How Groups Can Help Coalitions

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LAMAS & SR 2021 (online), 3–4 May

The paper

The talk is based on

Coalition and Relativised Group Announcement Logic. In *Journal of Logic, Language and Information*. 2021.

PAL: The Existence of Santa

The Existence of Santa

Three children — Alice (a), Bobby (b), and Claire (c) — are arguing about the existence of Santa Claus. Alice knows that Santa exists (p), while Bobby and Claire consider it possible that Santa does not exist ($\neg p$). Also, children do not know about each other's beliefs.

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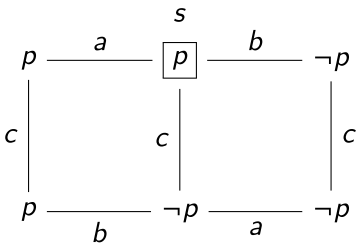

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$$M_s \models \Box_a p \wedge \neg \Box_b p$$
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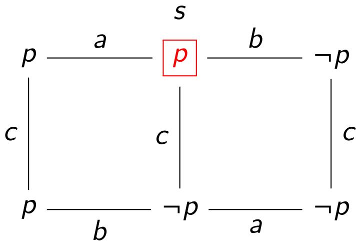

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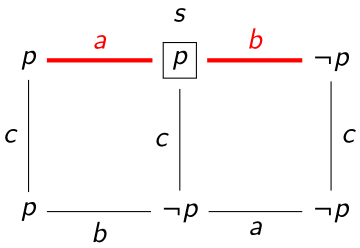

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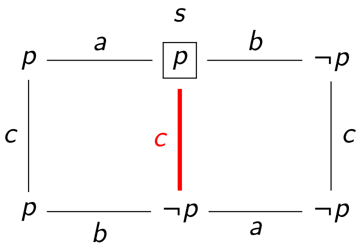

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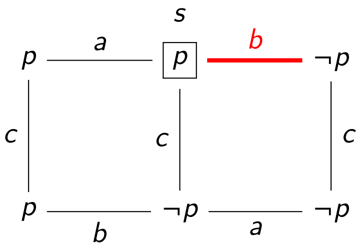

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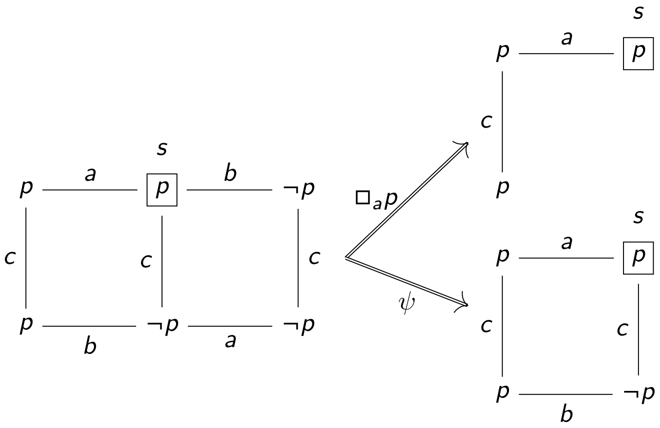
PAL: Learning about Santa

What agents know may change once new fact become known.
$\psi := \neg p \to \Diamond_c p$



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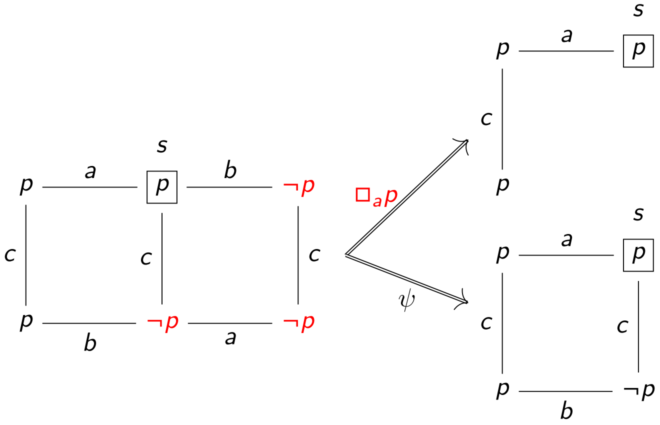

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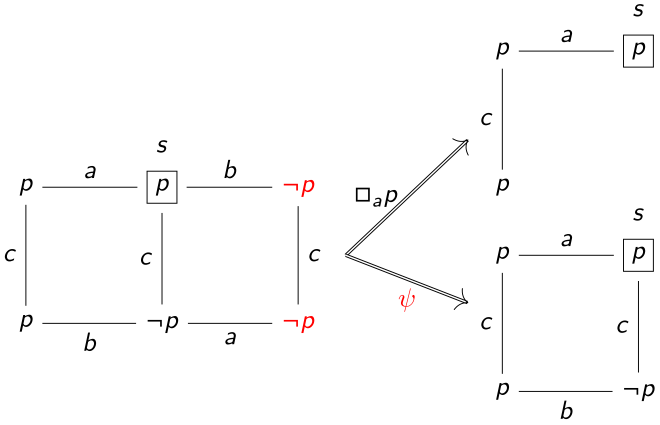

PAL: Syntax and Semantics

$$\mathcal{PAL} \ni \varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \Box_a \varphi \mid [\varphi]\varphi$$
$$\mathcal{EL} \ni \varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \Box_a \varphi$$

Definition (Semantics)

An announcement of φ in a pointed model $M_s = (S, \sim, V)$ results in an updated pointed model M_s^φ containing only φ-states:

- $S^\varphi = [\![\varphi]\!]_M$,
- $\sim_a^\varphi = \sim_a \cap (S^\varphi \times S^\varphi)$,
- $V^\varphi(p) = V(p) \cap S^\varphi$.

$$\begin{array}{lll} M_s \models \Box_a \varphi & \text{iff} & \forall t \in S : s \sim_a t \text{ implies } M_t \models \varphi \\ M_s \models [\varphi]\psi & \text{iff} & M_s \models \varphi \text{ implies } M_s^\varphi \models \psi \\ M_s \models \langle\varphi\rangle\psi & \text{iff} & M_s \models \varphi \text{ and } M_s^\varphi \models \psi \end{array}$$

PAL: Properties

For each $\varphi \in \mathcal{PAL}$ there is an equivalent $t(\psi) \in \mathcal{EL}$, where t is a translation function.

Theorem

PAL is sound and complete.

Theorem

PAL and EL are equally expressive.

GAL: (Almost) Strategic Santa

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Reconsider the Santa example from a more strategic point of view. Can Alice inform everyone that Santa does exist? Can she inform only Bobby and leave Claire in ignorance?

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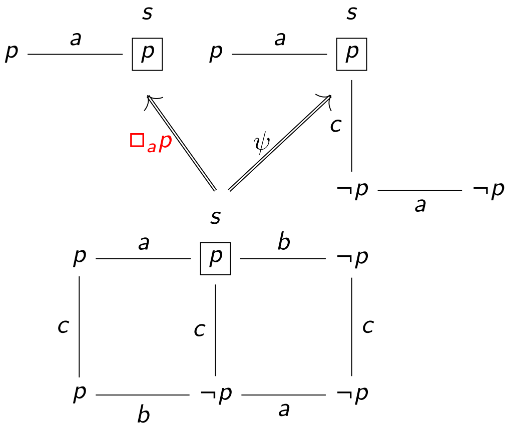

GAL: (Almost) Strategic Santa

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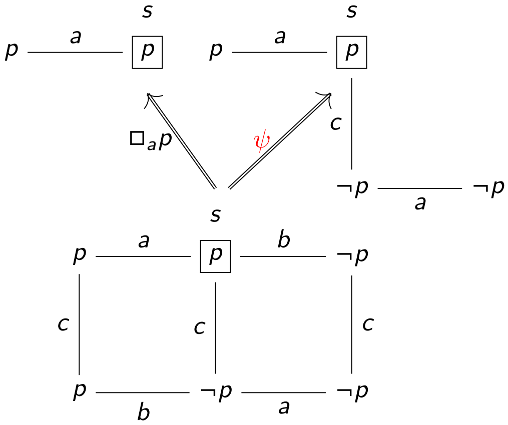
Reconsider the Santa example from a more strategic point of view.

Can she inform only Bobby and leave Claire in ignorance?

$\psi := \Box_a((p \to \Box_b p \wedge \Box_c p) \wedge (\neg p \to \Box_b \neg p \wedge \Box_c \neg p))$



GAL: Syntax and Semantics

Group Announcement Logic (GAL) = PAL + $\{[G]\varphi, \langle G\rangle\varphi\}$

$\langle G\rangle\varphi$: 'agents from G have a joint announcement such that φ holds in the resulting model'

$[G]\varphi$: 'whatever agents from G announce, they cannot avoid φ'

Let $\psi_G := \bigwedge_{a\in G} \Box_a\psi_a$, where ψ_a is an epistemic formula (truthfulness).

Definition (Semantics)

$$M_s \models [G]\varphi \quad \text{iff} \quad \forall \psi_G : M_s \models [\psi_G]\varphi$$
$$M_s \models \langle G\rangle\varphi \quad \text{iff} \quad \exists \psi_G : M_s \models \langle\psi_G\rangle\varphi$$

GAL: Properties

PAL	$[G]\varphi \to [\psi_G]\varphi$
From $\forall \psi_G : \eta([\psi_G]\varphi)$ infer $\eta([G]\varphi)$	From φ infer $[G]\varphi$

The axiomatisation is infinitary

Theorem

GAL is sound and complete.

Properties

$\langle G \rangle \langle G \rangle \varphi \leftrightarrow \langle G \rangle \varphi$ A multi-step strategy is reducible to a single-step strategy.
$\langle G \cup H \rangle \varphi \not\to \langle G \rangle \langle H \rangle \varphi$ It is not the case for different groups
$\langle G \rangle [H] \varphi \not\to [H] \langle G \rangle \varphi$ Confluence does not hold

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Can Alice inform only Bobby that Santa exists if other children are allowed to spoil her announcements?

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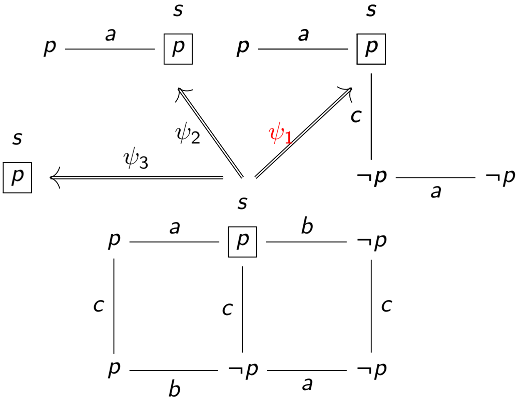
Strategic Santa

Can Alice inform only Bobby that Santa exists if other children are allowed to spoil her announcements? This is up to Bobby and Claire. They may keep silent (announce $\top$).

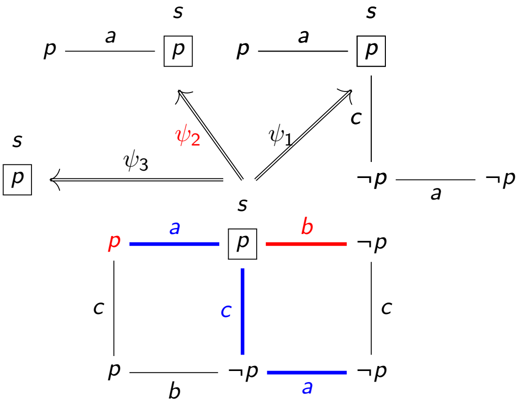
$\psi_1 := \psi_a \wedge \Box_b \top \wedge \Box_c \top$



CAL: Strategic Santa

Strategic Santa

Can Alice inform only Bobby that Santa exists if other children are allowed to spoil her announcements? This is up to Bobby and Claire. Bobby may announce ψ_b. $\psi_2 := \psi_a \wedge \psi_b$



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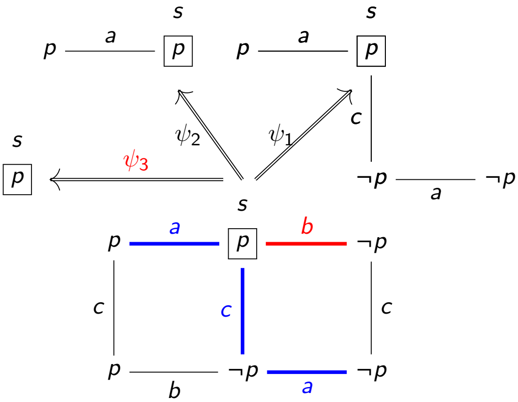


CAL: Syntax and Semantics

Coalition Announcement Logic (CAL) = PAL + $\{[\langle G \rangle]\varphi, \langle [G] \rangle\varphi\}$

$\langle [G] \rangle\varphi$: 'there exists a joint announcement by the agents from G such that no matter what other agents announce at the same time, φ holds '

$[\langle G \rangle]\varphi$: 'whatever agents from G announce, there is a simultaneous announcement by the anti-coalition such that φ holds'

Let $\psi_G := \bigwedge_{a \in G} \Box_a \psi_a$, where ψ_a is an epistemic formula.

Definition (Semantics)

$$M_s \vDash [\langle G \rangle]\varphi \quad \text{iff} \quad \forall \psi_G \exists \chi_{\overline{G}} : M_s \vDash \psi_G \to \langle \psi_G \land \chi_{\overline{G}} \rangle \varphi$$
$$M_s \vDash \langle [G] \rangle\varphi \quad \text{iff} \quad \exists \psi_G \forall \chi_{\overline{G}} : M_s \vDash \psi_G \land [\psi_G \land \chi_{\overline{G}}] \varphi$$

CAL: Properties

Properties

$\langle\!\![G \cup H]\!\!\rangle\varphi \not\to \langle\!\![G]\!\!\rangle\langle\!\![H]\!\!\rangle\varphi$ Agents spoil each other's strategies
$\langle\!\![G]\!\!\rangle[\underline{\langle} H\rangle]\varphi \not\to [\langle H\rangle]\langle\!\![G]\!\!\rangle\varphi$ Confluence does not hold
$\langle G\rangle[\overline{G}]\varphi \not\to \langle\!\![G]\!\!\rangle\varphi$ Coalition announcements are not reducible to group ones

The axiomatisation is unknown. Problem: double quantification in $\langle\!\![G]\!\!\rangle\varphi$. We use special group announcements to break it down.

Relativised Group Announcements

$\langle G, \psi \rangle \varphi$: given announcement ψ, there is a simultaneous announcement by agents from G such that φ holds

$[G, \psi]\varphi$: given announcement ψ, whatever agents from G simultaneously announce, φ holds

Definition (Semantics)

$$M_s \models \langle G, \chi \rangle \varphi \quad \text{iff} \quad M_s \models \chi \Rightarrow \exists \psi_G : M_s \models \langle \psi_G \land \chi \rangle \varphi$$
$$M_s \models \langle\!\![G]\!\!\rangle \varphi \quad \text{iff} \quad \exists \psi_G : M_s \models [\overline{G}, \psi_G]\varphi$$

No double quantification anymore!

Coalition and Relativised Group Announcement Logic

$\text{CoRGAL} = \text{PAL} + \{\langle G, \psi \rangle \varphi, \langle\![G]\!\rangle \varphi\}$

PAL
$[G, \chi]\varphi \to [\psi_G \land \chi]\varphi$
$[\langle G \rangle]\varphi \to \langle \overline{G}, \psi_G \rangle \varphi$
From $\forall \psi_G : \eta(\chi \land [\psi_G \land \chi]\varphi)$ infer $\eta([G, \chi]\varphi)$
From $\forall \psi_G : \eta(\langle \overline{G}, \psi_G \rangle \varphi)$ infer $\eta([\langle G \rangle]\varphi)$

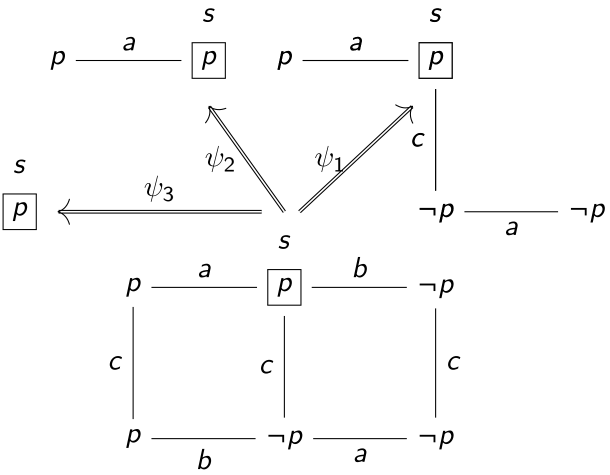
CoRGAL is sound and complete.

Completeness via extended models, where there are transitions for each public announcement from each state.

Definition (Semantics)

$$M_s \vDash \langle \psi \rangle \varphi \quad \text{iff} \quad \exists t \in S : s \xrightarrow{\psi} t \text{ and } M_t \vDash \varphi$$

Completeness Proof Idea



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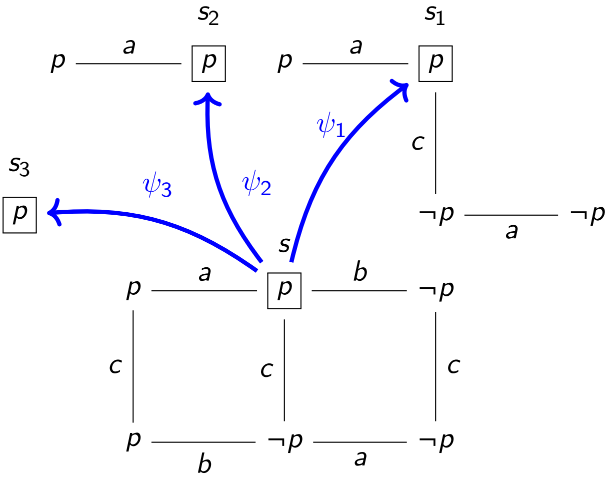

Overview

- CAL allows us to reason about what coalitions of agents are able to achieve through public communication in the presence of adversaries who may 'spoil' coalitions' announcements.
- We extended the language of CAL with relativised group announcements in order to split a coalition's announcement and the anti-coalition's response.
- The resulting logic is sound and complete. The only known complete axiomatisation that involves coalition announcements.
- Public announcements were treated as transitions in an extended model rather than model updates

CAL as a Strategic Logic

CAL was inspired by Coalition Logic (CL) but so far studied as a DEL. In (epistemic) CL modalities $\langle\langle G \rangle\rangle\varphi$ mean that there is an action by agents in coalition G that forces a φ-state no matter what $\overline{G}$ choose to do.

$\psi_1 := \psi_a \wedge \psi_b \wedge \psi_c$, $\psi_2 := \psi_a \wedge \psi_b \wedge \Box_c \top$