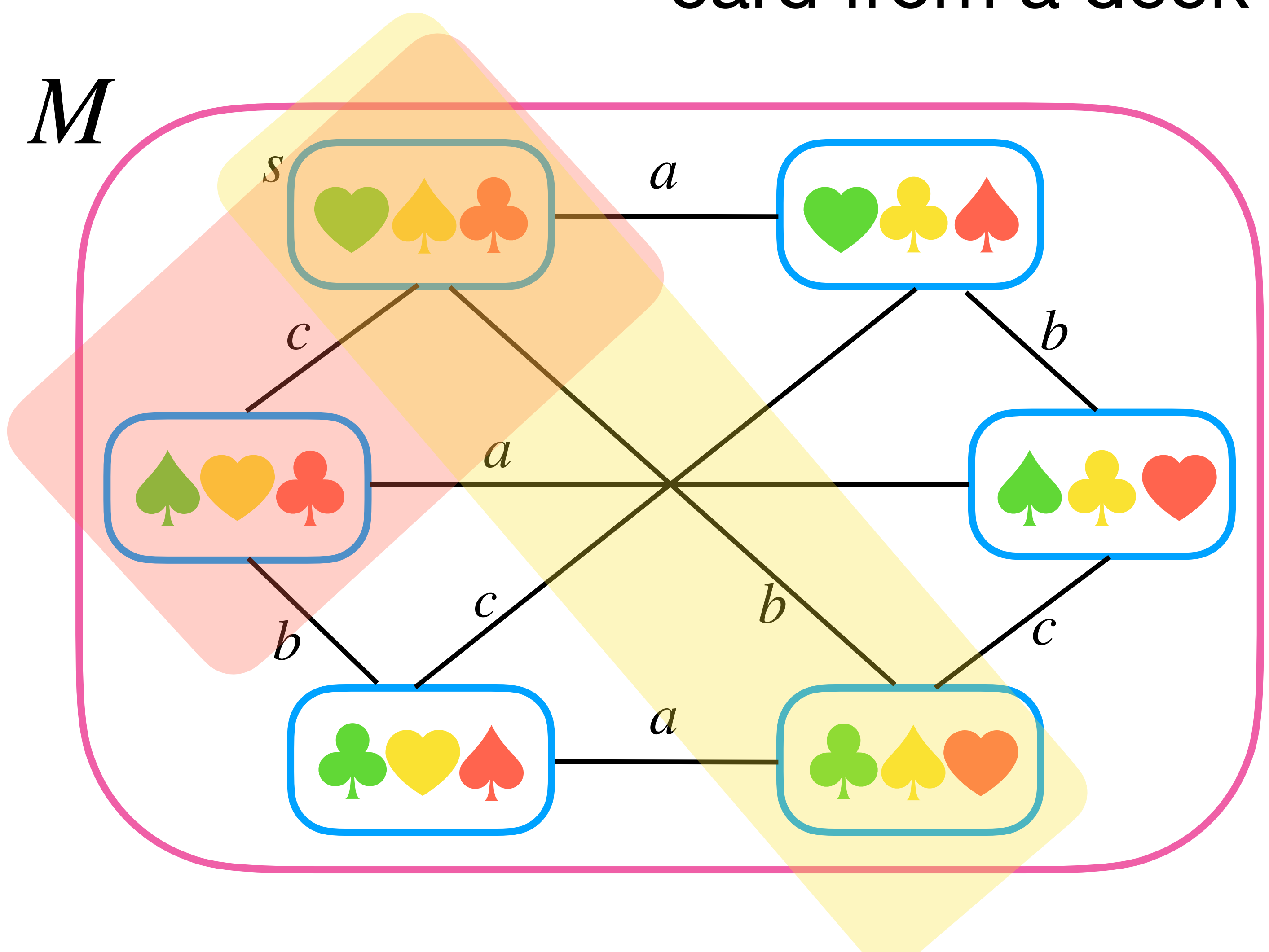
Card Example

Three agents, Alice, Bob, and Carol, have each drawn one card from a deck of $\{\heartsuit\ \spadesuit\ \clubsuit\}$



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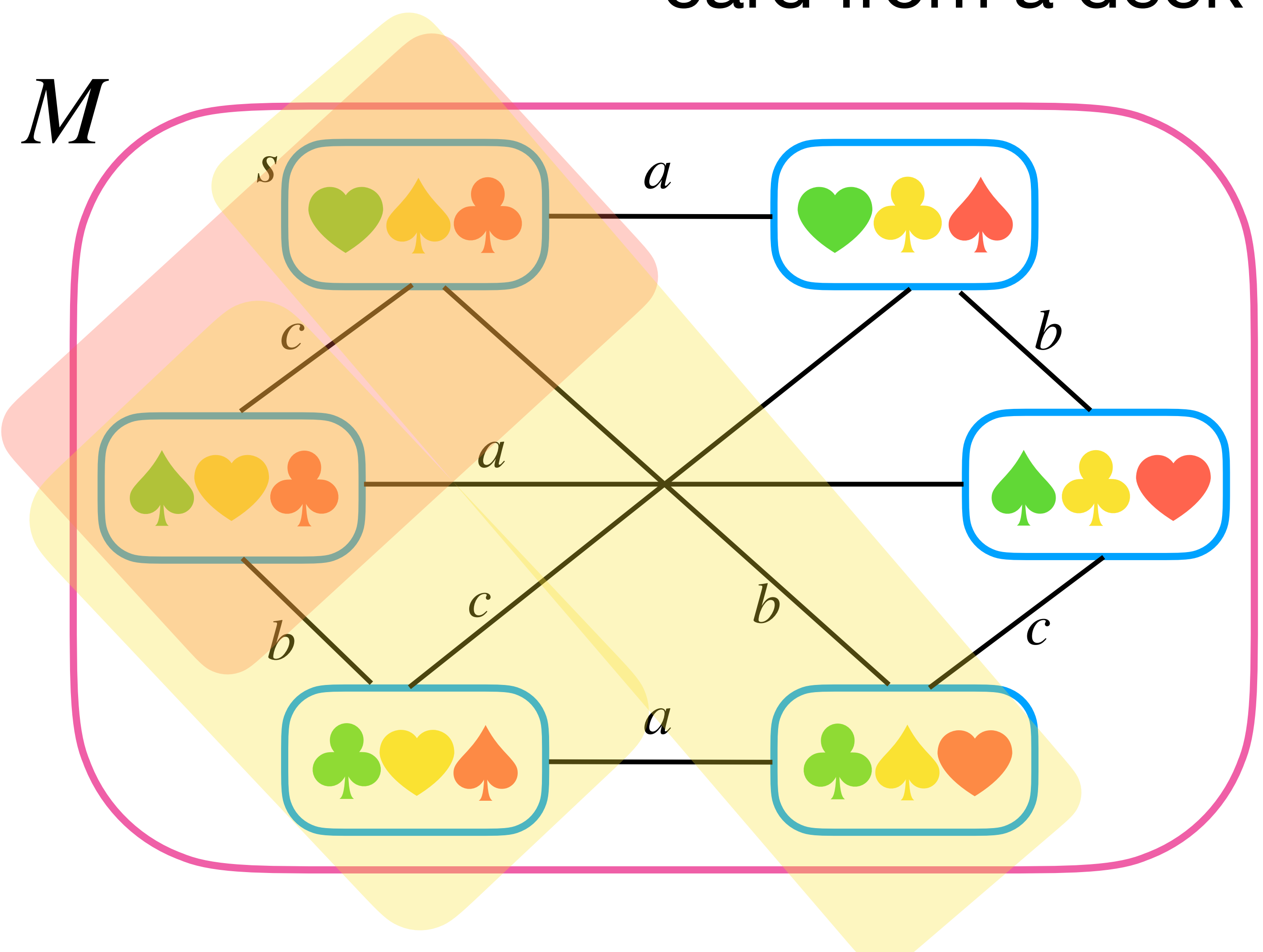

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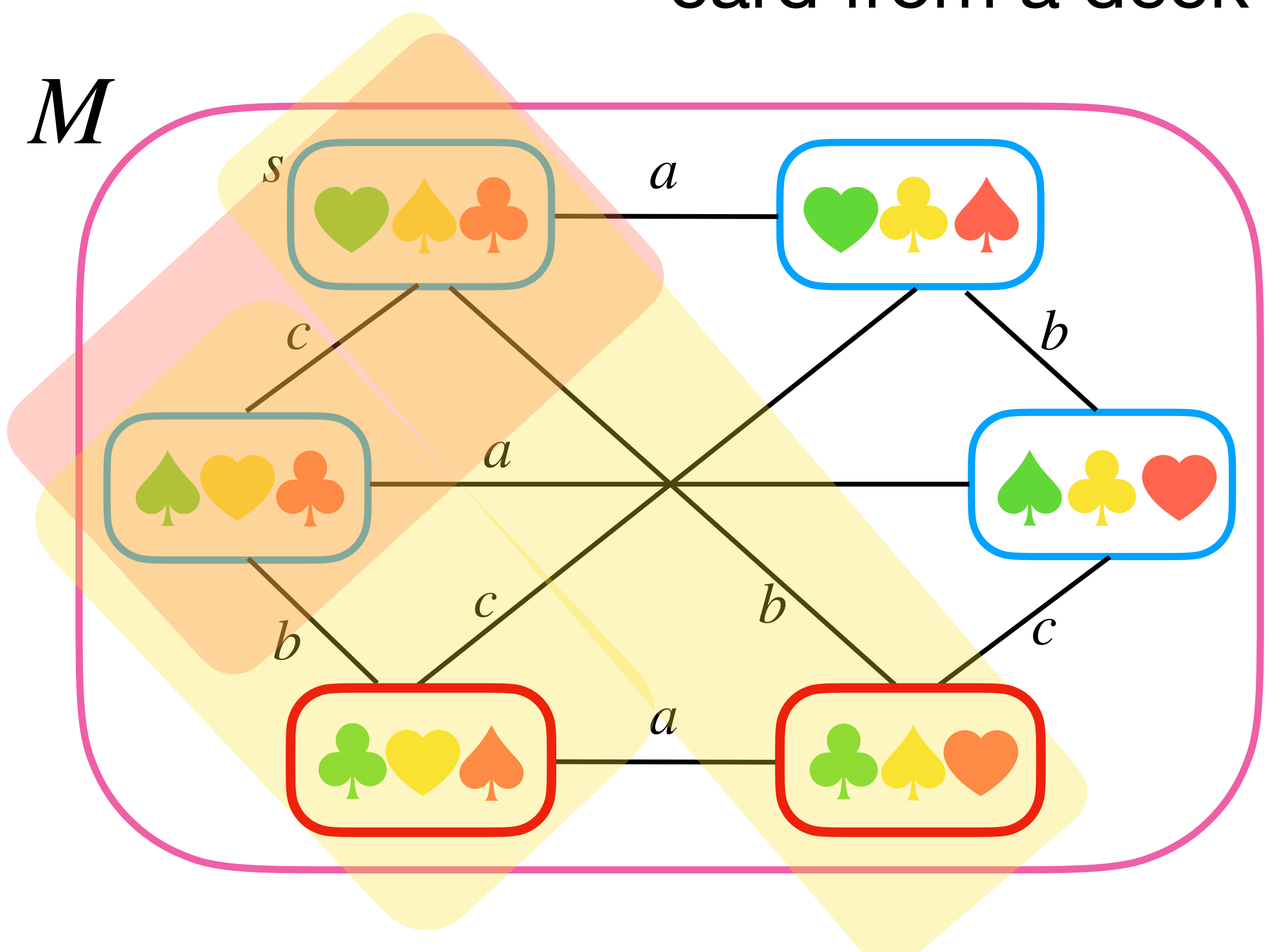

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Epistemic Logic

Agents and propositions	Let A and P be countable sets of agents and propositional variables
Language of EL	$\mathscr{E}\mathscr{L} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi$
Epistemic models	An epistemic model M is a tuple $(S, \sim, V)$, where • $S \neq \emptyset$ is a set of states; • $\sim: A \to 2^{S\times S}$ is an indistinguishability function with each $\sim_a$ being an equivalence relation; • $V : P \to 2^S$ is the valuation function.
Pointed model	A pair of M and $s \in S$ is called a pointed model and is denoted as M_s

Semantics of EL

$M_s \models p$ iff $s \in V(p)$

$M_s \models \neg\varphi$ iff $M_s \not\models \varphi$

$M_s \models \varphi \wedge \psi$ iff $M_s \models \varphi$ and $M_s \models \psi$

$M_s \models \Box_a \varphi$ iff $\forall t \in S : s \sim_a t$ implies $M_t \models \varphi$

$M_s \models \Diamond_a \varphi$ iff $\exists t \in S : s \sim_a t$ and $M_t \models \varphi$

Note that $\Diamond_a \varphi$ is equivalent to $\neg \Box_a \neg \varphi$

Axiomatisation of EL

Propositional tautologies

$\Box_a(\varphi \to \psi) \to (\Box_a\varphi \to \Box_a\psi)$

$\Box_a\varphi \to \varphi$ Reflexivity

$\Box_a\varphi \to \Box_a\Box_a\varphi$

$\neg\Box_a\varphi \to \Box_a\neg\Box_a\varphi$

From φ, $\varphi \to \psi$ infer ψ

From φ infer $\Box_a\varphi$

Halpern, Moses. *A guide to completeness and complexity for modal logics of knowledge and belief*, 1992.

Axiomatisation of EL

Propositional tautologies

$\Box_a(\varphi \to \psi) \to (\Box_a\varphi \to \Box_a\psi)$

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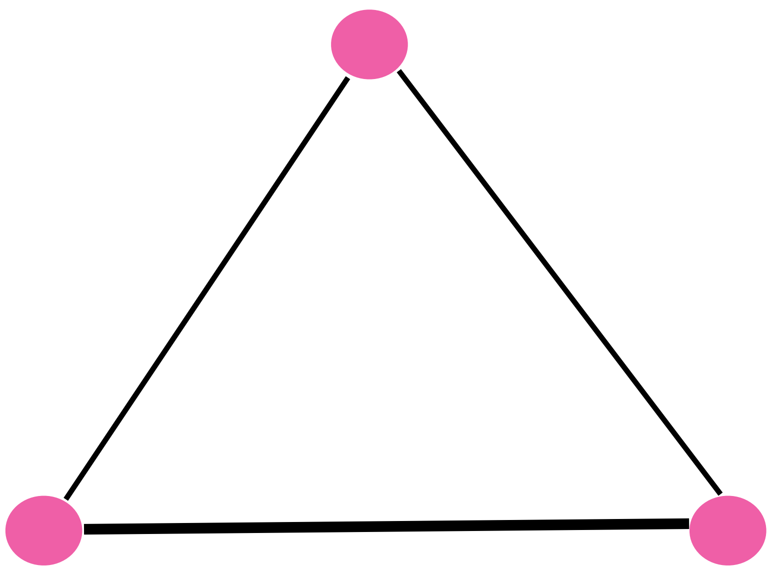

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From φ, $\varphi \to \psi$ infer ψ

From φ infer $\Box_a \varphi$

Theorem. EL is sound and complete

Theorem. Complexity of SAT-EL is PSPACE-complete

Satisfiability: for a given φ, determine whether there is a M_s such that $M_s \models \varphi$

Halpern, Moses. *A guide to completeness and complexity for modal logics of knowledge and belief*, 1992.

Axiomatisation of EL

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From φ, $\varphi \to \psi$ infer ψ

From φ infer $\Box_a\varphi$

Theorem. EL is sound and complete

Theorem. Complexity of SAT-EL is PSPACE-complete

Theorem. Complexity of MC-EL is P-complete

Model checking: for a given φ and M_s, determine whether $M_s \vDash \varphi$

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Card Example

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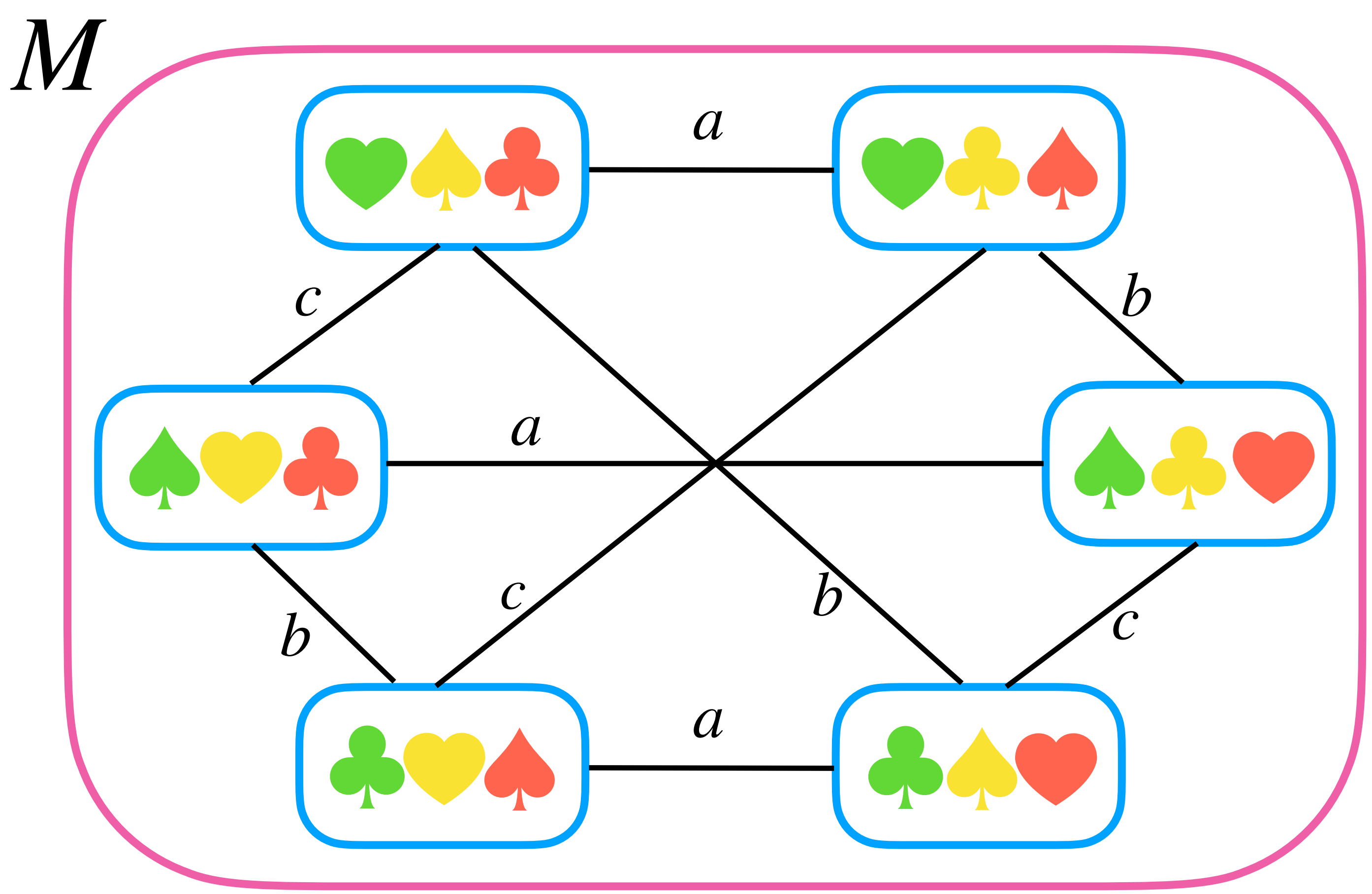

Alice says that she does not have clubs: $\neg ♣_a$

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

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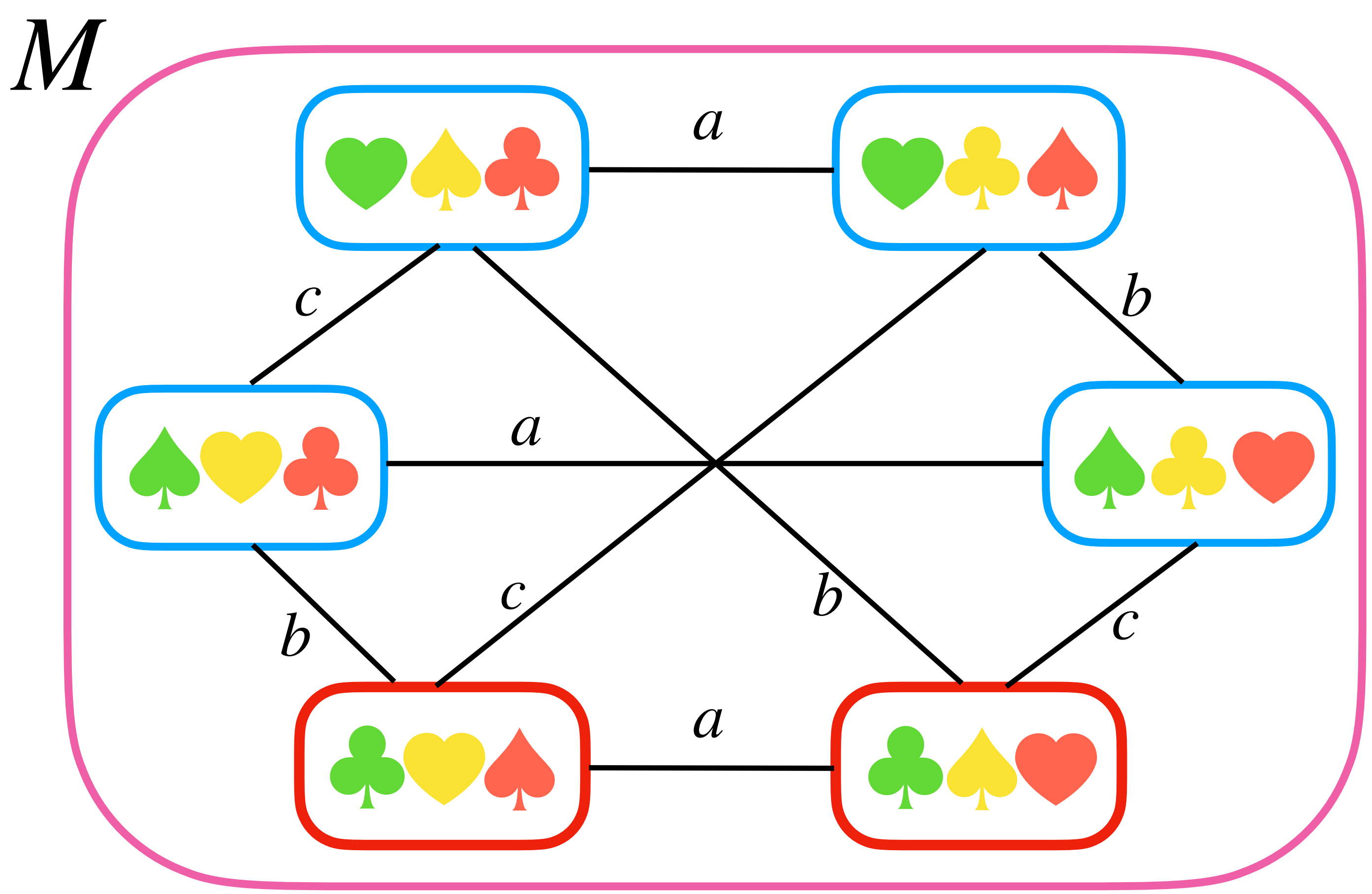

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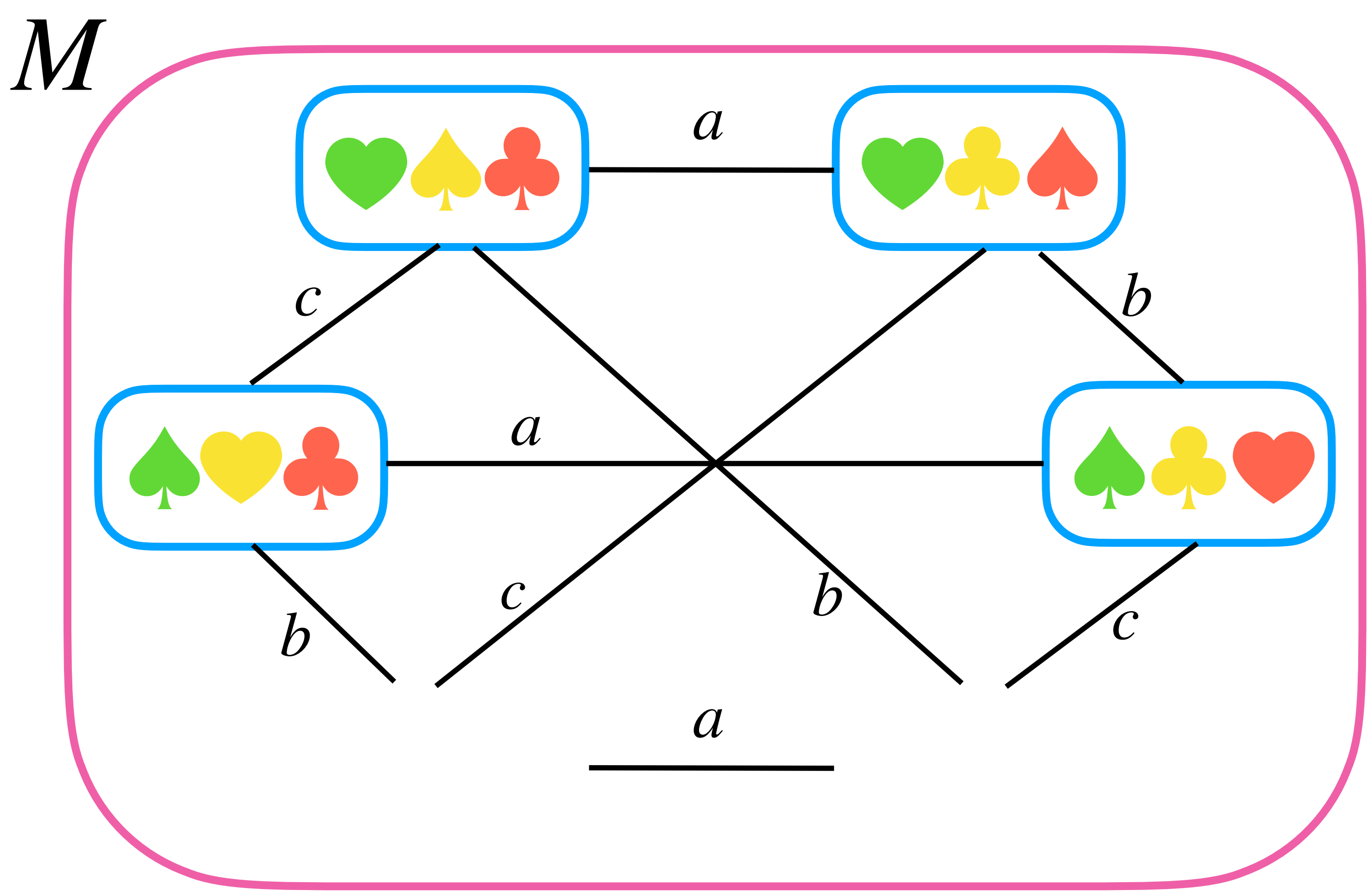

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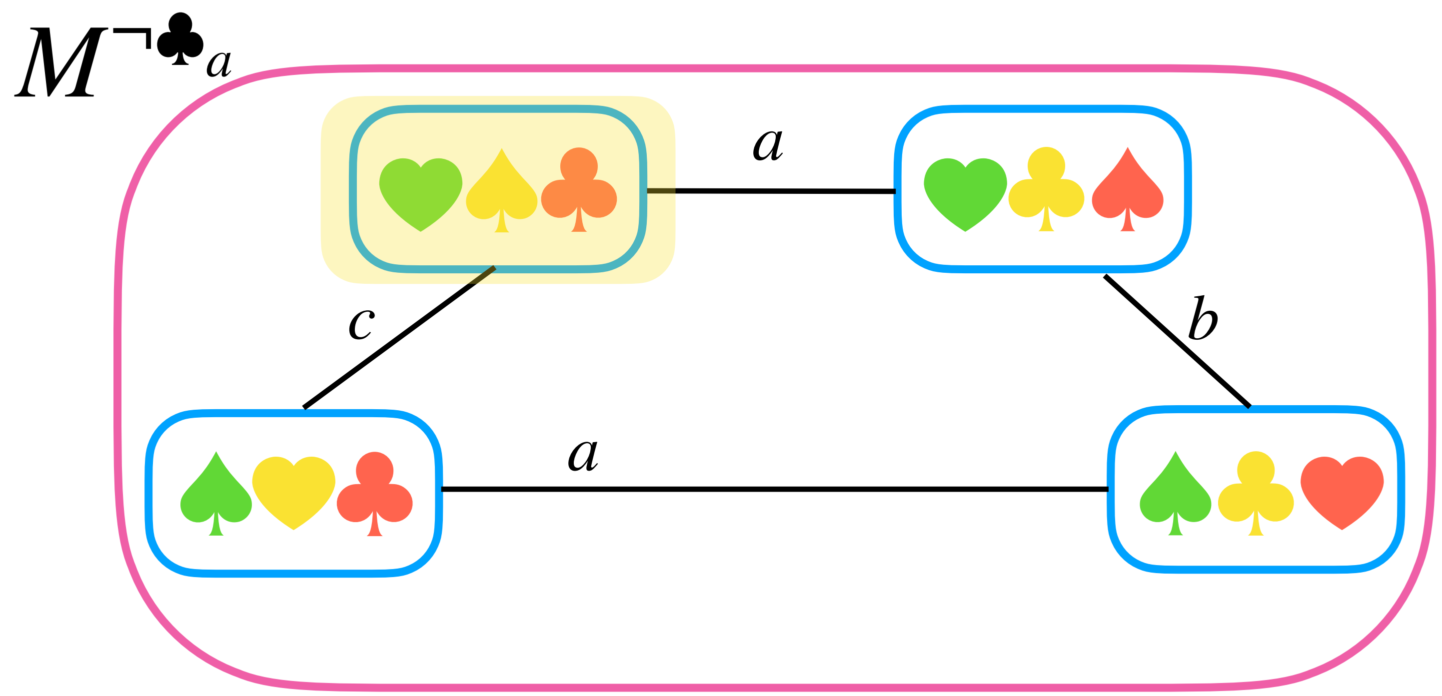

Bob says that he now knows that Carol has clubs: $\Box_b \clubsuit_c$

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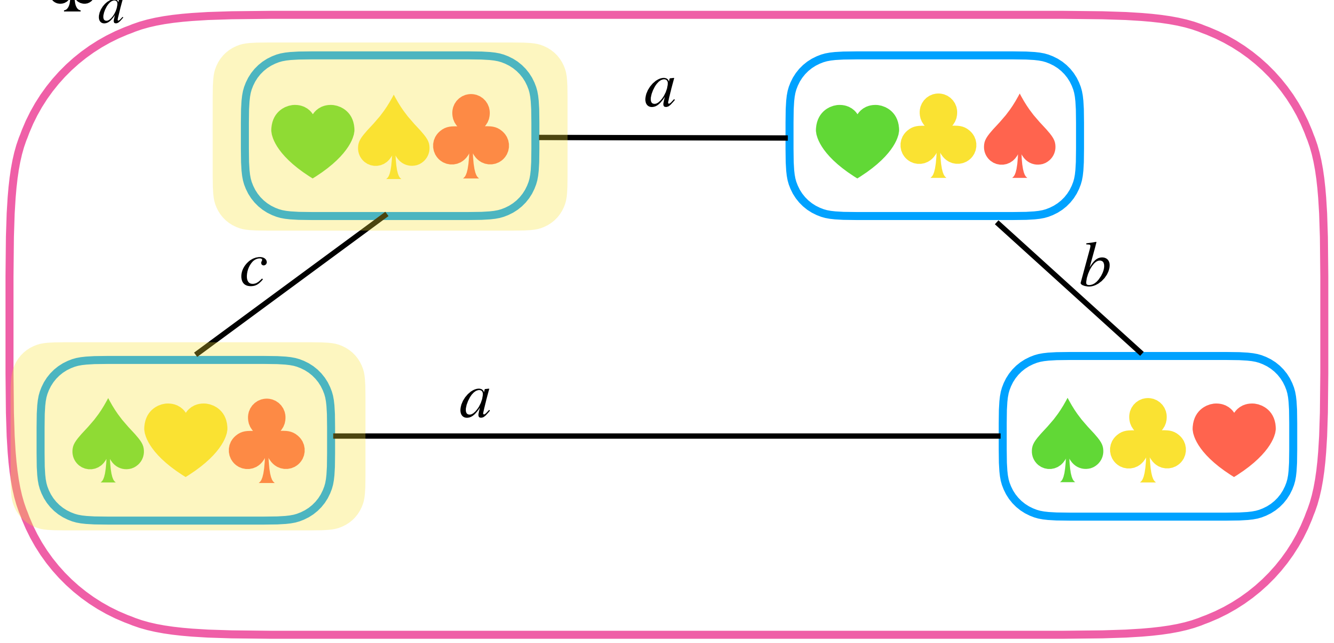
Three agents, Alice, Bob, and Carol, have each drawn one card from a deck of {♥ ♠ ♣}, and then Alice says that she does not have clubs

$M^{\neg \clubsuit_a}$



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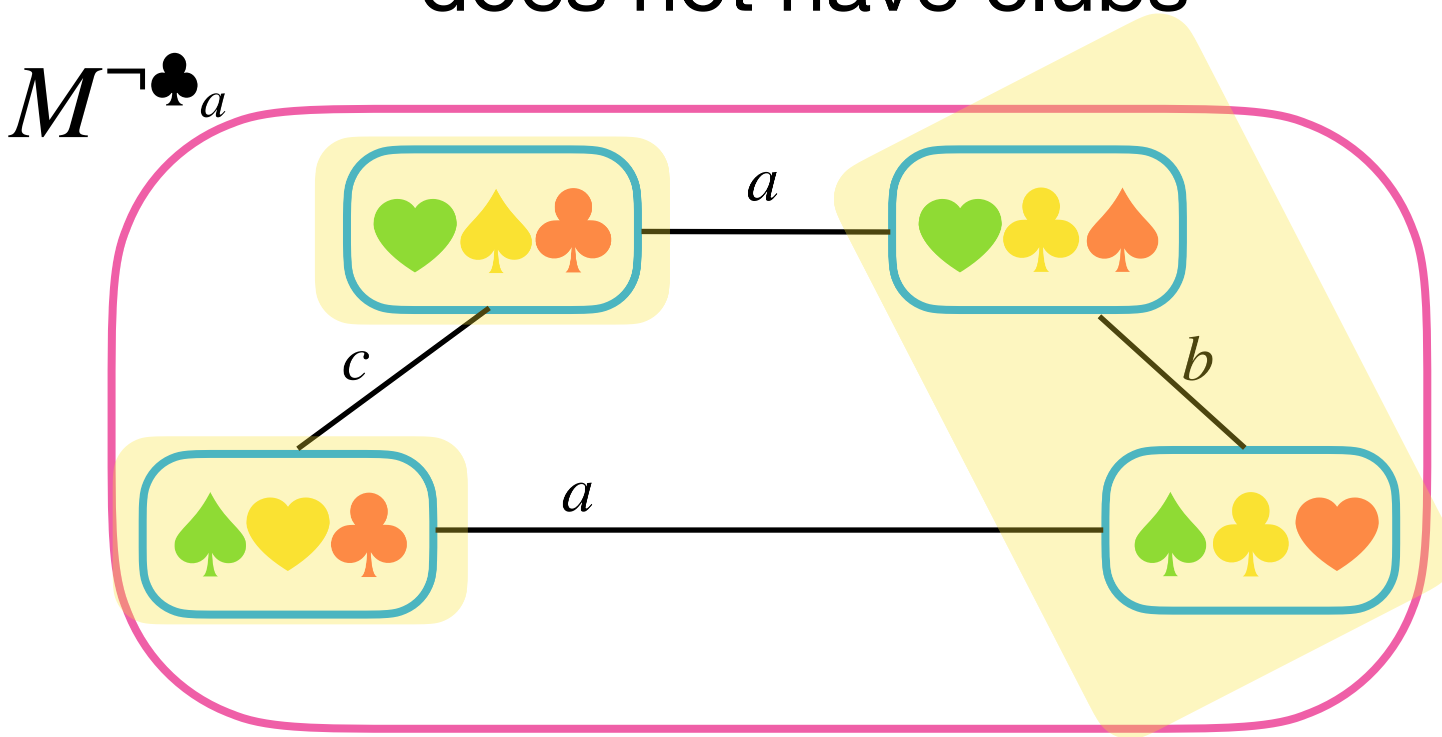

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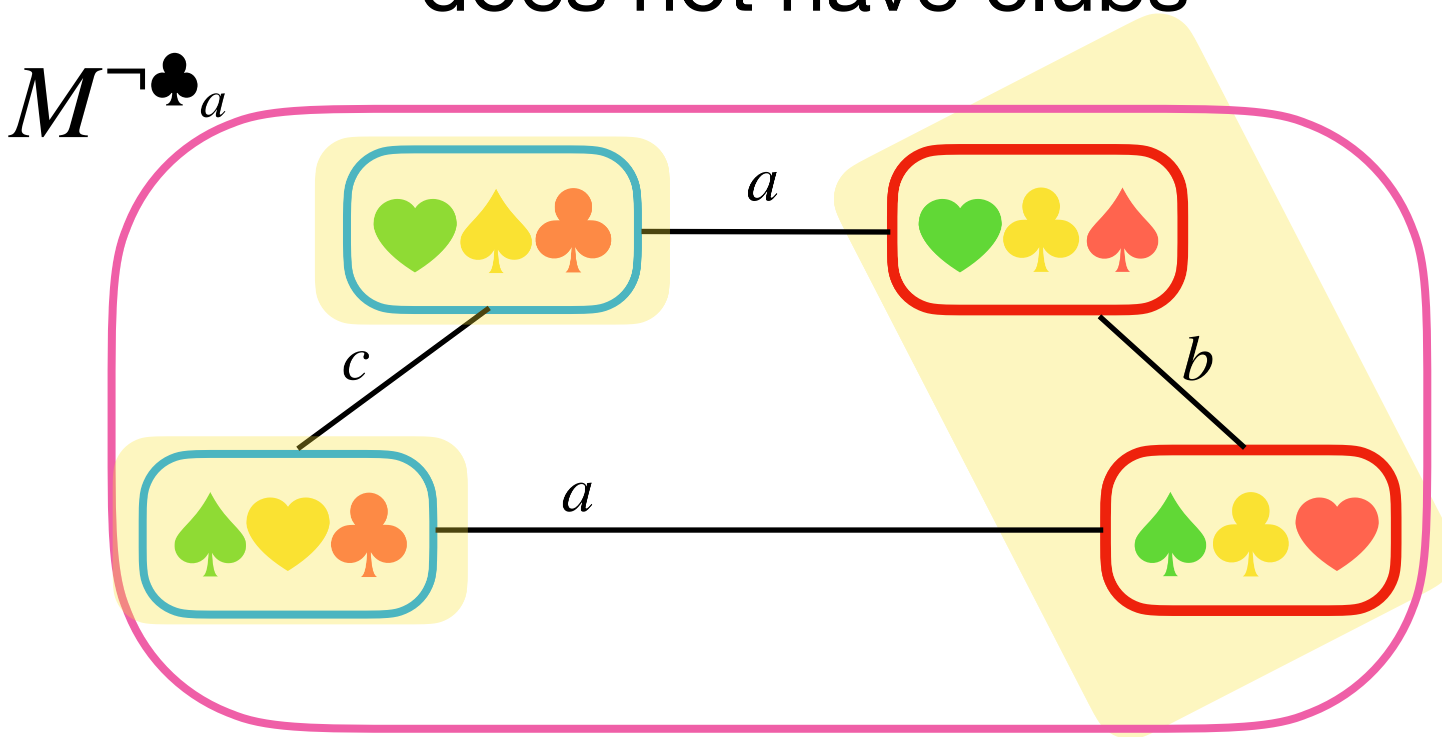

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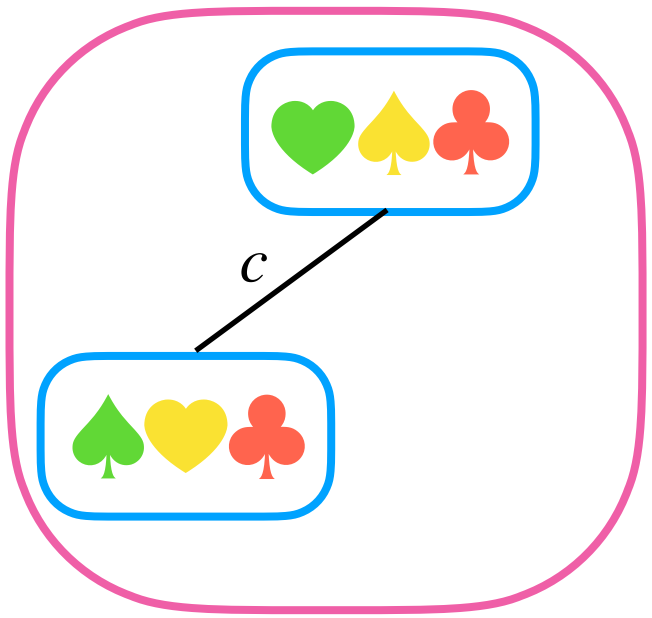
$M^{\neg\clubsuit_a, \Box_b \clubsuit_c}$



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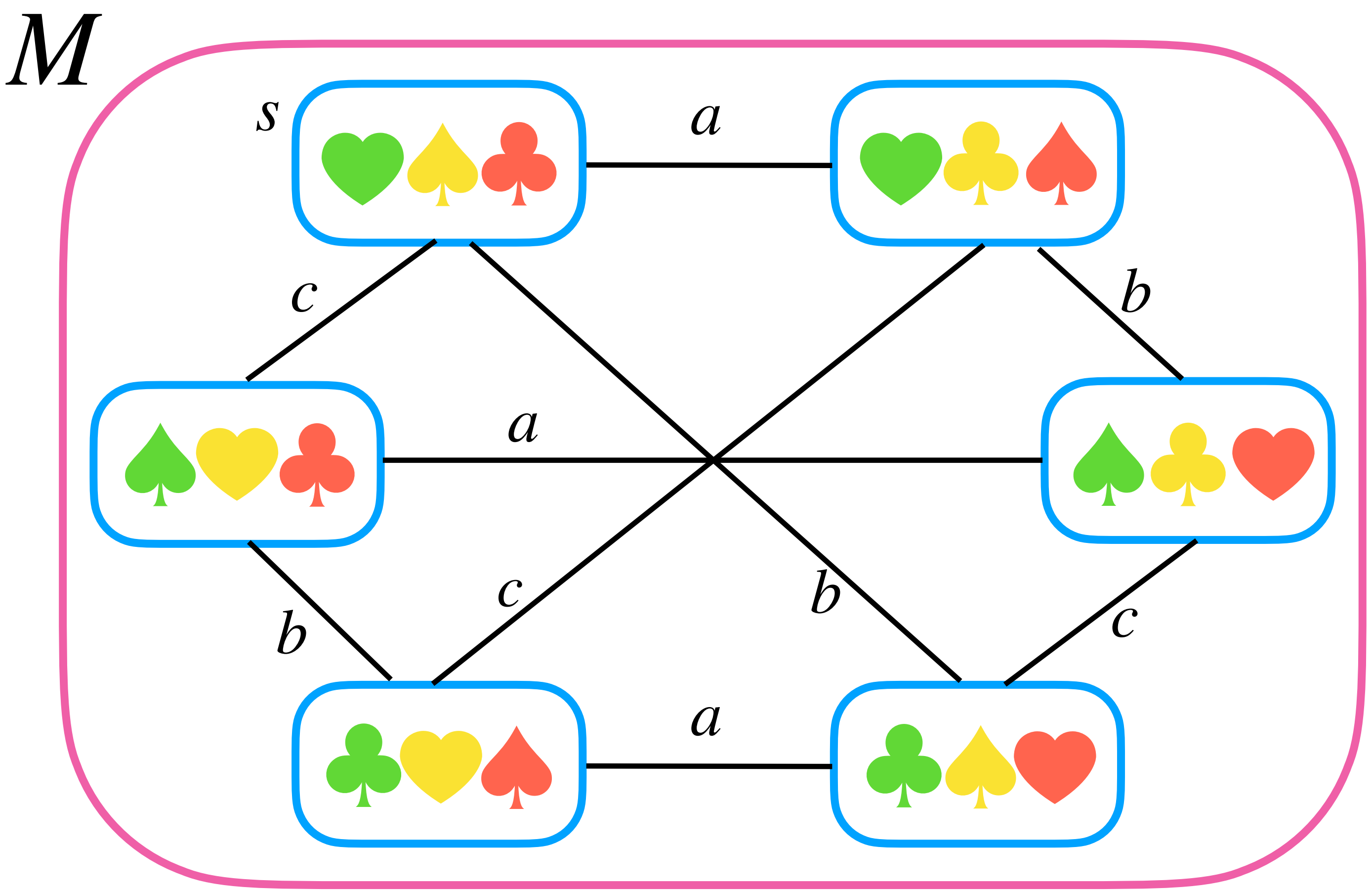

$M_s \vDash [\neg ♣_a]\Box_b(♥_a \wedge ♠_b \wedge ♣_c)$

$[\psi]\varphi$: after public announcement of ψ, φ is true

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

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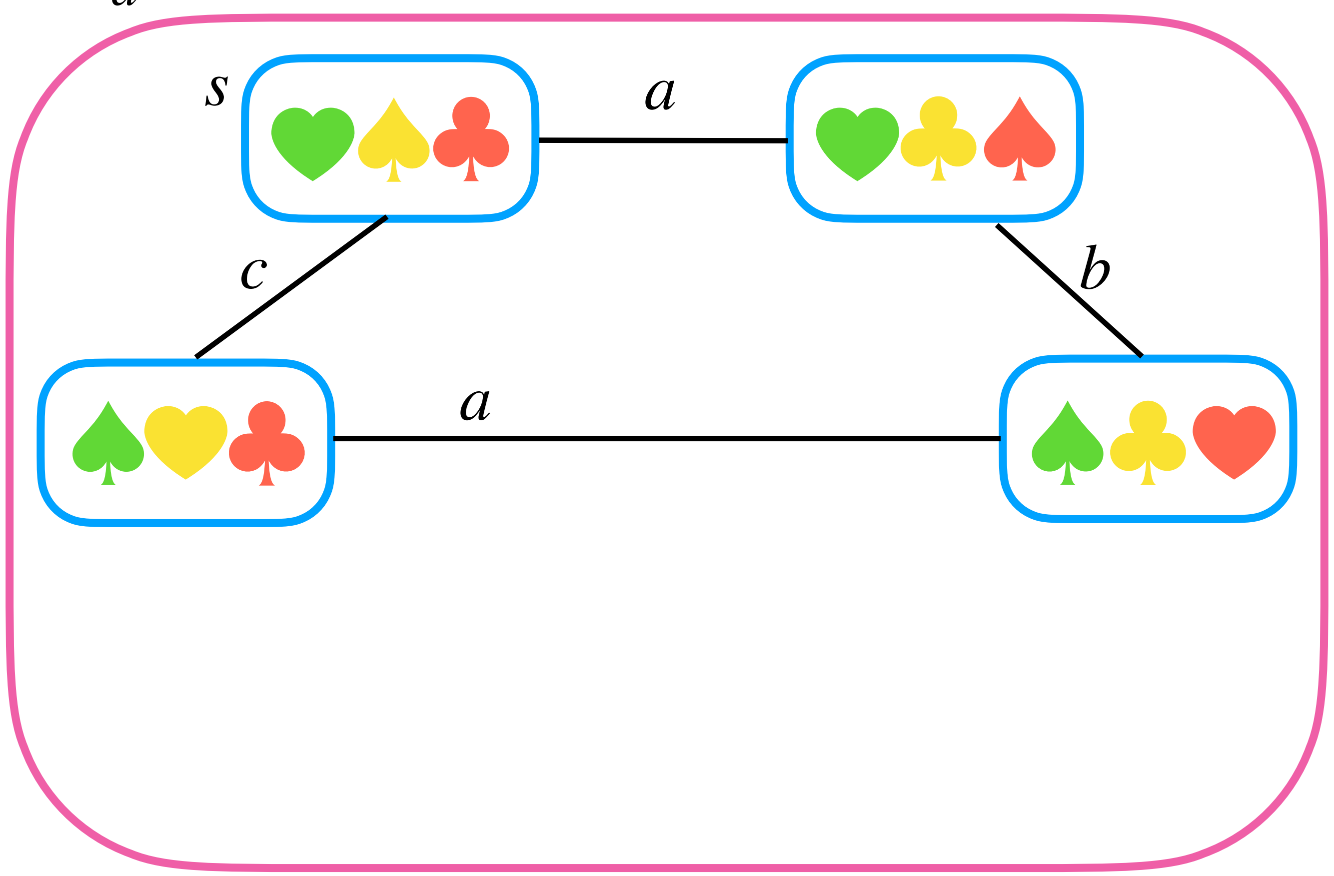

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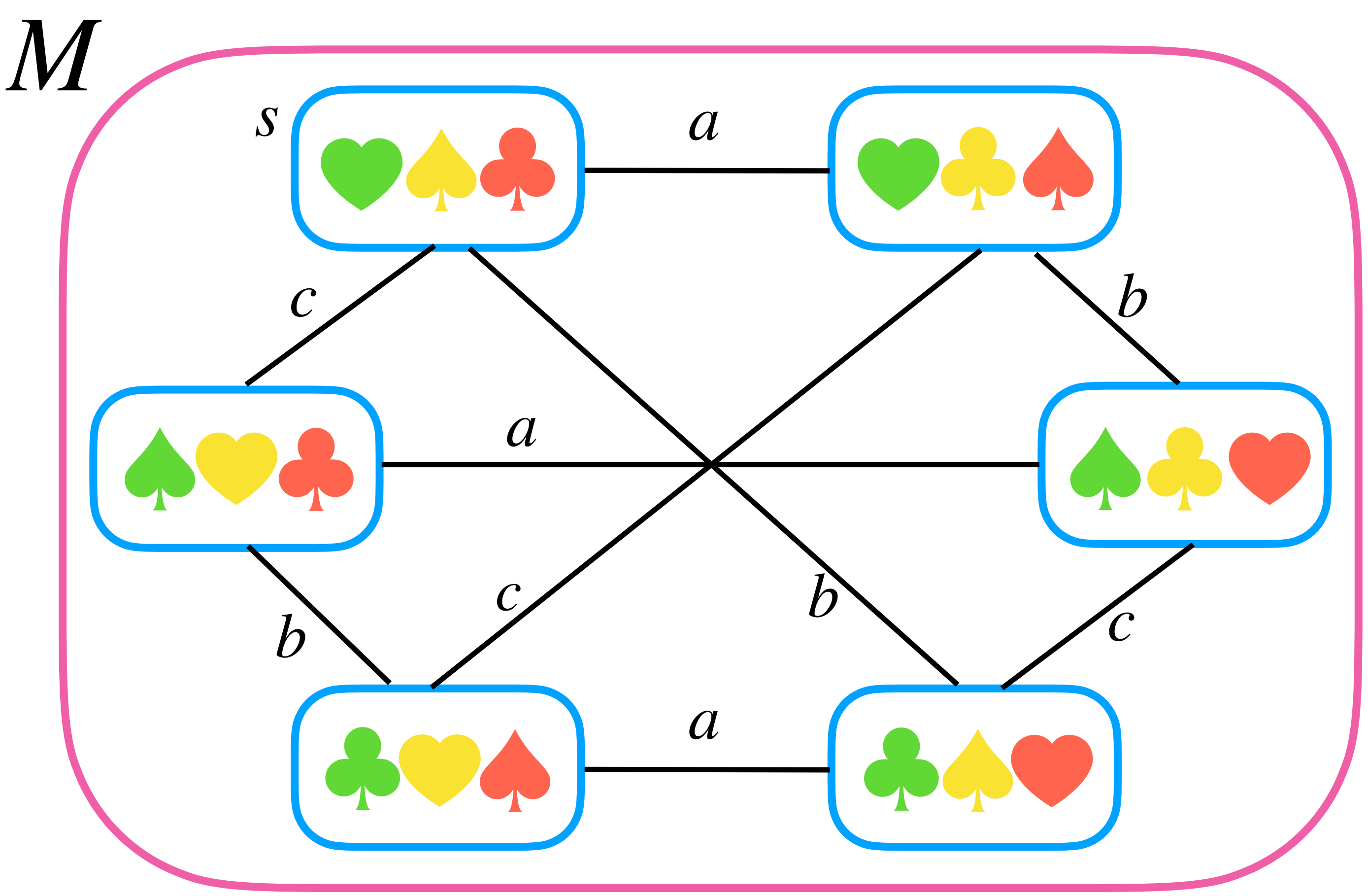

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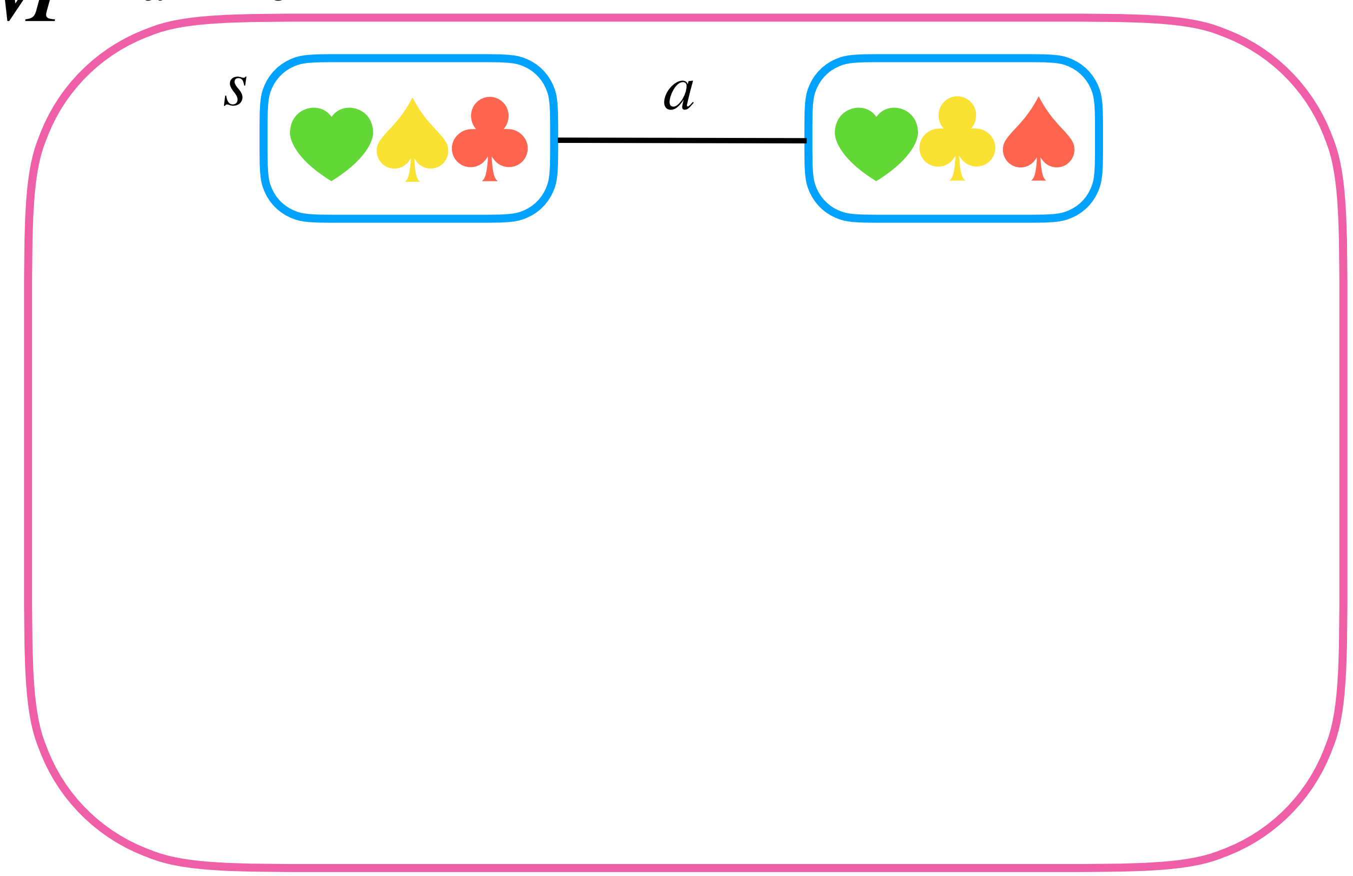

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Public Announcement Logic

Language of PAL

$$\mathscr{PAL} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\varphi]\varphi$$

Semantics

$M_s \vDash [\psi]\varphi$ iff $M_s \vDash \psi$ implies $M_s^\psi \vDash \varphi$

$M_s \vDash \langle\psi\rangle\varphi$ iff $M_s \vDash \psi$ and $M_s^\psi \vDash \varphi$

Updated model

Let $M = (S, \sim, V)$ and $\varphi \in \mathscr{PAL}$. An updated model M^φ is a tuple $(S^\varphi, \sim^\varphi, V^\varphi)$, where

- $S^\varphi = \{s \in S \mid M_s \vDash \varphi\}$;
- $\sim_a^\varphi = \sim_a \cap (S^\varphi \times S^\varphi)$;
- $V^\varphi(p) = V(p) \cap S^\varphi$.

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

Axiomatisation of PAL

Axioms of EL

$[\varphi]p \leftrightarrow (\varphi \to p)$

$[\varphi]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi]\psi)$

$[\varphi](\psi \wedge \chi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\chi)$

$[\varphi]\Box_a\psi \leftrightarrow (\varphi \to \Box_a[\varphi]\psi)$

$[\varphi][\psi]\chi \leftrightarrow ([\varphi \wedge [\varphi]\psi]\chi)$

From φ infer $[\psi]\varphi$

Theorem. PAL and EL are equally expressive

Theorem. PAL is sound and complete

Observe that axioms of PAL allow one to rewrite any formula of PAL into a formula of EL

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

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Theorem. Complexity of SAT-PAL is PSPACE-complete

Lutz. Complexity and Succinctness of Public Announcement Logic, 2006.

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

Axiomatisation of PAL

Axioms of EL

$[\varphi]p \leftrightarrow (\varphi \rightarrow p)$

$[\varphi]\neg\psi \leftrightarrow (\varphi \rightarrow \neg[\varphi]\psi)$

$[\varphi](\psi \wedge \chi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\chi)$

$[\varphi]\Box_a\psi \leftrightarrow (\varphi \rightarrow \Box_a[\varphi]\psi)$

$[\varphi][\psi]\chi \leftrightarrow ([\varphi \wedge [\varphi]\psi]\chi)$

From φ infer $[\psi]\varphi$

Theorem. PAL and EL are equally expressive

Theorem. PAL is sound and complete

Theorem. Complexity of SAT-PAL is PSPACE-complete

Theorem. Complexity of MC-PAL is P-complete

Van Benthem, Kooi. *Reduction axioms for epistemic actions*, 2004.

Lutz. *Complexity and Succinctness of Public Announcement Logic*, 2006.

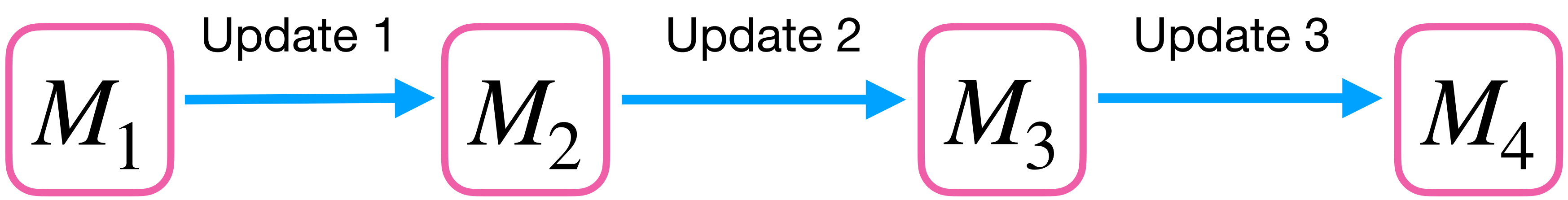
Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

Part II

Introduction to Arbitrary Public Announcement Logic

Open Problem I and a partial solution

Dynamic Epistemic Logic



Some extensions

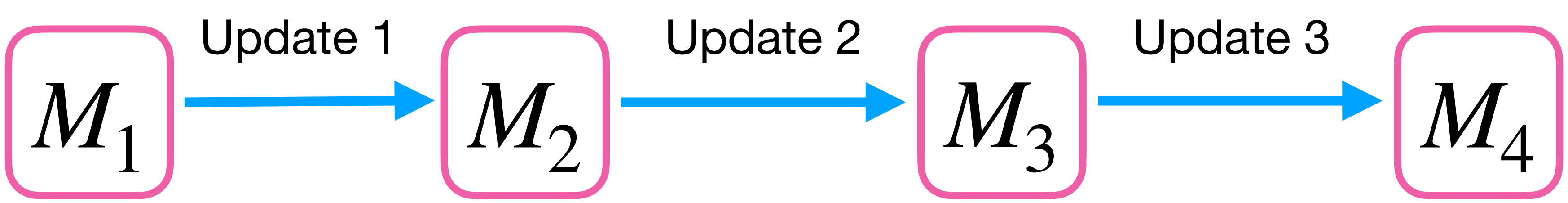
Making epistemic actions more expressive
(e.g. adding ontic changes, etc.)

Adding temporal operators

Adding group knowledge

Allowing quantification over epistemic actions

Dynamic Epistemic Logic



Some extensions

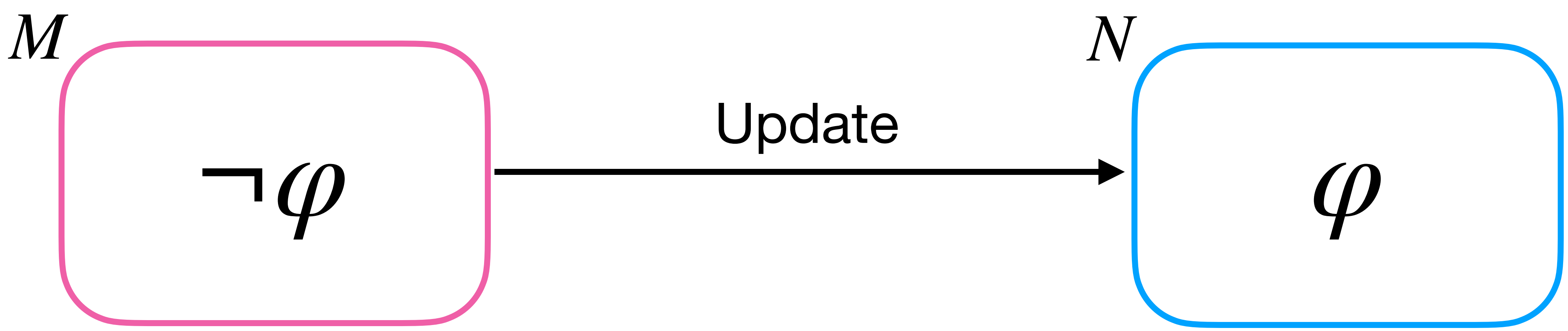
Making epistemic actions more expressive
(e.g. adding ontic changes, etc.)

Adding temporal operators

Adding group knowledge

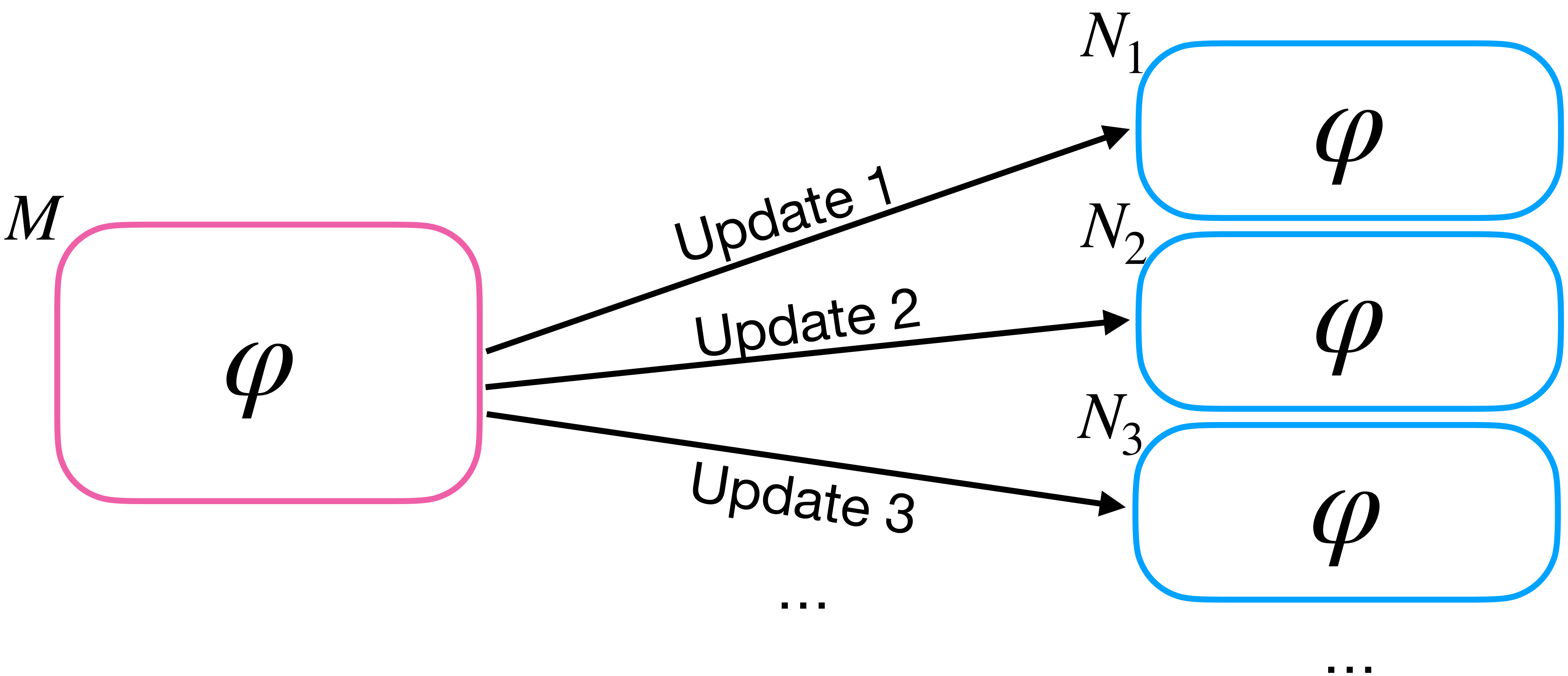
Allowing quantification over epistemic actions

Quantifying Over Public Announcements



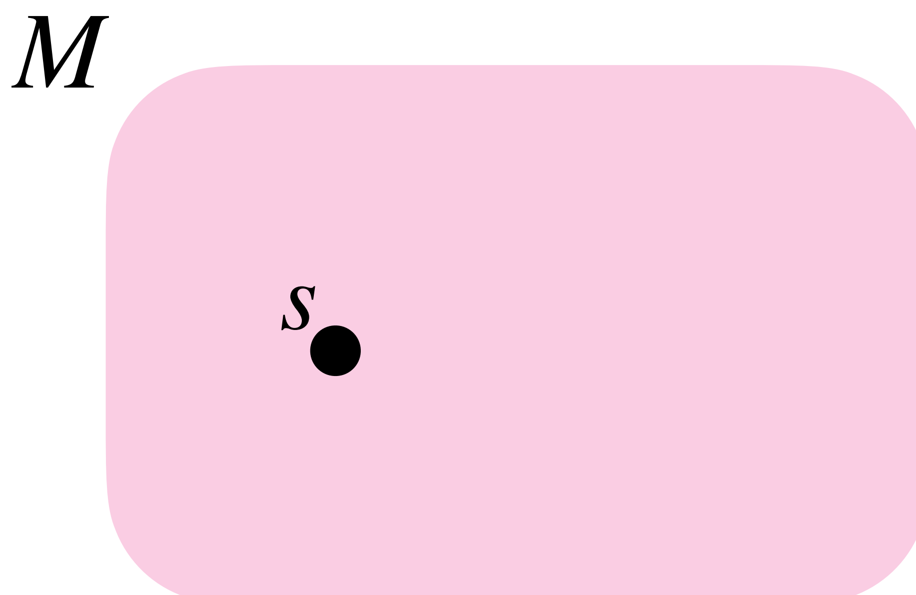
Existence: Having a starting configuration M and a property φ we would like to have, there is an epistemic action that results in configuration N satisfying φ

Quantifying Over Public Announcements



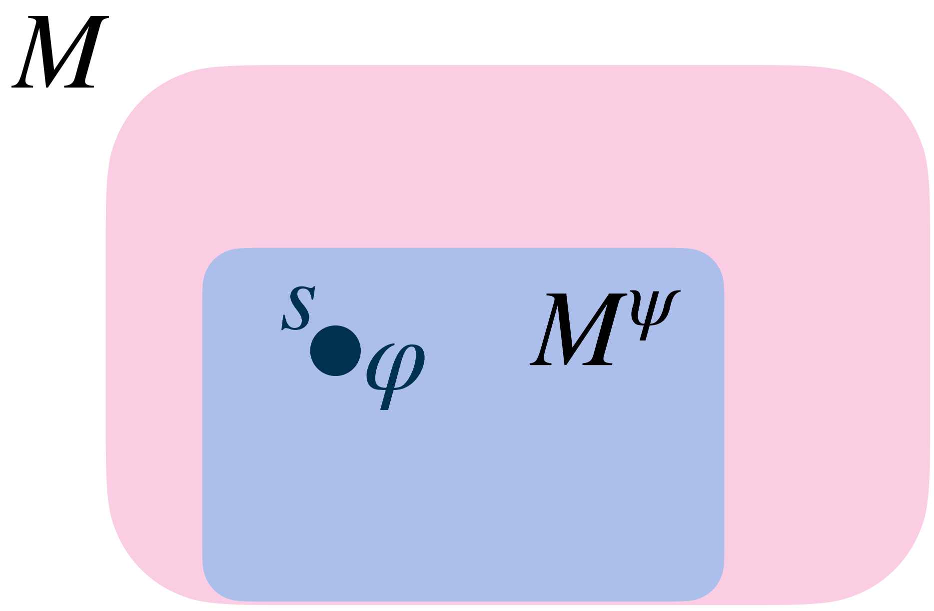
Universality: Having a starting configuration M satisfying φ, we would like to ensure that all epistemic actions result in some configuration N satisfying φ

Quantifying Over Public Announcements



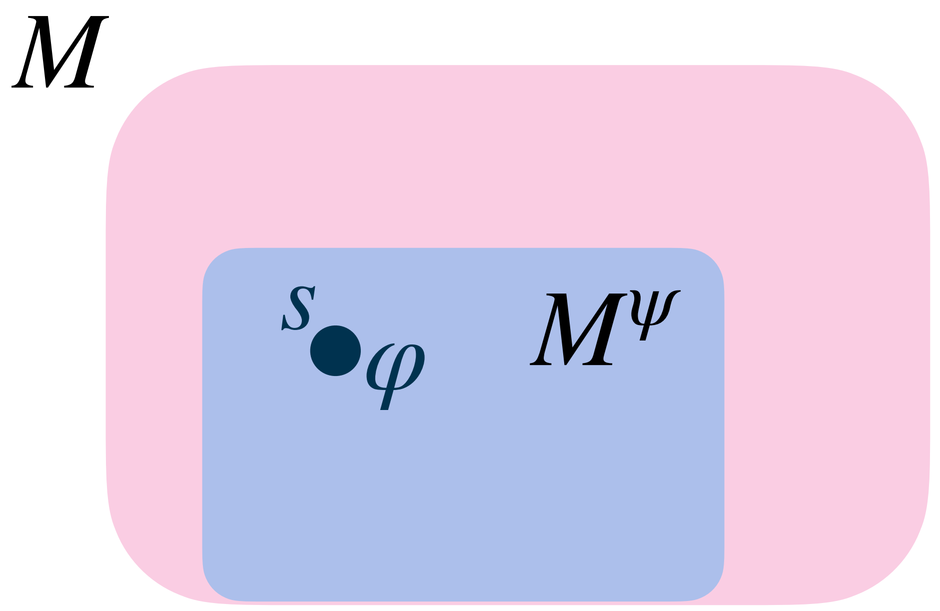
$\langle ! \rangle \varphi$: There is a public announcement, after which φ is true

Quantifying Over Public Announcements



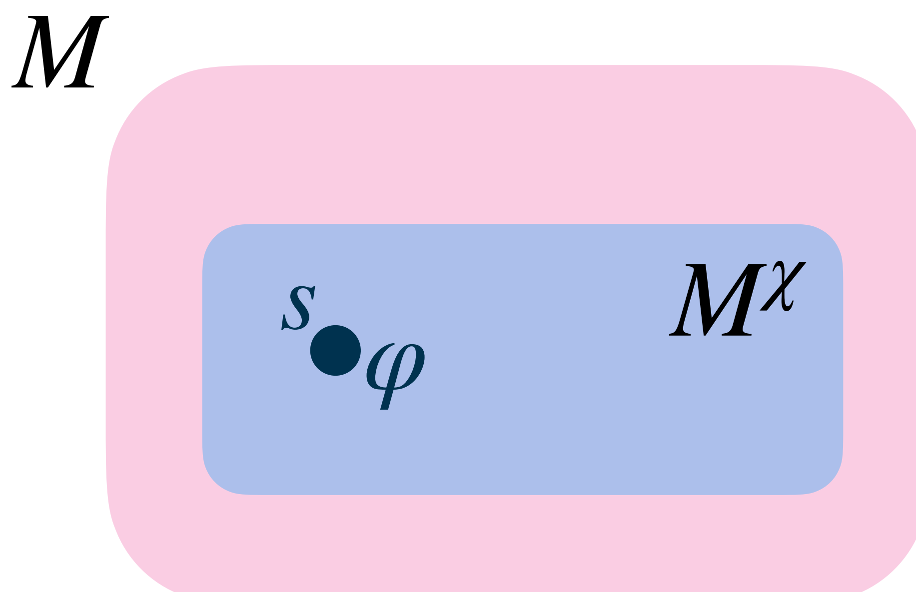
$\langle ! \rangle \varphi$: There is a public announcement, after which φ is true

Quantifying Over Public Announcements



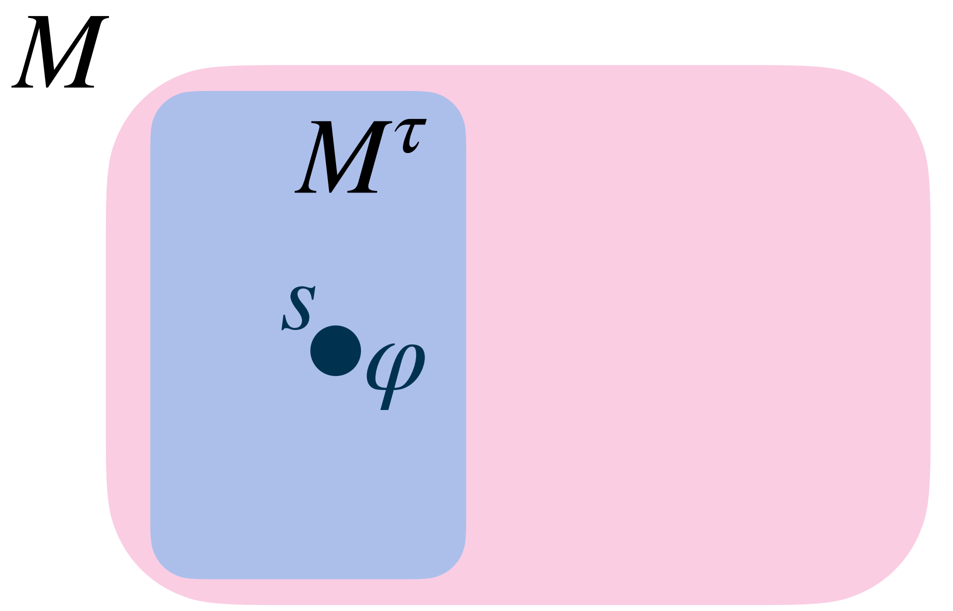
$[\,!\,]\varphi$: After all public announcements, φ is true

Quantifying Over Public Announcements



$[!]\varphi$: After all public announcements, φ is true

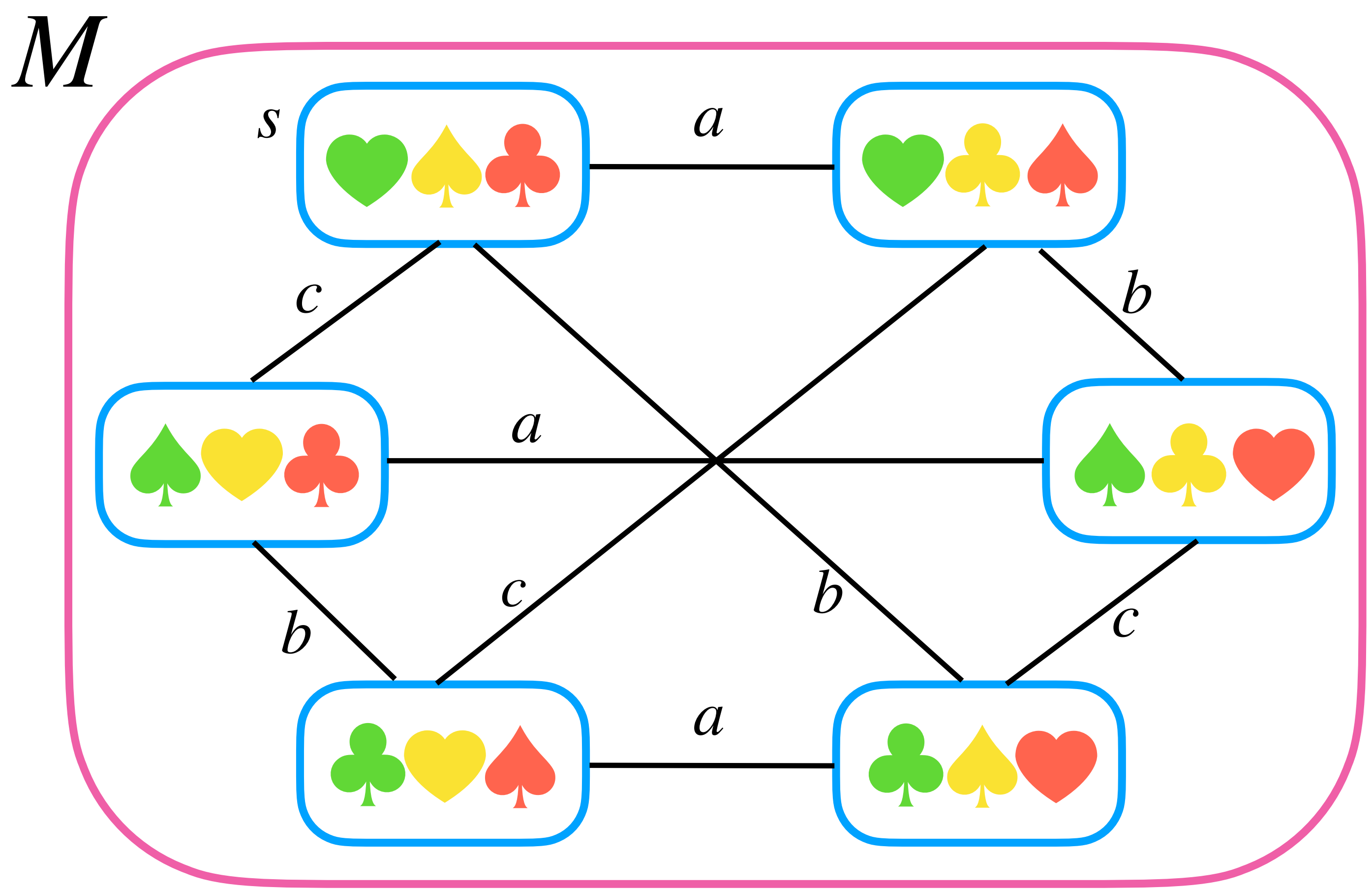
Quantifying Over Public Announcements



$[!]\varphi$: After all public announcements, φ is true

Card Example

There is an announcement such that Alice knows the deal, and Bob and Carol do not

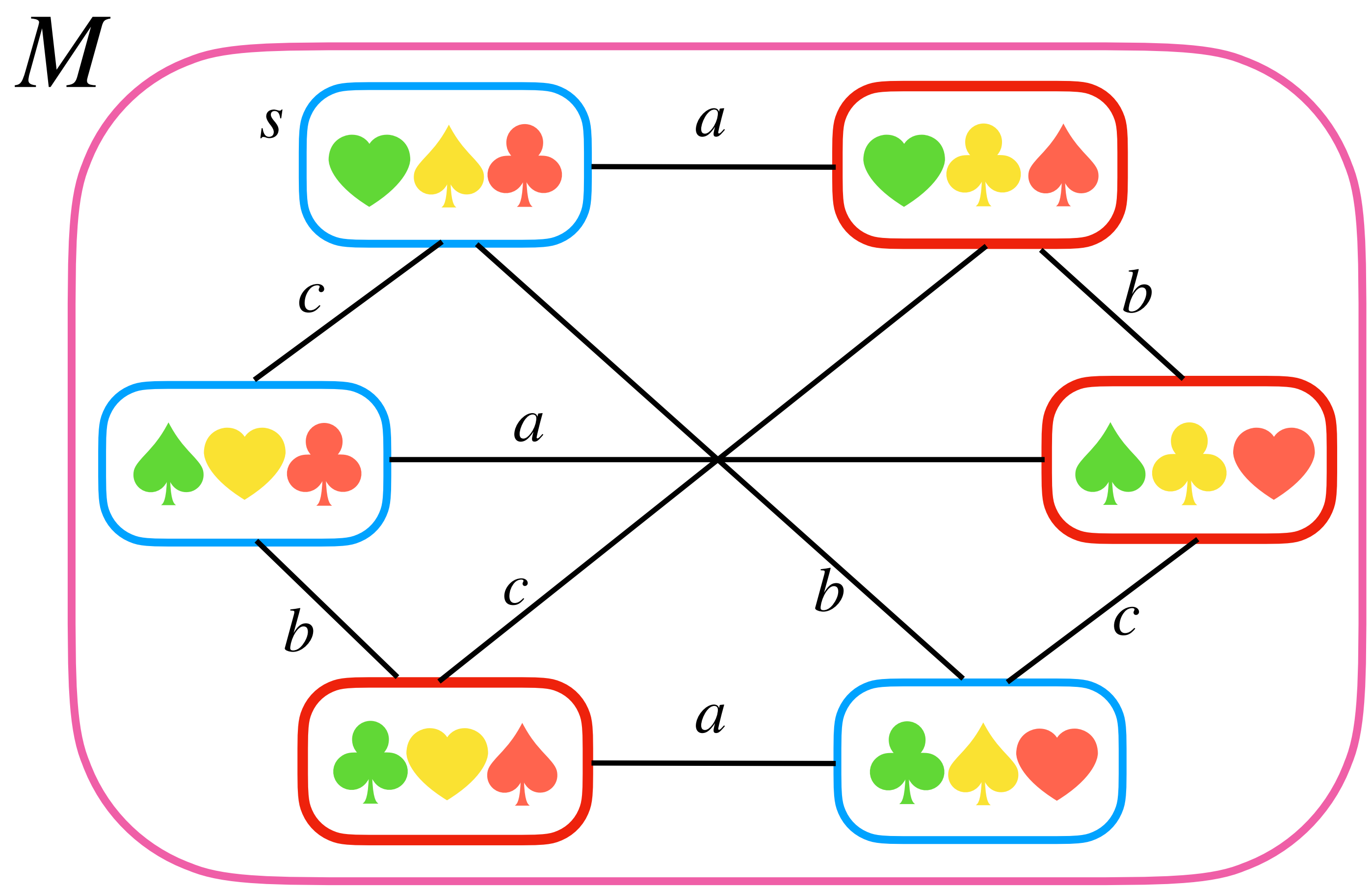


$$M_s \vDash \langle ! \rangle (\Box_a \text{deal} \wedge \neg \Box_b \text{deal} \wedge \neg \Box_c \text{deal})$$

$$\varphi := (\spadesuit_b \vee \heartsuit_b) \wedge (\clubsuit_c \vee \heartsuit_c)$$

Card Example

There is an announcement such that Alice knows the deal,
and Bob and Carol do not



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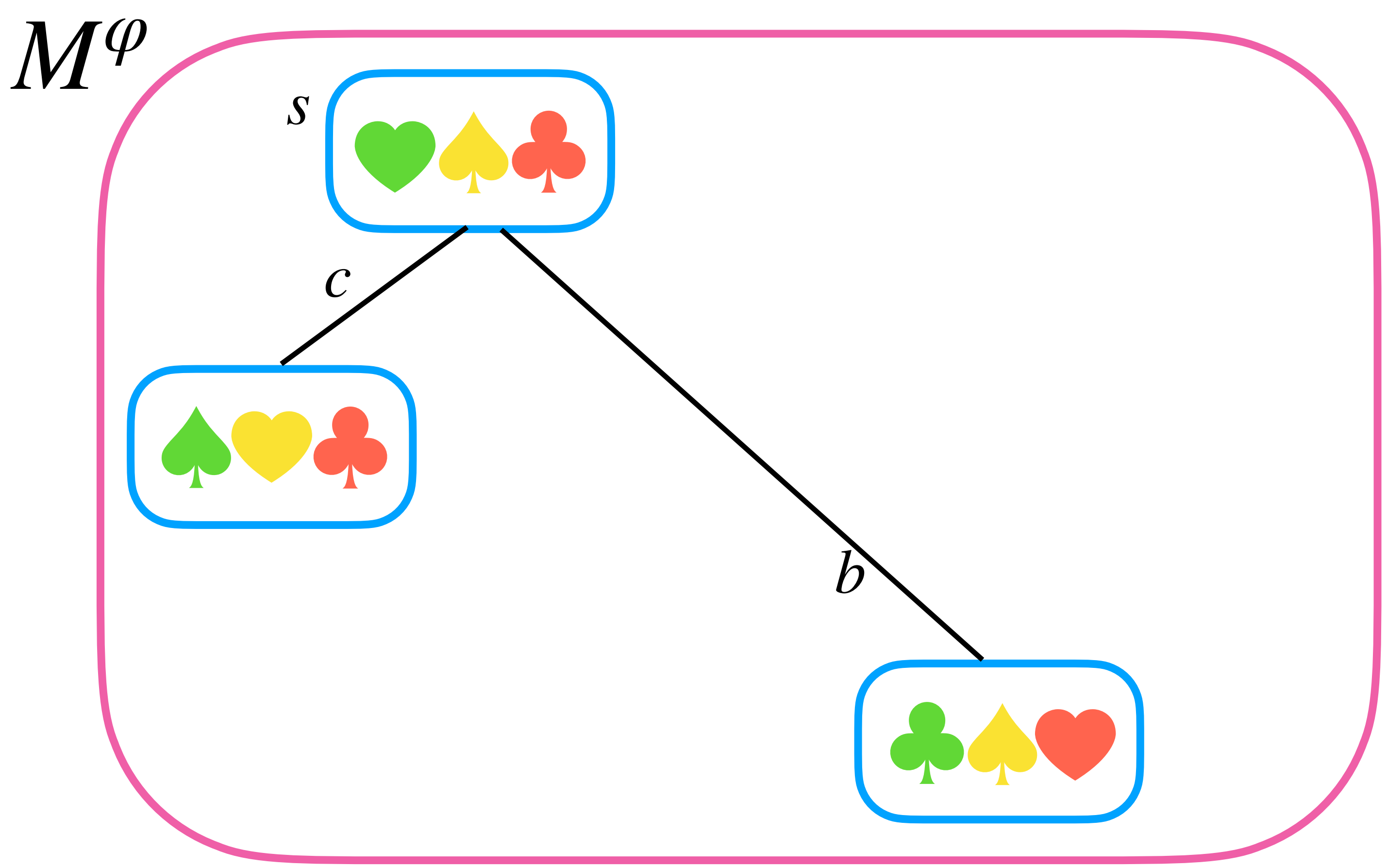
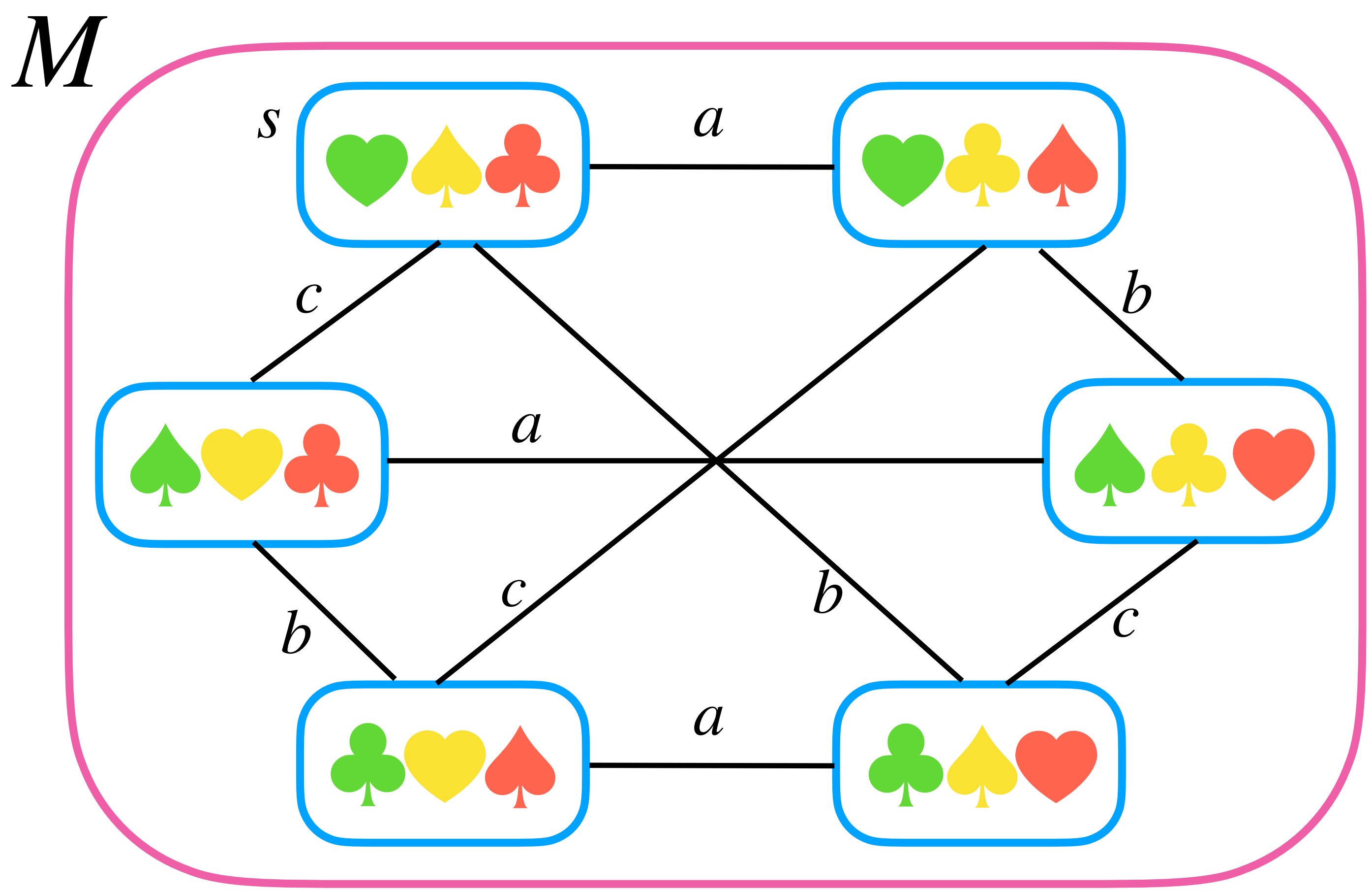


$$M_s \vDash \langle ! \rangle (\Box_a \text{deal} \land \neg \Box_b \text{deal} \land \neg \Box_c \text{deal})$$

$$\varphi := (\spadesuit_b \lor \heartsuit_b) \land (\clubsuit_c \lor \heartsuit_c)$$

Card Example

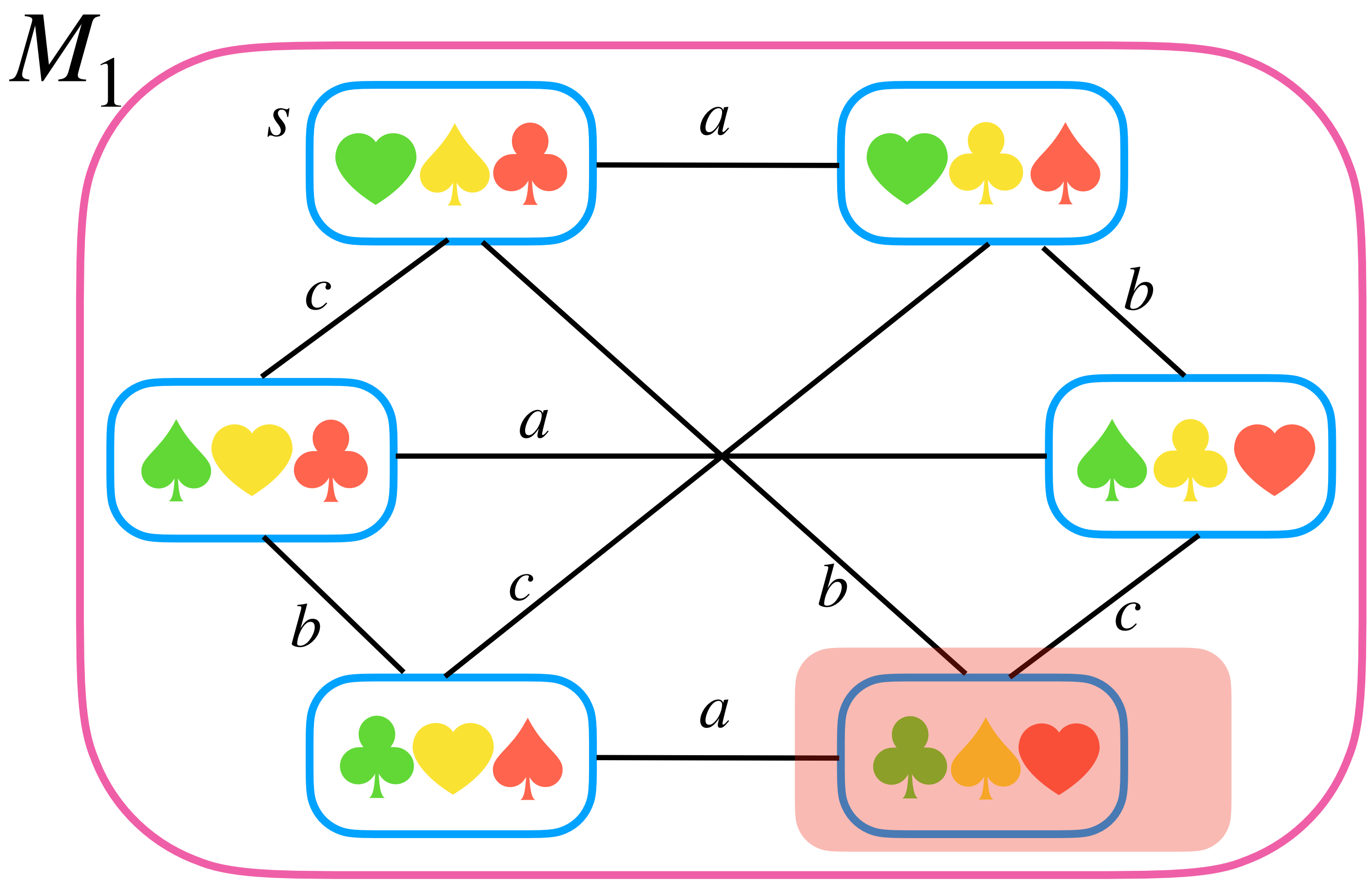
After any announcement, Alice has one of the cards



$$M_s \vDash [\,!\,](\heartsuit_a \vee \clubsuit_a \vee \spadesuit_a)$$

Card Example

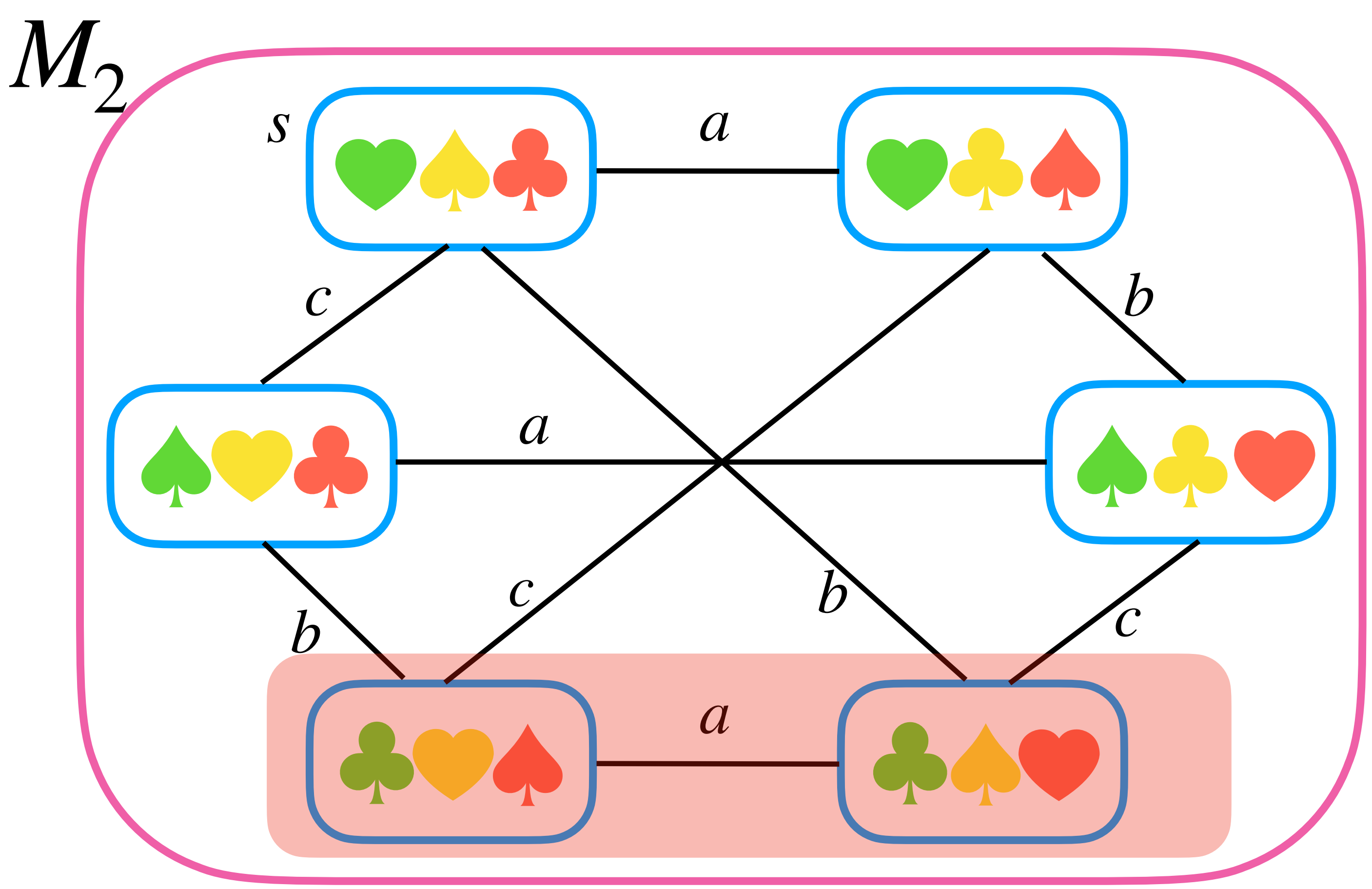
After any announcement, Alice has one of the cards



$$M_s \models [!](♥_a \vee ♣_a \vee ♠_a)$$

Card Example

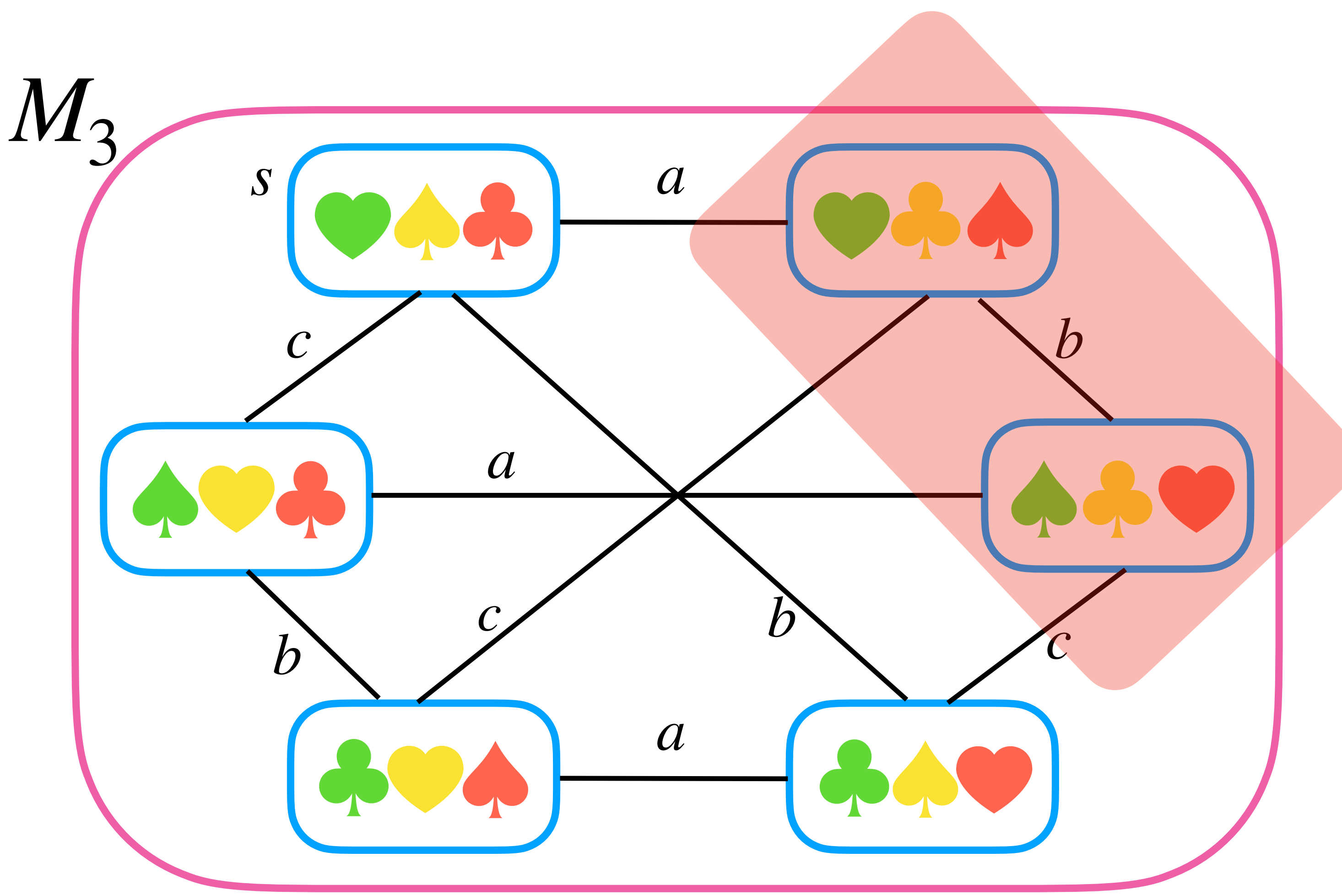
After any announcement, Alice has one of the cards



$$M_s \vDash [!](\heartsuit_a \lor \clubsuit_a \lor \spadesuit_a)$$

Card Example

After any announcement, Alice has one of the cards



$$M_s \models [\,!\,](\heartsuit_a \vee \clubsuit_a \vee \spadesuit_a)$$

Arbitrary PAL

Language of APAL

$$\mathscr{APAL} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\varphi]\varphi \mid [!]\varphi$$

Semantics

$$M_s \vDash [!]\varphi \text{ iff } \forall \psi \in \mathscr{PAL} : M_s \vDash [\psi]\varphi$$

$$M_s \vDash \langle ! \rangle\varphi \text{ iff } \exists \psi \in \mathscr{PAL} : M_s \vDash \langle \psi \rangle\varphi$$

Some validities

$$\langle \psi \rangle\varphi \to \langle ! \rangle\varphi \qquad [!]\varphi \to \varphi$$

$$\langle ! \rangle\varphi \leftrightarrow \langle ! \rangle\langle ! \rangle\varphi \qquad \langle ! \rangle[!]\varphi \leftrightarrow [!]\langle ! \rangle\varphi$$

Quantification is restricted to formulas of PAL in order to avoid circularity

Balbiani et al. *'Knowable' as 'Known After an Announcement'*, 2008.

Axiomatisation of APAL

Axioms of EL and PAL

$[\,!\,]\varphi \rightarrow [\psi]\varphi$ with $\psi \in \mathscr{PAL}$

From $\{\eta([\psi]\varphi) \mid \psi \in \mathscr{PAL}\}$

infer $\eta([\,!\,]\varphi)$

Infinitary number of premises

Open Problem I. Is there a finitary axiomatisation of APAL?

Theorem. APAL is more expressive than PAL

Theorem. APAL is sound and complete

Theorem. SAT-APAL is undecidable

Theorem. Complexity of MC-APAL is PSPACE-complete

Ågotnes et al. *Group announcement logic*, 2010.

French, Van Ditmarsch. *Undecidability for arbitrary public announcement logic*, 2008.

Balbiani, Van Ditmarsch. *A simple proof of the completeness of APAL*, 2015.

Backstabbing OP I

A logic has the finite model property (FMP) iff every formula of the logic that is true in some model is also true in a finite model

Finitary axiomatisation $\land$ FMP $\rightarrow$ Decidability

φ

Finitary axiomatisation

Finding the proof of $\neg\varphi$

If successful, φ is not satisfiable

FMP

Looking for a finite model of φ

If successful, φ is satisfiable

Urquhart. *Decidability and the Finite Model Property*, 1981.

Backstabbing OP I

A logic has the finite model property (FMP) iff every formula of the logic that is true in some model is also true in a finite model

Finitary axiomatisation $\wedge$ FMP $\rightarrow$ Decidability

$\equiv$

$\neg$Decidability $\rightarrow$ $\neg$Finitary axiomatisation $\vee$ $\neg$FMP

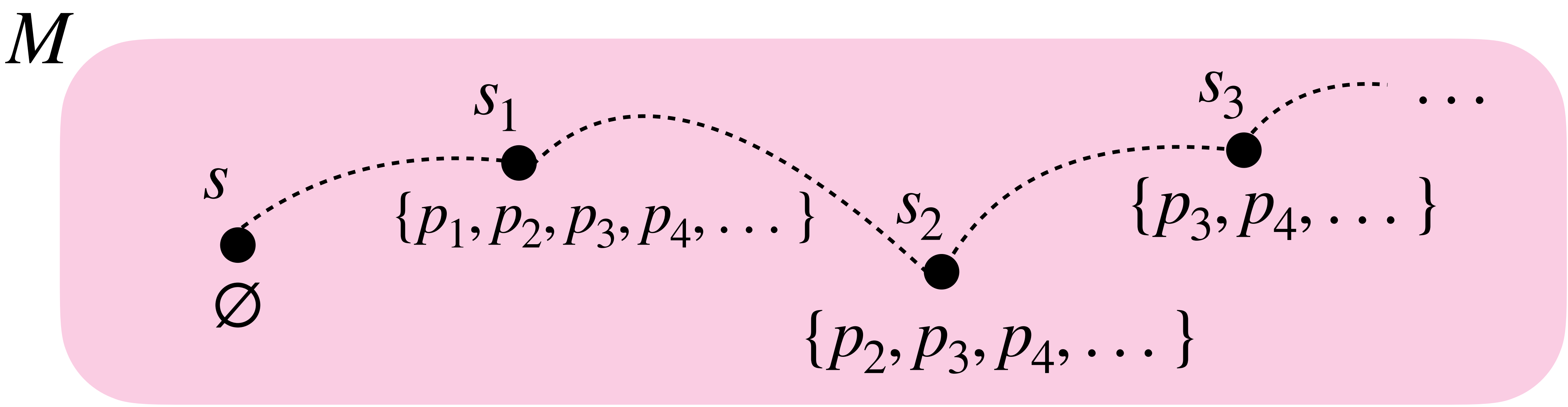
APAL is undecidable. If we show that APAL has the FMP, then we will know that it is not finitely axiomatisable…

Urquhart. *Decidability and the Finite Model Property*, 1981.

No FMP for APAL

$[!]\varphi$ is quite powerful as it quantifies over formulas with all propositional variables (even those not explicitly present in φ) and over formulas of arbitrary finite modal depth

However, it is not powerful enough to pick out all interesting submodes of a model



Example. Try removing all states apart from s using only propositional announcements

French, Van Ditmarsch, RG. *No Finite Model Property for Logics of Quantified Announcements*, 2021.

Back to OP I

$\neg$Decidability $\rightarrow$ $\neg$Finitary axiomatisation $\vee$ $\neg$FMP

Open Problem I. Is there a finitary axiomatisation of APAL?

French, Van Ditmarsch. *Undecidability for arbitrary public announcement logic*, 2008.
Urquhart. *Decidability and the Finite Model Property*, 1981.
French, Van Ditmarsch, RG. *No Finite Model Property for Logics of Quantified Announcements*, 2021.

Part III

Introduction to Group and Coalition Announcement Logics

Open Problems II and III and their partial solutions

Letting agents do the work

APAL allows quantification over all announcements

However, it does not specify whether such announcements can be made by any agent modelled in a system

$\langle G \rangle \varphi$: There is a truthful simultaneous announcement by agents from group G, such that φ is true after it

$[G]\varphi$: Whatever agents from group G truthfully and simultaneously announce, φ is true after it

Truthful part

$$\varphi_a := \Box_a \varphi$$

Simultaneous part

$$\varphi_G := \bigwedge_{a \in G} \varphi_a$$

Group Announcement Logic

Language of GAL

$$\mathcal{GAL} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\varphi]\varphi \mid [G]\varphi$$

Semantics

$$M_s \models [G]\varphi \text{ iff } \forall \psi_G \in \mathcal{PAL} : M_s \models [\psi_G]\varphi$$

$$M_s \models \langle G\rangle\varphi \text{ iff } \exists \psi_G \in \mathcal{PAL} : M_s \models \langle\psi_G\rangle\varphi$$

Some validities

$$\langle\psi_G\rangle\varphi \to \langle G\rangle\varphi \qquad [G]\varphi \to \varphi$$

$$\langle G\rangle\langle H\rangle\varphi \to \langle G \cup H\rangle\varphi \qquad \langle G \cup H\rangle\varphi \nrightarrow \langle G\rangle\langle H\rangle\varphi$$

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Ågotnes et al. *Group announcement logic*, 2010.

Axiomatisation of GAL

Axioms of EL and PAL

$[G]\varphi \to [\psi_G]\varphi$ with $\psi_G \in \mathscr{PAL}$

From $\{\eta([\psi_G]\varphi) \mid \psi_G \in \mathscr{PAL}\}$
infer $\eta([G]\varphi)$

Open Problem I. Is there a finitary axiomatisation of GAL?

Theorem. GAL lacks the FMP

Theorem. GAL is more expressive than PAL

Theorem. GAL is sound and complete

Theorem. SAT-GAL is undecidable

Theorem. Complexity of MC-GAL is PSPACE-complete

Ågotnes et al. *Group announcement logic*, 2010.
French, Van Ditmarsch, RG. *No Finite Model Property for Logics of Quantified Announcements*, 2021.
Ågotnes, French, Van Ditmarsch. *The Undecidability of Quantified Announcements*, 2016.

Strategic setting

In GAL only a specified group of agents makes an announcement

Following the lead of ATL, we can think of group announcements as one-step strategies to achieve an epistemic goal no matter what opponents do at the same time

$\langle [G] \rangle \varphi$: There is a truthful simultaneous announcement by agents from coalition G, such that no matter what agents in the anti-coalition announce at the same time, φ is true

$[\langle G \rangle] \varphi$: Whatever agents from coalition G announce, there is a counter-announcement by the anti-coalition, such that φ is true

Ågotnes, Van Ditmarsch. *Coalitions and Announcements*, 2008.
Alur, Henzinger, Kupferman. *Alternating-time Temporal Logic*, 2002.

Coalition Announcement Logic

Language of CAL

$$\mathscr{CAL} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\varphi]\varphi \mid [\langle G \rangle]\varphi$$

Semantics

$$M_s \models [\langle G \rangle]\varphi \text{ iff}$$
$$\forall \psi_G \exists \chi_{A \setminus G} : M_s \models \psi_G \rightarrow \langle \psi_G \wedge \chi_{A \setminus G} \rangle \varphi$$
$$M_s \models \langle [G] \rangle\varphi \text{ iff}$$
$$\exists \psi_G \forall \chi_{A \setminus G} : M_s \models \psi_G \wedge [\psi_G \wedge \chi_{A \setminus G}] \varphi$$

Some validities

$$\neg\langle[\varnothing]\rangle\neg\varphi \rightarrow \langle[A]\rangle\varphi \quad \langle[G]\rangle\varphi \rightarrow [\langle A \setminus G \rangle]\varphi$$

$$\langle[G]\rangle \top$$

$$\langle[G]\rangle[\langle H \rangle]\varphi \nrightarrow [\langle H \rangle]\langle[G]\rangle\varphi$$

RG. *Coalition and Relativised Group Announcement Logic*, 2021.
Ågotnes, Van Ditmarsch. *Coalitions and Announcements*, 2008.

Axiomatisation of CAL

Theorem. CAL lacks the FMP

Open Problem II. Is there an axiomatisation, finitary or infinitary, of CAL?

Theorem. CAL is more expressive than PAL

Theorem. SAT-CAL is undecidable

Theorem. Complexity of MC-CAL is PSPACE-complete

Alechina et al. *Verification and Strategy Synthesis for Coalition Announcement Logic*, 2021.
French, Van Ditmarsch, RG. *No Finite Model Property for Logics of Quantified Announcements*, 2021.
Ågotnes, French, Van Ditmarsch. *The Undecidability of Quantified Announcements*, 2016.

Why OP II is hard

While proving completeness using the canonical model construction, one usually has to prove a Lindenbaum type lemma

Recall APAL

$M_s \models [!]\varphi$ iff $\forall \psi \in \mathscr{PAL} : M_s \models [\psi]\varphi$

$M_s \models \langle ! \rangle\varphi$ iff $\exists \psi \in \mathscr{PAL} : M_s \models \langle\psi\rangle\varphi$

$[!]\varphi$ →

MCS

$[!]\varphi \to [\psi_1]\varphi$

$[!]\varphi \to [\psi_2]\varphi$

$[!]\varphi \to [\psi_3]\varphi$

$\dots$

Instances of an axiom schema

Why OP II is hard

While proving completeness using the canonical model construction, one usually has to prove a Lindenbaum type lemma

Recall APAL

$$M_s \models [!]\varphi \text{ iff } \forall \psi \in \mathscr{PAL} : M_s \models [\psi]\varphi$$

$$M_s \models \langle ! \rangle\varphi \text{ iff } \exists \psi \in \mathscr{PAL} : M_s \models \langle \psi \rangle\varphi$$

MCS $[!]\varphi$

$[\psi_1]\varphi$

$[\psi_2]\varphi$

$[\psi_3]\varphi$

$\ldots$

By closure under MP

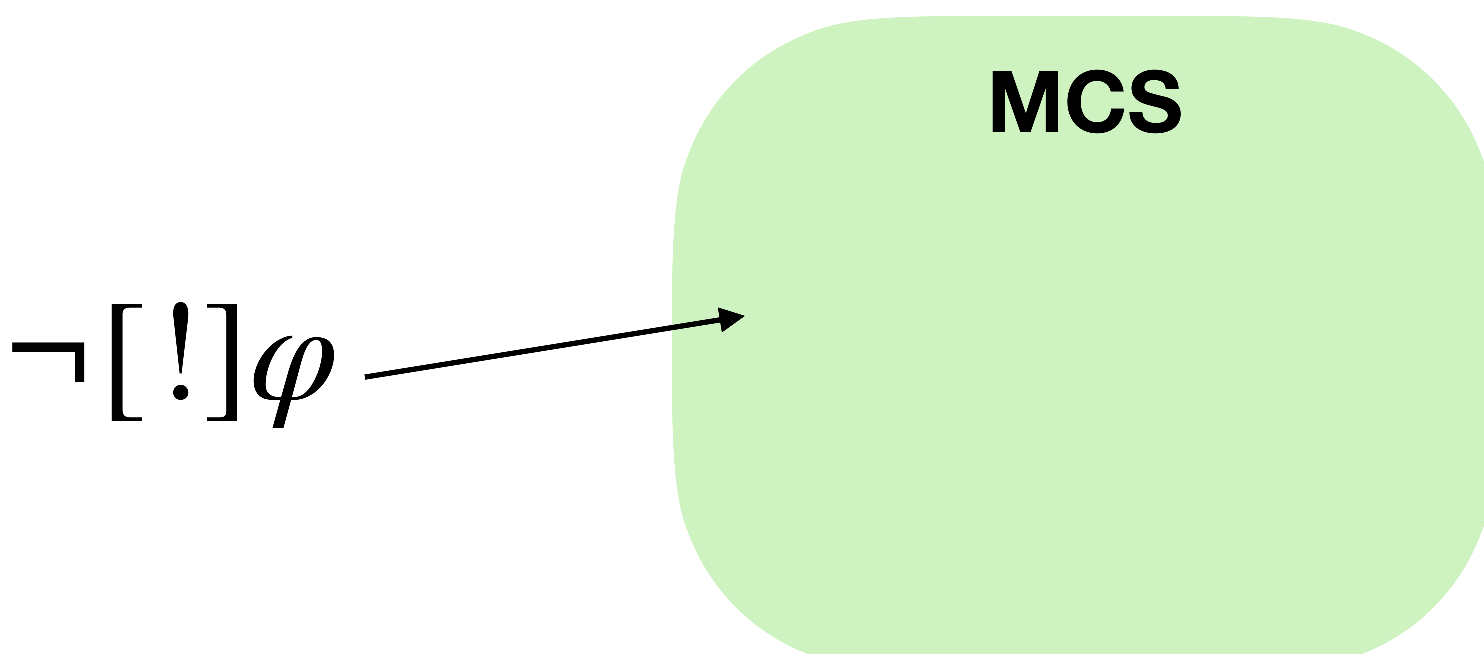
Why OP II is hard

While proving completeness using the canonical model construction, one usually has to prove a Lindenbaum type lemma

Recall APAL

$$M_s \vDash [!]\varphi \text{ iff } \forall \psi \in \mathcal{PAL} : M_s \vDash [\psi]\varphi$$

$$M_s \vDash \langle ! \rangle\varphi \text{ iff } \exists \psi \in \mathcal{PAL} : M_s \vDash \langle \psi \rangle\varphi$$



Add a witness

Why OP II is hard

While proving completeness using the canonical model construction, one usually has to prove a Lindenbaum type lemma

Recall APAL

$$M_s \models [!]\varphi \text{ iff } \forall \psi \in \mathscr{PAL} : M_s \models [\psi]\varphi$$

$$M_s \models \langle ! \rangle \varphi \text{ iff } \exists \psi \in \mathscr{PAL} : M_s \models \langle \psi \rangle \varphi$$

MCS

$$\neg [!]\varphi$$

$$\neg [\psi_n]\varphi$$

Add a witness

Why OP II is hard

While proving completeness using the canonical model construction, one usually has to prove a Lindenbaum type lemma

Recall CAL

$$M_s \models [\langle G \rangle]\varphi \text{ iff}$$
$$\forall \psi_G \exists \chi_{A \setminus G} : M_s \models \psi_G \rightarrow \langle \psi_G \wedge \chi_{A \setminus G} \rangle \varphi$$
$$M_s \models \langle [G] \rangle \varphi \text{ iff}$$
$$\exists \psi_G \forall \chi_{A \setminus G} : M_s \models \psi_G \wedge [\psi_G \wedge \chi_{A \setminus G}]\varphi$$

Note double quantification in both box and diamond operators

It is not clear how to deal with the double quantification

Why OP II is hard

While proving completeness using the canonical model construction, one usually has to prove a Lindenbaum type lemma

Recall CAL

$$M_s \models [\langle G \rangle]\varphi \text{ iff}$$
$$\forall \psi_G \exists \chi_{A \setminus G} : M_s \models \psi_G \to \langle \psi_G \wedge \chi_{A \setminus G} \rangle \varphi$$
$$M_s \models \langle [G] \rangle \varphi \text{ iff}$$
$$\exists \psi_G \forall \chi_{A \setminus G} : M_s \models \psi_G \wedge [\psi_G \wedge \chi_{A \setminus G}] \varphi$$

$[\langle G \rangle]\varphi$ → **MCS** ???

For each ψ_G there may be a unique corresponding χ_G

Why OP II is hard

While proving completeness using the canonical model construction, one usually has to prove a Lindenbaum type lemma

Recall CAL

$$M_s \vDash [\langle G \rangle]\varphi \text{ iff}$$
$$\forall \psi_G \exists \chi_{A \setminus G} : M_s \vDash \psi_G \to \langle \psi_G \wedge \chi_{A \setminus G} \rangle \varphi$$
$$M_s \vDash \langle [G] \rangle \varphi \text{ iff}$$
$$\exists \psi_G \forall \chi_{A \setminus G} : M_s \vDash \psi_G \wedge [\psi_G \wedge \chi_{A \setminus G}]\varphi$$

$\neg [\langle G \rangle]\varphi$ → **MCS** ???

We need to add an infinite number of witnesses

Partial Solution

We can use additional operators to split the quantification in CAL modalities

$[G, \chi]\varphi$: given a true announcement χ, whatever agents from coalition G announce in conjunction with χ, φ is true

$\langle G, \chi \rangle \varphi$: given any announcement χ, there is a simultaneous announcement by agents from coalition G, such that φ is true

Observe only single quantifiers

Formula χ is used as a placeholder (or memory) for announcements by a coalition

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Coalition and Relativised GAL

Language of CoRGAL

$$\mathscr{CORGAL} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\varphi]\varphi \mid [G, \varphi]\varphi \mid [\langle G \rangle]\varphi$$

Semantics

$$M_s \models [G, \chi]\varphi \text{ iff } \forall \psi_G : M_s \models \chi \wedge [\chi \wedge \psi_G]\varphi$$
$$M_s \models \langle G, \chi \rangle\varphi \text{ iff } \exists \psi_G : M_s \models \chi \to \langle \chi \wedge \psi_G \rangle\varphi$$
$$M_s \models [\langle G \rangle]\varphi \text{ iff } \forall \psi_G : M_s \models \langle A \backslash G, \psi_G \rangle\varphi$$
$$M_s \models \langle [G] \rangle\varphi \text{ iff } \exists \psi_G : M_s \models [A \backslash G, \psi_G]\varphi$$

Coalition operators now have only one quantifier

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Axiomatisation of CoRGAL

Axioms of EL and PAL

$[G,\chi]\varphi \to \chi \wedge [\psi_G \wedge \chi]\varphi$ with $\psi_G \in \mathscr{PAL}$

$[\langle G \rangle]\varphi \to \langle A \backslash G, \psi_G \rangle \varphi$ with $\psi_G \in \mathscr{PAL}$

From $\{\eta(\chi \wedge [\psi_G \wedge \chi]\varphi) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([G,\chi]\varphi)$

From $\{\eta(\langle A \backslash G, \psi_G \rangle) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([\langle G \rangle]\varphi)$

$[\langle G \rangle]\varphi$ → **MCS**

$[\langle G \rangle]\varphi \to \langle A \backslash G, \psi_G^1 \rangle \varphi$

$[\langle G \rangle]\varphi \to \langle A \backslash G, \psi_G^2 \rangle \varphi$

$[\langle G \rangle]\varphi \to \langle A \backslash G, \psi_G^3 \rangle \varphi$

$\ldots$

Instances of an axiom schema

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Axiomatisation of CoRGAL

Axioms of EL and PAL

$[G,\chi]\varphi \to \chi \land [\psi_G \land \chi]\varphi$ with $\psi_G \in \mathscr{PAL}$

$[\langle G \rangle]\varphi \to \langle A \backslash G, \psi_G \rangle \varphi$ with $\psi_G \in \mathscr{PAL}$

From $\{\eta(\chi \land [\psi_G \land \chi]\varphi) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([G,\chi]\varphi)$

From $\{\eta(\langle A \backslash G, \psi_G \rangle) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([\langle G \rangle]\varphi)$

MCS $[\langle G \rangle]\varphi$

$\langle A \backslash G, \psi_G^1 \rangle \varphi$

$\langle A \backslash G, \psi_G^2 \rangle \varphi$

$\langle A \backslash G, \psi_G^3 \rangle \varphi$

$\ldots$

Closure under MP

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Axiomatisation of CoRGAL

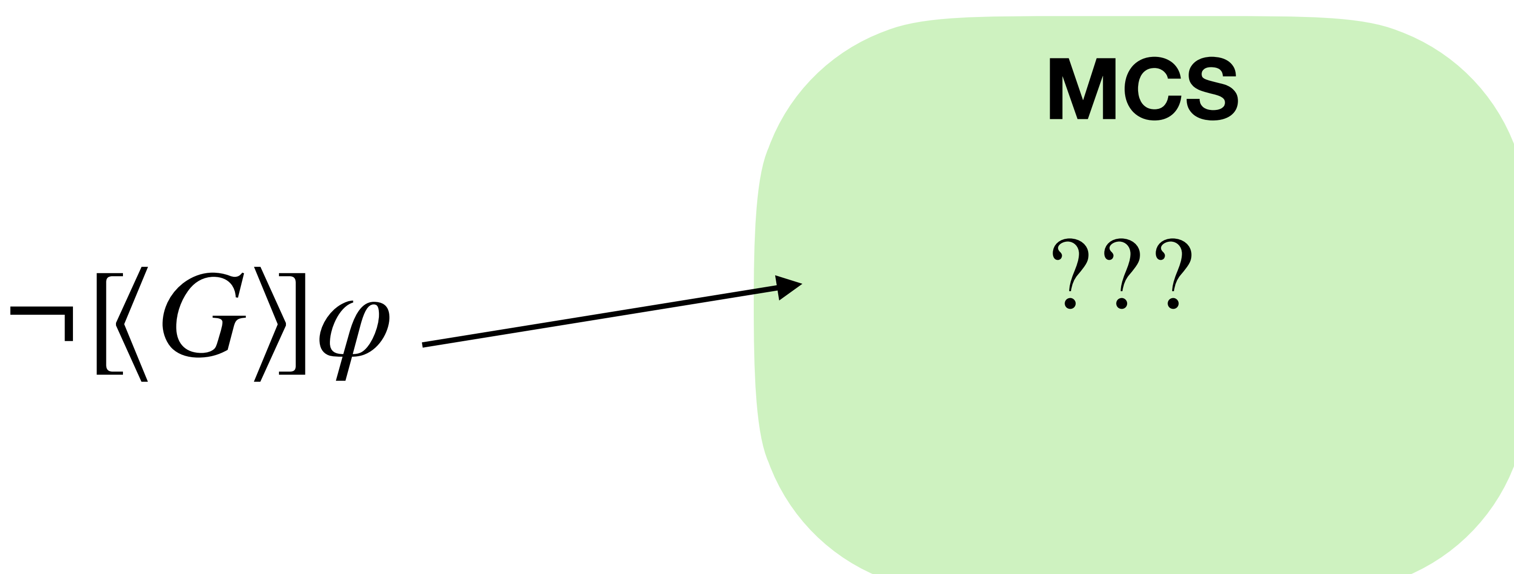
Axioms of EL and PAL

$[G,\chi]\varphi \to \chi \land [\psi_G \land \chi]\varphi$ with $\psi_G \in \mathscr{PAL}$

$[\![\langle G\rangle]\!]\varphi \to \langle A \backslash G, \psi_G\rangle\varphi$ with $\psi_G \in \mathscr{PAL}$

From $\{\eta(\chi \land [\psi_G \land \chi]\varphi) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([G,\chi]\varphi)$

From $\{\eta(\langle A \backslash G, \psi_G\rangle) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([\![\langle G\rangle]\!]\varphi)$



RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Axiomatisation of CoRGAL

Axioms of EL and PAL

$[G, \chi]\varphi \to \chi \wedge [\psi_G \wedge \chi]\varphi$ with $\psi_G \in \mathscr{PAL}$

$[\langle G \rangle]\varphi \to \langle A \backslash G, \psi_G \rangle \varphi$ with $\psi_G \in \mathscr{PAL}$

From $\{\eta(\chi \wedge [\psi_G \wedge \chi]\varphi) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([G, \chi]\varphi)$

From $\{\eta(\langle A \backslash G, \psi_G \rangle) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([\langle G \rangle]\varphi)$

MCS

$\neg [\langle G \rangle]\varphi$

$\neg \langle A \backslash G, \psi_G^n \rangle \varphi$

Add a witness

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Back to OP II

CoRGAL, a logic with coalition modalities, is sound and complete

Open Problem II. Is there an axiomatisation, finitary or infinitary, of CAL (without additional modalities)?

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Logics of Quantified Announcements

APAL. $[\,!\,]\varphi$: quantifies of all formulas of PAL

GAL. $[G]\varphi$: quantifies over $\psi_G := \bigwedge_{a \in G} \psi_a$ with

$$\psi_a := \Box_a \psi$$

CAL. $[\langle G \rangle]\varphi$: quantifies over ψ_G and $\chi_{A \setminus G}$

Open Problem III. Relative expressivity of APAL, GAL, and CAL

Partial results

APAL is incomparable with both GAL and CAL

APAL can force any* submodel of a given model, while GAL and CAL can force only G-definable submodels

Reasoning about GAL vs. CAL is a bit trickier…

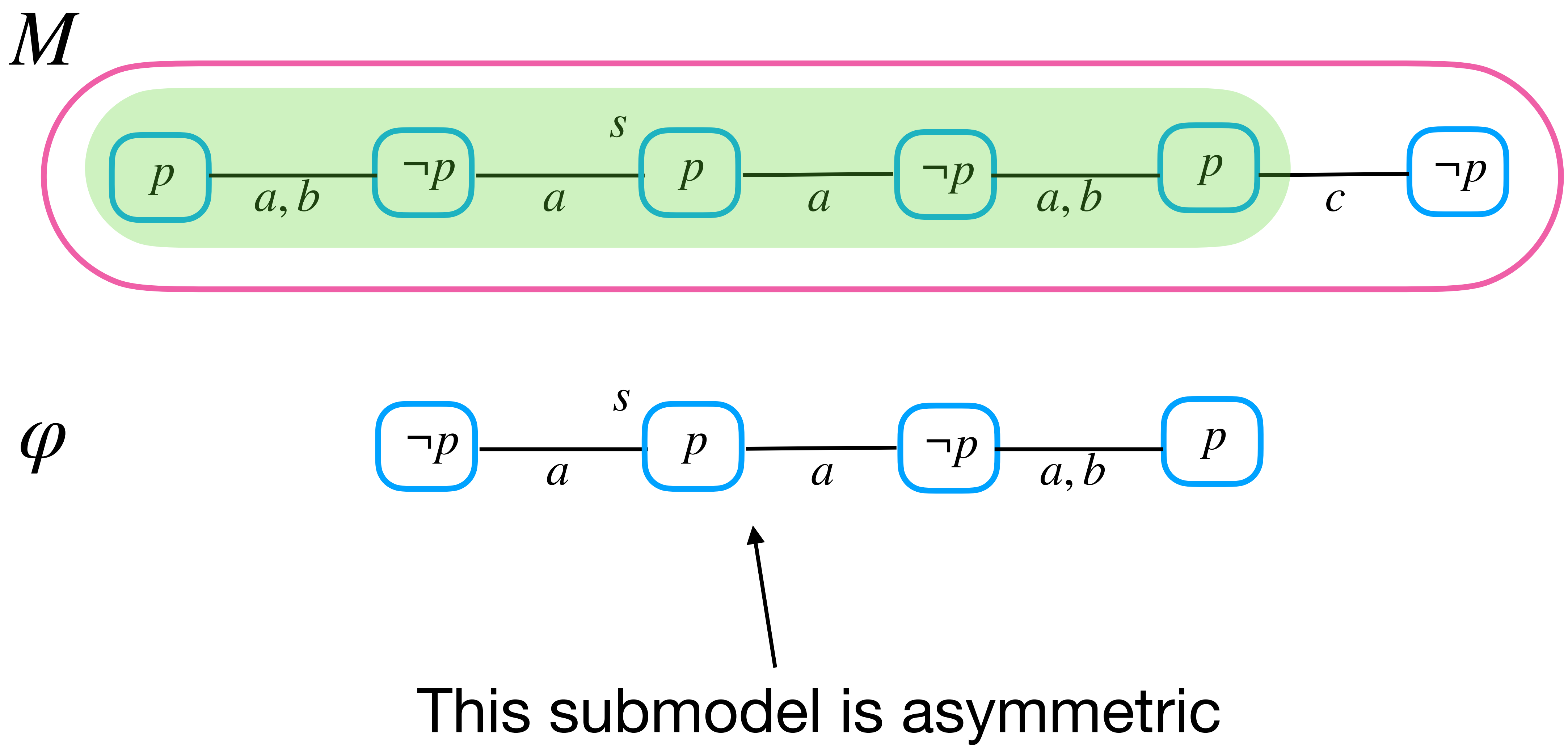
An intuitive definition of CAL modalities through GAL modalities

$$\langle\!\![G]\!\!\rangle\varphi \leftrightarrow \langle G\rangle[A\backslash G]\varphi$$

Ågotnes et al. *Group announcement logic*, 2010.
Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

Partial results

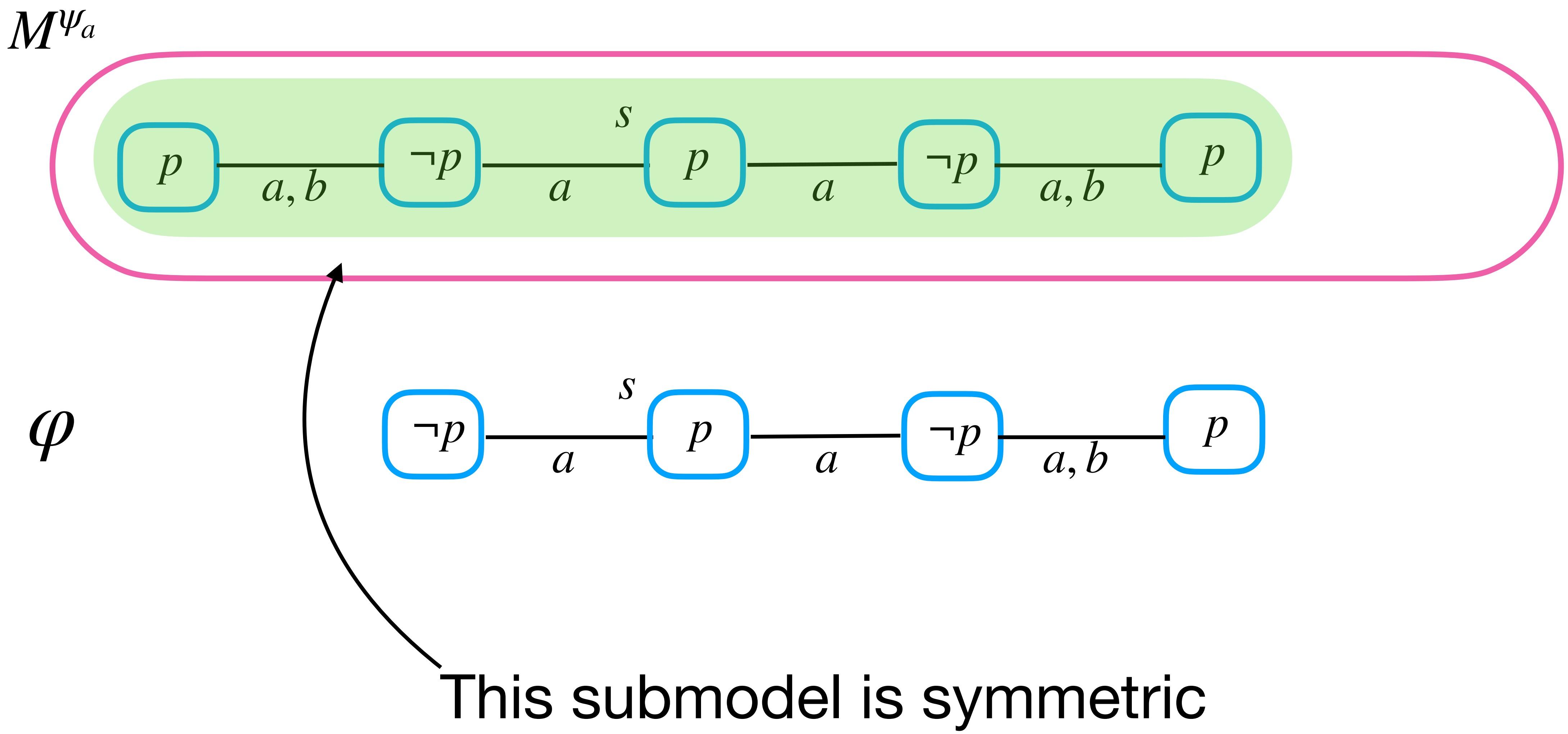
$$\langle a\rangle[b,c]\neg\varphi \rightarrow \langle\!\langle a\rangle\!\rangle\varphi$$



Ågotnes et al. *Group announcement logic*, 2010.
Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

Partial results

$$\langle a \rangle [b, c] \neg \varphi \rightarrow \langle\!\lbrack a \rbrack\!\rangle \varphi$$

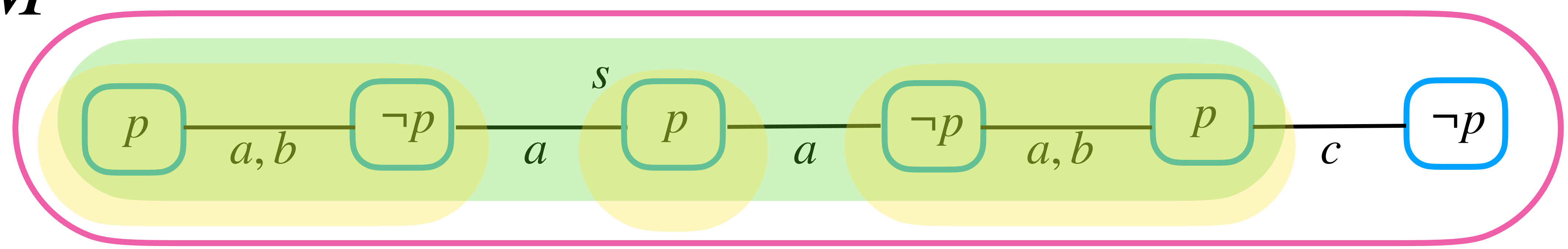


Ågotnes et al. *Group announcement logic*, 2010.
Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

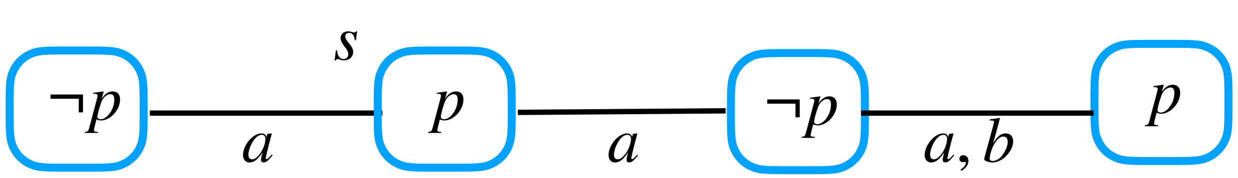
Partial results

$$\langle a \rangle [b, c] \neg \varphi \to \langle\!\![a]\!\!\rangle \varphi$$

M



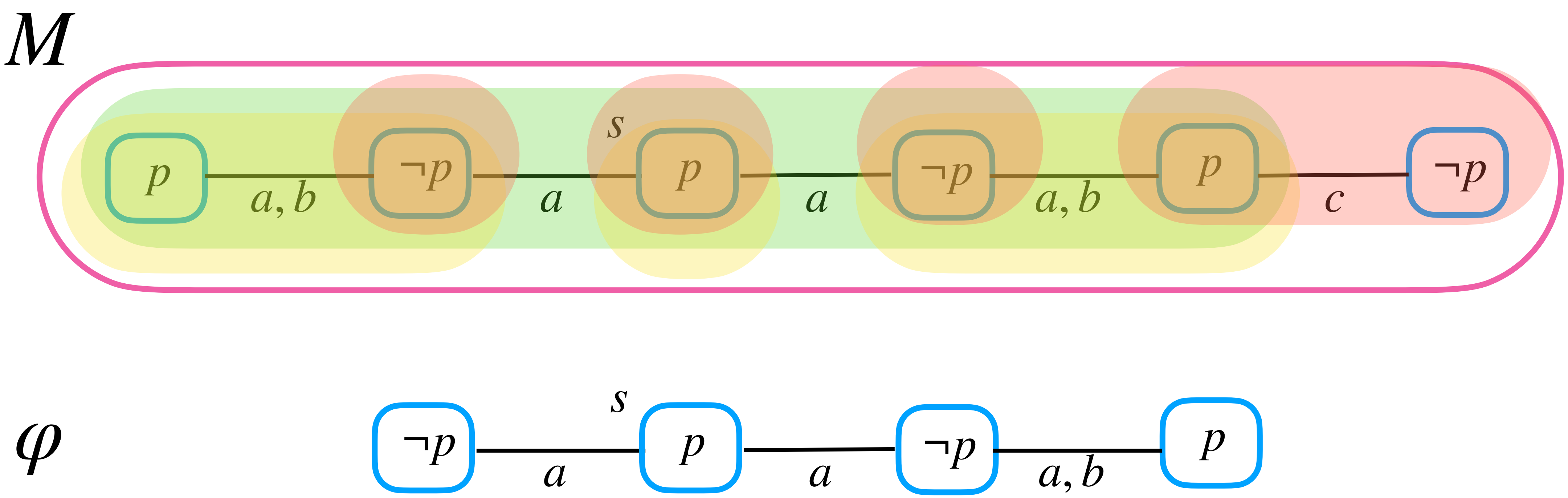
φ



Ågotnes et al. *Group announcement logic*, 2010.
Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

Partial results

$$\langle a\rangle[b,c]\neg\varphi \not\Rightarrow \langle\!\langle a\rangle\!\rangle\varphi$$



Ågotnes et al. *Group announcement logic*, 2010.
Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

Logics of Quantified Announcements

APAL is incomparable to GAL

There are some classes of models that GAL can distinguish and CAL cannot

There are some classes of models that APAL can distinguish and CAL cannot

Open Problem III (Refined). Are there classes of models that CAL can distinguish and APAL and GAL cannot?

Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

Recap of Open Problems

Open Problem I. Is there a finitary axiomatisation of APAL?

Partial Solution. APAL (and GAL and CAL) lack the FMP

Open Problem II. Is there an axiomatisation of CAL?

Partial Solution. There is an axiomatisation of a logic with CAL modalities (and relativised group announcements)

Open Problem III. Expressivity of APAL, GAL, and CAL

Partial Solution. CAL is not at least as expressive as GAL or APAL; APAL and GAL are incomparable