Logics with Group Announcements and Distributed Knowledge: Completeness and Expressive Power *

Thomas Ågotnes† Natasha Alechina‡ Rustam Galimullin§

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Abstract

Public announcement logic (PAL) is an extension of epistemic logic with dynamic operators that model the effects of all agents simultaneously and publicly acquiring the same piece of information. One of the extensions of PAL, group announcement logic (GAL), allows quantification over (possibly joint) announcements made by agents. In GAL, it is possible to reason about what groups can achieve by making such announcements. It seems intuitive that this notion of coalitional ability should be closely related to the notion of distributed knowledge, the implicit knowledge of a group. Thus, we study the extension of GAL with distributed knowledge, and in particular possible interaction properties between GAL operators and distributed knowledge. The perhaps surprising result is that, in fact, there are no interaction properties, contrary to intuition. We make this claim precise by providing a sound and complete axiomatisation of GAL with distributed knowledge. We also consider several natural variants of GAL with distributed knowledge, as well as some other related logic, and compare their expressive power.

1 Introduction

There has recently been considerable interest in epistemic logics with quantifiers over information-changing actions. See, for example, formal systems proposed in [3, 7, 25, 10, 14], and a recent survey [11]. Arguably the most studied formalisms of this kind are extensions of *public announcement logic* (PAL) [29] with quantification over announcements. The notable extensions are arbitrary public announcement logic (APAL) [7], group announcement logic (GAL) [2], and coalition announcement logic (CAL) [3].

*This is an extended version of the LORI paper [21]. Compared to the latter, we consider two new ways of extending GAL with distributed knowledge, and expand Sections 3, 4, and 6. Sections 5 and 7 are entirely new.

†Southwest University, Chongqing, China, and University of Bergen, Bergen, Norway; thomas.agotnes@uib.no

‡University of Utrecht, Utrecht, The Netherlands; n.a.alechina@uu.nl

§University of Bergen, Bergen, Norway; rustam.galimullin@uib.no

APAL extends PAL with a modality that quantifies over all possible (truthful) announcements. In GAL, there are modalities for each group of agents G, and these modalities quantify over all possible joint announcements the group can make. Modalities of CAL are similar to those of GAL with the additional property that the agents outside of group G also make a simultaneous announcement. Thus GAL and CAL can be seen as logics of *coalitional ability* [28] in terms of epistemic consequences of publicly observable joint actions.

Distributed knowledge [16] is a standard notion of group knowledge that captures the total, combined, knowledge in a group. While there has been a renewed interest in the dynamics of distributed knowledge [32, 1, 5], extensions of logics of quantifed actions with distributed knowledge have not been studied in detail. In this paper we investigate some extensions of the latter type, with the focus on the interaction of group announcements and distributed knowledge.

In addition to filling the obvious gap in the literature, the main motivation for studying these two modalities is their apparent connectedness: distributed knowledge is often understood as a state of knowledge agents in a group have the ability to bring about if they share their individual knowledge [16]. Careful analysis of this intuition [26, 30, 5] shows that this strong relationship does not always hold. This fact only begs the question of what exactly the relationship between the two types of modalities is, particularly in the form of interaction axioms.

We start out by considering several intuitively plausible candidates for such interaction axioms, and show that none of them are actually valid. Then we show that in fact, contrary to intuition, *there are no (non-trivial) interaction axioms at all*: the axiom system obtained by the independent combination of axioms for epistemic logic with distributed knowledge and GAL is *complete*.

From the semantical perspective, adding distributed knowledge to GAL (and some other logics of quantified announcements) is not straightforward, since it allows for three meaningful extensions. The first extension, which we denote as GALD$^{pa\text{-}D}$, is the most conservative one. Note that in GAL, under the standard assumption, there is a public announcement operator for each formula in the language. In GALD$^{pa\text{-}D}$ we do not add any new public announcement operators. Syntactically, this means that formulas with distributed knowledge do not occur inside public announcement operators.

The next two extensions deal with the domain of quantification of group announcements. One of them, GALD, keeps the semantics of group announcement operators as in GAL, i.e. they quantify over the purely epistemic language[1]. Compared to GALD$^{pa\text{-}D}$, in GALD we have *more* public announcement operators since formulas with distributed knowledge are allowed to occur inside of them.

Finally, in the third extension, GALD^{ga+D}, we change the semantics of group announcements to allow agents to announce formulas with distributed knowledge[2]. In order to get

[1] The quantification range of group announcements does not include formulas with group announcements to avoid circularity. We also exclude public announcements for simplicity, since they do not add expressivity to epistemic logic [29].

[2] The point made above regarding public announcements still hold: epistemic logic with distributed

GALD^{ga+D}, we substitute classic group announcements in GALD with this new type of operators.

One of the reasons to consider the three variants is to understand the significance of distributed knowledge in the context of public communication. As it turns out, the relationship between the variants is quite surprising, as their relative expressive power is not strictly increasing. For example, the very conservative fragment $\mathrm{GALD}^{pa\text{-}D}$ can capture some properties of models that cannot be captured by the seemingly more expressive GALD^{ga+D}. On the other hand, sound and complete axiomatisations of each of these three variants are quite similar.

Having dealt with group announcements and distributed knowledge, we also briefly consider two other logical variants. First, instead of the classic distributed knowledge, which falls short of capturing the intuition of 'pooling knowledge together', we discuss extending GAL with *resolved distributed knowledge* [5] that is a better approximation of the intuition. Second, we consider extending CAL, which is quite different from GAL in many aspects, with distributed knowledge, and propose some preliminary results. These results indicate that, at least expressivity-wise, there are some parallels between GAL and CAL.

The paper is organised as follows. In the next section we set the stage by defining the syntax and semantics of GAL with distributed knowledge, including the variants mentioned above, as well as some other background information and preliminary results. In Section 3 we look at some potential interaction axioms relating group announcements and group knowledge. In Section 4 we present a Hilbert-style axiomatic system for group announcement logic with distributed knowledge, and show that it is sound and complete. In Section 5 we investigate the relative expressive power of $\mathrm{GALD}^{pa\text{-}D}$, GALD, and GALD^{ga+D}. Operators for resolving distributed knowledge are discussed in Section 6, and CAL with distributed knowledge is considered in Section 7. We conclude in Section 8.

2 Syntax and Semantics

In this section, we introduce the main logical languages and semantics we consider in the paper, together with some additional basic tools.

2.1 Languages

All languages in the paper are defined relative to a finite set of agents A and a countable set of propositional variables P. The distinctions between the following languages correspond partly to the subtle distinctions in semantics discussed in the introduction and will be clearer shortly.

Definition 1 (Languages)**.** Languages $\mathcal{L}_{\mathrm{EL}}$ of *epistemic logic*, $\mathcal{L}_{\mathrm{PAL}}$ of *public announcement logic*, $\mathcal{L}_{\mathrm{GAL}}$ of *group announcement logic*, and extensions thereof with distributed

knowledge and public announcements is equally expressive as the one without public announcements [32].

knowledge, are recursively defined by the following grammars:

$$
\begin{array}{ll}
\mathcal{L}_{\mathrm{EL}} & \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \\
\mathcal{L}_{\mathrm{ELD}} & \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid D_G\varphi \\
\mathcal{L}_{\mathrm{PAL}} & \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid [\varphi]\varphi \\
\mathcal{L}_{\mathrm{PALD}} & \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid D_G\varphi \mid [\varphi]\varphi \\
\mathcal{L}_{\mathrm{GAL}} & \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid [\varphi]\varphi \mid [G]\varphi \\
\mathcal{L}_{\mathrm{GALD}^{ga+D}} & \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid D_G\varphi \mid [\varphi]\varphi \mid [G]^{\triangle}\varphi \\
\mathcal{L}_{\mathrm{GALD}} & \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid D_G\varphi \mid [\varphi]\varphi \mid [G]\varphi \\
\mathcal{L}_{\mathrm{GALD}^{pa\text{-}D}} & \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid D_G\varphi \mid [\psi]\varphi \mid [G]\varphi
\end{array}
$$

where $p \in P$, $\psi \in \mathcal{L}_{\mathrm{GAL}}$, $a \in A$, $G \subseteq A$, and all the usual abbreviations of propositional logic (e.g. $\varphi \vee \psi$, $\varphi \to \psi$, and $\varphi \leftrightarrow \psi$) and conventions for deleting parentheses hold. Duals are defined as $\widehat{K}_a\varphi := \neg K_a \neg\varphi$, $\langle\varphi\rangle\psi := \neg[\varphi]\neg\psi$, $\langle G\rangle\varphi := \neg[G]\neg\varphi$, and $\langle G\rangle^{\triangle}\varphi := \neg[G]^{\triangle}\neg\varphi$.

The intuitive meaning of formulas is as follows: $K_a\varphi$ means that agent a knows that φ; $D_G\varphi$ means that G has distributed knowledge of φ (φ is true in the set of states that all agents in G consider possible); $[\varphi]\psi$ means that if φ is true, then after it is announced (and everyone's knowledge updated by removing states not satisfying φ), ψ is true; $[G]\varphi$ and $[G]^{\triangle}\varphi$ mean that after any joint announcement by agents in G of formulas they know, φ is true.

The quantification in the latter modalities are intended to be over conjunctions of formulas of $\mathcal{L}_{\mathrm{EL}}$ for $[G]$ and $\mathcal{L}_{\mathrm{ELD}}$ for $[G]^{\triangle}$. The different scope of quantification is the reason we distinguish syntactically between the group modality $[G]^{\triangle}$ of $\mathcal{L}_{\mathrm{GALD}^{ga+D}}$ and $[G]$ of $\mathcal{L}_{\mathrm{GALD}}$. The meaning of $[G]$ is the same in $\mathcal{L}_{\mathrm{GALD}^{pa\text{-}D}}$ as in $\mathcal{L}_{\mathrm{GALD}}$, but note that in the former distributed knowledge formulas are not allowed inside public announcement operators. As discussed in the introduction, the distinction is relevant because $\mathcal{L}_{\mathrm{GALD}^{pa\text{-}D}}$ has exactly the same set of public announcement modalities as $\mathcal{L}_{\mathrm{GAL}}$.

2.2 Models and Bisimulation

Definition 2 (Epistemic Model)**.** An *epistemic model* M is a triple $(S, \sim, V)$, where S is a non-empty set of states, $\sim\colon A \to 2^{S\times S}$ assigns to each agent an equivalence relation, and $V : P \to 2^S$ is a valuation. For a group $G \subseteq A$, $\sim_G$ denotes $\bigcap_{a\in G} \sim_a$. If necessary, we refer to the elements of the tuple as S^M, $\sim^M$, and V^M. A model M with a designated state $s \in S$ is called a *pointed model* and denoted by M_s.

Model M is called *finite* if S is finite. Also, we write $M \subseteq N$ if $S^M \subseteq S^N$, $\sim^M$ and V^M are results of restricting $\sim^N$ and V^N to S^M, and call M a *submodel* of N.

Let $M_s = (S, \sim, V)$, and $X \subseteq S$ such that $X \neq \emptyset$. An *updated model* M_s^X is $(S^X, \sim^X, V^X)$, where $s \in X$, $S^X = X$, $\sim_a^X = \sim_a \cap (X \times X)$ for all $a \in A$, and $V^X(p) = V(p) \cap X$.

Definition 3 (Collective Bisimulation)**.** Let $M = (S^M, \sim^M, V^M)$ and $N = (S^N, \sim^N, V^N)$ be two models. A non-empty binary relation $Z \subseteq S^M \times S^N$ is called a *collective bisimulation* if and only if for all $s \in S^M$ and $u \in S^N$ with $(s, u) \in Z$:

- for all $p \in P$, $s \in V^M(p)$ if and only if $u \in V^N(p)$;
- for all $G \subseteq A$ and all $t \in S^M$: if $s \sim_G^M t$, then there is a $v \in S^N$ such that $u \sim_G^N v$ and $(t, v) \in Z$;
- for all $G \subseteq A$ and all $v \in S^N$: if $u \sim_G^N v$, then there is a $t \in S^M$ such that $s \sim_G^M t$ and $(t, v) \in Z$.

The notion of an *individual bisimulation* (or just *bisimulation*) is defined in exactly the same way as a collective bisimulation, except that the two last conditions are only required to hold for singleton groups G. Thus, a collective bisimulation is also an individual bisimulation, but not necessarily the other way around

If there is a bisimulation between models M and N linking states s and u, we say that M_s and N_u are bisimilar, and write $M_s \rightleftarrows N_u$. For collective bisimulation we write $M_s \rightleftarrows^C N_u$, and say that M_s and N_u are collectively bisimilar.

If a (collective) bisimulation between M_s and N_u is over $P \setminus \{p\}$ for some $p \in P$, we say that M_s and N_u are *(collectively) bisimilar except for* p.

Definition 4 (Bisimulation contraction)**.** Let $M = (S, \sim, V)$ be an epistemic model. The *bisimulation contraction* of M is the model $\|M\| = (\|S\|, \|\sim\|, \|V\|)$, where $\|S\| = \{[s] \mid s \in S\}$ and $[s] = \{t \in S \mid M_s \leftrightarrows M_t\}$, $[s]\|\sim\|_a[t]$ if and only if $\exists s' \in [s]$, $\exists t' \in [t]$ such that $s' \sim_a t'$ in M, and $[s] \in \|V\|(p)$ if and only if $\exists s' \in [s]$ such that $s' \in V(p)$.

Intuitively, the bisimulation contraction is the most compact representation of a model. It is a known result that $M_s \leftrightarrows \|M\|_{[s]}$ [24].

2.3 Semantics

Let $\mathcal{L}_{\mathrm{EL}}^G = \{\bigwedge_{i \in G} K_i \psi_i \mid \psi_i \in \mathcal{L}_{\mathrm{EL}}\}$ with typical elements ψ_G be the set of formulas describing individual knowledge of members of group G. Similarly, we fix $\mathcal{L}_{\mathrm{ELD}}^G = \{\bigwedge_{i \in G} K_i \psi_i \mid \psi_i \in \mathcal{L}_{\mathrm{ELD}}\}$.

Definition 5 (Semantics)**.** Let $M = (S, \sim, V)$, $s \in S$, and M_s be a pointed epistemic model. The *semantics* is defined recursively as follows :

$$
\begin{array}{lll}
M_s \models p & \text{iff} & s \in V(p) \\
M_s \models \neg\varphi & \text{iff} & M_s \not\models \varphi \\
M_s \models \varphi \wedge \psi & \text{iff} & M_s \models \varphi \text{ and } M_s \models \psi \\
M_s \models K_a\varphi & \text{iff} & M_t \models \varphi \text{ for all } t \in S \text{ such that } s \sim_a t \\
M_s \models D_G\varphi & \text{iff} & M_t \models \varphi \text{ for all } t \in S \text{ such that } s \sim_G t \\
M_s \models [\psi]\varphi & \text{iff} & M_s \models \psi \text{ implies } M_s^X \models \varphi, \text{ where } X = \{t \in S \mid M_t \models \psi\} \\
M_s \models [G]\varphi & \text{iff} & M_s \models [\psi_G]\varphi \text{ for all } \psi_G \in \mathcal{L}_{\mathrm{EL}}^G \\
M_s \models [G]^{\triangle}\varphi & \text{iff} & M_s \models [\psi_G]\varphi \text{ for all } \psi_G \in \mathcal{L}_{\mathrm{ELD}}^G
\end{array}
$$

Whenever $X = \{t \in S \mid M_t \models \psi\}$, we will write M_s^ψ for M_s^X. Observe that in order to avoid circularity, the quantification in the definition of the semantics of both group announcement operators $[G]\varphi$ and $[G]^\triangle\varphi$ is restricted to formulas without group announcement operators.

Formula φ is *valid* if and only if for any pointed model M_s it holds that $M_s \models \varphi$.

For convenience, let us also provide the semantics for the diamond versions of the announcement operators.

$$\begin{array}{lll} M_s \models \langle\psi\rangle\varphi & \text{iff} & M_s \models \psi \text{ and } M_s^X \models \varphi, \text{ where } X = \{t \in S \mid M_t \models \psi\} \\ M_s \models \langle G\rangle\varphi & \text{iff} & M_s \models \langle\psi_G\rangle\varphi \text{ for some } \psi_G \in \mathcal{L}_{\mathrm{EL}}^G \\ M_s \models \langle G\rangle^\triangle\varphi & \text{iff} & M_s \models \langle\psi_G\rangle\varphi \text{ for some } \psi_G \in \mathcal{L}_{\mathrm{ELD}}^G \end{array}$$

We will use GALD to refer to the logic with language $\mathcal{L}_{\mathrm{GALD}}$ and semantics as given above, and so on for the other logical languages we consider.

The next proposition states that collective bisimulation implies modal equivalence.

Proposition 1. Let $\varphi \in (\mathcal{L}_{\mathrm{GALD}} \cup \mathcal{L}_{\mathrm{GALD}^{ga+D}} \cup \mathcal{L}_{\mathrm{GALD}^{pa\text{-}D}})$, and M_s and N_t be epistemic models. If $M_s \rightleftarrows^C N_t$, then $M_s \models \varphi$ if and only if $N_t \models \varphi$.

Proof. By a straightforward induction following the corresponding proof for $\mathcal{L}_{\mathrm{ELD}}$ from [30]. $\square$

2.4 The Positive Fragment

Positive formulas can be considered as a particularly well behaved fragment of public announcement logic [15]. In particular, they remain true after an announcement.

Definition 6 (Positive Fragments)**.** *Positive fragments* of languages of group announcement logic with distributed knowledge are defined as follows:

$$\begin{array}{ll} \mathcal{L}^+_{\mathrm{GALD}^{ga+D}} & \varphi ::= p \mid \neg p \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid K_a\varphi \mid D_G\varphi \mid [\neg\varphi]\varphi \mid [G]^\triangle\varphi \\ \mathcal{L}^+_{\mathrm{GALD}} & \varphi ::= p \mid \neg p \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid K_a\varphi \mid D_G\varphi \mid [\neg\varphi]\varphi \mid [G]\varphi \\ \mathcal{L}^+_{\mathrm{GALD}^{pa\text{-}D}} & \varphi ::= p \mid \neg p \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid K_a\varphi \mid D_G\varphi \mid [\neg\psi]\varphi \mid [G]\varphi \end{array}$$

where $p \in P$, $\psi \in \mathcal{L}^+_{GAL}$, $a \in A$, and $G \subseteq A$. We will abbreviate $\mathcal{L}^+_{\mathrm{GALD}} \cup \mathcal{L}^+_{\mathrm{GALD}^{pa\text{-}D}} \cup \mathcal{L}^+_{\mathrm{GALD}^{ga+D}}$ as $\mathcal{L}^+$.

Definition 7 (Preservation)**.** A formula φ is *preserved* under submodels if and only if $M_s \models \varphi$ implies $N_s \models \varphi$ for any pointed models M_s and N_s such that $N_s \subseteq M_s$.

In the following proposition we show that formulas of any of the positive fragments remain true under submodels. Particularly, this means that if a positive formula is true in a model, then no matter what agents announce, the formula will remain true (see more on positive formulas in the context of quantified announcements in [12]).

Proposition 2. Formulas of $\mathcal{L}^+$ are preserved under submodels.

Proof. Let $M = (S^M, \sim^M, V^M)$ and $N = (S^N, \sim^N, V^N)$ be models such that $s \in S^M$, $s \in S^N$, and $N_s \subseteq M_s$. Boolean cases, case $K_a\varphi$, and case $[\neg\psi]\varphi$ are proved in [15, Proposition 8]. We show the remaining two cases $D_G\varphi$ and $[G]\varphi$.

Induction hypothesis. If $M_s \models \varphi$, then $N_s \models \varphi$.

Case $D_G\varphi$. Let $M_s \models D_G\varphi$. By the definition of semantics, this is equivalent to the fact that $M_t \models \varphi$ for all $t \in S^M$ such that $s \sim_G t$. The latter implies $M_t \models \varphi$ for every $t \in S^N$ such that $s \sim_G t$. By the induction hypothesis, we have that $N_t \models \varphi$ for all $t \in S^N$ such that $s \sim_G t$, which is equivalent to $N_s \models D_G\varphi$ by the semantics.

Case $[G]\varphi$. Assume towards a contradiction that $M_s \models [G]\varphi$ and $N_s \not\models [G]\varphi$. By the duality of group announcements, this is equivalent to $N_s \models \langle G\rangle\neg\varphi$, and by the definition of semantics, the latter is equivalent to $N_s \models \langle\psi_G\rangle\neg\varphi$ for some $\psi_G \in \mathcal{L}_{\mathrm{ELD}}$, which, in turn, equals to $N_s \models \psi_G$ and $N_s^{\psi_G} \not\models \varphi$. Now observe that $N_s^{\psi_G} \subseteq N_s \subseteq M_s$. From that and the contraposition of the induction hypothesis, it follows that $M_s \not\models \varphi$. However, $M_s \models [G]\varphi$ implies that $M_s \models [\bigwedge_{i\in G} K_i(p \vee \neg p)]\varphi$. It is immediate that $M_s \models [\bigwedge_{i\in G} K_i(p \vee \neg p)]\varphi$ is equivalent to $M_s \models \varphi$, which contradicts $M_s \not\models \varphi$.

Case $[G]^{\triangle}\varphi$. Similar to $[G]\varphi$. □

3 Ability, announcements, and group knowledge

Distributed knowledge is often described as potential individual (or even common) knowledge that members of a group can establish 'through communication' or by 'pooling their knowledge together'. However, this intuition is in fact not correct [5]. For example, a group can have distributed knowledge of a formula of the form $p \wedge \neg K_a p$ (sometimes called a *Moore sentence* [27]), which can never become individual knowledge in a group that contains agent a [5]. Nevertheless, that doesn't mean that there are no interaction properties between group announcements and group knowledge. We consider some candidates in this section. Note that all the properties mentioned below hold in GALD, GALD$^{pa\text{-}D}$ and GALD^{ga+D}, with $\langle G\rangle^{\triangle}\varphi$ substituted for $\langle G\rangle\varphi$, with the exception of Proposition 3.

It is known that the following potential axioms are not valid [2]:

- $\langle G\rangle\varphi \to D_G\langle G\rangle\varphi$
- $D_G\langle G\rangle\varphi \to \langle G\rangle D_G\varphi$

It is also known that the following are valid:

- $\langle G\rangle D_G\varphi \to D_G\langle G\rangle\varphi$ (implied by Proposition 28 of [2] and the fact that knowledge de re implies knowledge de dicto)
- $D_G\langle G\rangle\varphi \to \langle G\rangle\varphi$ (distributed knowledge is veridical)

Consider weaker properties which involve 'everybody knows' operator E_G, where $E_G\varphi := \bigwedge_{i\in G} K_i\varphi$. These properties encapsulate the intuition that distributed knowledge can be made explicit through public communication. It is known that the following is not valid:

- $D_G\varphi \to \langle G\rangle E_G\varphi$ (take $\varphi := p \wedge \neg K_a p$ where $a \in G$ [5])

The other direction also does not hold:

Fact 1. $\langle G\rangle E_G\varphi \to D_G\varphi$ is not valid.

Proof. Let $\varphi := K_b p \vee K_b \neg p$ and $\psi_{\{a,b\}} := K_a(p \to K_b p)$, and consider Figure 1.

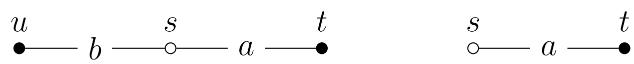

Figure 1: Models M (left) and $M^{\psi_{\{a,b\}}}$ (right). Propositional variable p is true in black states.

We have $M_s \models \langle\psi_{\{a,b\}}\rangle E_{\{a,b\}}\varphi$, which is equivalent to $M_s \models \psi_{\{a,b\}}$ and $M_s^{\psi_{\{a,b\}}} \models E_{\{a,b\}}\varphi$. On the other hand, it is easy to verify that $M_s \not\models D_{\{a,b\}}\varphi$ as the only $\sim_{\{a,b\}}$-accessible state is s itself, and $M_s \not\models \varphi$. $\square$

In general, distributed knowledge of a group cannot be made known to members of the group via public communication. This is contrary to the intuition that distributed knowledge is a kind of knowledge that members can attain through public communication. Thus it is interesting to know what are the requirements on formulas and models so that this intuition would be true. We argue that positive formulas can be made known on *bisimulation contracted models* (this restriction is not surprising given analysis in [26]).

Fact 2. $D_G\varphi \to \langle G\rangle E_G\varphi$ with $\varphi \in \mathcal{L}^+$ is valid on finite bisimulation contracted models.

Proof. Let $M_s \models D_G\varphi$ for an arbitrary finite bisimulation contracted M_s. Since distributed knowledge is veridical, the latter implies $M_s \models \varphi$. Now let us a consider the maximally informative announcement by agents from G. Since M_s is finite and bisimulation contracted, each state in the model can be uniquely described by a characteristic formula. Moreover, disjunctions of these formulas correspond to sets of states. Agents from G can announce characteristic formulas that describe their equivalence classes and include s, i.e. $\bigcap_{i\in G}[s]_i$ for all $i \in G$ (see [4, 22] for details). In the resulting model $M_s^{\psi_G}$, relation $\sim_G$ on set of states S^{ψ_G} is universal. Moreover, since $M_s \models D_G\varphi$ and φ is preserved under submodels, we have that $M_s^{\psi_G} \models E_G\varphi$, and, consequently, $M_s \models \langle G\rangle E_G\varphi$. $\square$

The restriction to finite bisimulation contracted models is essential in the previous proposition.

Fact 3. $D_G\varphi \to \langle G\rangle E_G\varphi$ with $\varphi \in \mathcal{L}^+$ is not valid.

Proof. Consider the model in Figure 2. It is easy to check that $M_s \models D_{\{a,b\}}p$. In order to see that $M_s \not\models \langle\{a,b\}\rangle E_{\{a,b\}}p$, observe that $M_s \leftrightarrows M_v$ and $M_t \leftrightarrows M_u$. Thus, any announcement by a that preserves $\{s,t\}$ also preserves $\{u,v\}$. The same holds for agent b and $\{s,u\}$ and $\{t,v\}$. $\square$

Finally, we consider the relation between $[G]\varphi$ and $[G]^{\triangle}\varphi$. Recall that $[G]$ is the standard group announcement operator, where announcements by agents belong to a fragment of epistemic logic without distributed knowledge. On the other hand, in $[G]^{\triangle}$ agents' announcements may include distributed knowledge. In Proposition 3 we show that if agents in group G cannot avoid φ by any announcement involving distributed knowledge, then they cannot avoid φ by announcing purely epistemic formulas. Also, we show that the other direction of this conditional is not valid, i.e. allowing agents to make announcements with distributed knowledge increases their ability to force certain submodels of a model.

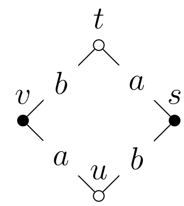

Figure 2: Model M. Propositional variable p is true in black states.

Proposition 3. Consider a unified language $\mathcal{L} = \mathcal{L}_{\mathrm{GALD}} \cup \mathcal{L}_{\mathrm{GALD}^{ga+D}}$. We have that $[G]^{\triangle}\varphi \to [G]\varphi$ is valid and $[G]\varphi \to [G]^{\triangle}\varphi$ is not valid.

Proof. Validity of $[G]^{\triangle}\varphi \to [G]\varphi$ follows from the fact that $\mathcal{L}_{\mathrm{EL}} \subseteq \mathcal{L}_{\mathrm{ELD}}$.

To show that $[G]\varphi \to [G]^{\triangle}\varphi$ is not valid we present a counterexample in Figure 3.

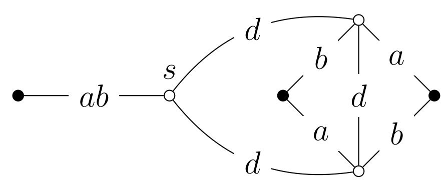

Figure 3: Model M. Propositional variable p is true in black states.

Also, consider two submodels of M depicted in Figure 4.

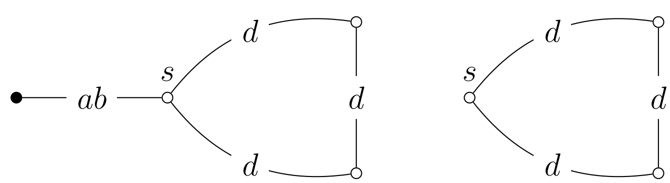

Figure 4: Submodels, N (left) and O (right), of model M.

Let $\varphi := \neg p \wedge \widehat{K}_d(K_a\neg p \wedge K_b\neg p) \wedge \widehat{K}_a K_d p \wedge \widehat{K}_b K_d p$, and consider formula $[\{d\}]\neg\varphi \in \mathcal{L}_{\mathrm{GALD}}$. Observe that $N_s \models \varphi$, $M_s \not\models \varphi$, and $O_s \not\models \varphi$.

In model M all black states are bisimilar to each other, and thus d does not have an announcement such that it would remove some of the black states and leave the other. Thus, the only possible submodels of M_s that d can enforce are O_s and M_s itself, which implies that $M_s \models [\{d\}]\neg\varphi$. On the other hand, $M_s \models \langle\{d\}\rangle^{\triangle}\varphi$; in particular, $M_s^{K_d\neg D_{\{a,b\}}p} \models \varphi$ since $K_d\neg D_{\{a,b\}}p$ holds in every state but two rightmost black ones, yielding N_s. $\square$

4 Proof System

In this section, we first provide an axiomatic system for GALD, and then prove that it is sound and complete. The system and the proof are then easily adapted to GALD$^{pa\text{-}D}$ and GALD^{ga+D}. The completeness proof is based on adaptions of definitions, results and proof techniques from [8, 32].

4.1 Axiomatisation of group announcement logic with distributed knowledge

Similarly to the axiomatisation of GAL, the system we provide here is infinitary (it contains rules with infinitely many premises), and it is defined using necessity forms [23].

Definition 8. *Necessity forms* are defined by the following grammar:

$$\eta(\sharp) ::= \sharp \mid \varphi \to \eta(\sharp) \mid K_a\eta(\sharp) \mid D_G\eta(\sharp) \mid [\varphi]\eta(\sharp)$$

where $\varphi \in \mathcal{L}_{\text{GALD}}$. The result of substituting φ for $\sharp$ in η is denoted by $\eta(\varphi)$.

Observe that $\sharp$ has a unique occurrence in $\eta(\sharp)$.

Definition 9. The axiomatisation of GALD comprises axiom systems for EL [16], PAL [13], GAL [2], and PALD [32].

$(A0)$	Propositional tautologies	$(A11)$	$[\varphi]p \leftrightarrow (\varphi \to p)$
$(A1)$	$K_a(\varphi \to \psi) \land K_a\varphi \to K_a\psi$	$(A12)$	$[\varphi]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi]\psi)$
$(A2)$	$K_a\varphi \to \varphi$	$(A13)$	$[\varphi](\psi \land \chi) \leftrightarrow ([\varphi]\psi \land [\varphi]\chi)$
$(A3)$	$K_a\varphi \to K_aK_a\varphi$	$(A14)$	$[\varphi]K_a\psi \leftrightarrow (\varphi \to K_a[\varphi]\psi)$
$(A4)$	$\neg K_a\varphi \to K_a\neg K_a\varphi$	$(A15)$	$[\varphi]D_G\psi \leftrightarrow (\varphi \to D_G[\varphi]\psi)$
$(A5)$	$D_G(\varphi \to \psi) \land D_G\varphi \to D_G\psi$	$(A16)$	$[\varphi][\psi]\chi \leftrightarrow [\varphi \land [\varphi]\psi]\chi$
$(A6)$	$D_G\varphi \to \varphi$	$(A17)$	$[G]\varphi \to [\psi_G]\varphi,$ where $\psi_G \in \mathcal{L}^G_{\text{EL}}$
$(A7)$	$D_G\varphi \to D_GD_G\varphi$	$(R0)$	From $\varphi \to \psi$ and $\varphi,$ infer ψ
$(A8)$	$\neg D_G\varphi \to D_G\neg D_G\varphi$	$(R1)$	From $\varphi,$ infer $K_a\varphi$
$(A9)$	$D_{\{a\}}\varphi \leftrightarrow K_a\varphi$	$(R2)$	From $\varphi,$ infer $[\psi]\varphi$
$(A10)$	$D_G\varphi \to D_H\varphi,$ if $G \subseteq H$	$(R3)$	From $\{\eta([\psi_G]\varphi) \mid \psi_G \in \mathcal{L}^G_{\text{EL}}\},$ infer $\vdash \eta([G]\varphi)$

We denote by **GALD** the smallest set that contains all instances of $A0$–$A17$ and is closed under $R0$–$R3$. Elements of **GALD** are called *theorems*.

Lemma 1. Rule $R3$ is truth-preserving.

Proof. The proof is by induction on the construction of η. Let M_s be a pointed epistemic model. We show only the case $D_H\eta(\sharp)$, and other cases are similar.

Case $D_H\eta(\sharp)$. Let $M_s \models D_H\eta([\psi_G]\varphi)$ for all $\psi_G \in \mathcal{L}^G_{\text{EL}}$. By the semantics this means that for every ψ_G, $M_t \models \eta([\psi_G]\varphi)$ for all t such that $s \sim_H t$. Pick any t such that $s \sim_H t$. By the induction hypothesis we have $M_t \models \eta([G]\varphi)$. Since t was arbitrary, $M_t \models \eta([G]\varphi)$ for all t such that $s \sim_H t$. The latter is equivalent to $M_s \models D_H\eta([G]\varphi)$. □

Theorem 1. The axiomatisation of GALD is sound.

Proof. Follows from the soundness of PALD [32], GAL [2], and Lemma 1. □

4.2 Completeness

Following the technique from [32, 33], we prove the completeness of GALD by making a detour through pre- and pseudo models, where distributed knowledge operators are treated as classic knowledge modalities.

Definition 10 (Pre- and pseudo models)**.** An *epistemic pre-model* is a tuple $\mathcal{M} = (S, \sim, V)$, where $\sim$ maps every agent a and every subset $G \subseteq A$ to an element of $2^{S\times S}$. A pre-model is called a *pseudo model* (and is written $\mathfrak{M}$) if for all a it holds that $\sim_{\{a\}} = \sim_a$, and for all $G, H \subseteq A$: if $G \subseteq H$, then $\sim_H \subseteq \sim_G$.

Next, we define theories that will be used for the construction of the canonical model.

Definition 11 (Theories)**.** A set x of formulas of $\mathcal{L}_{\text{GALD}}$ is called a *theory*, if it contains all theorems and is closed under $R0$ and $R3$. A theory is *consistent* if for all φ, $\varphi \wedge \neg\varphi \notin x$. A theory is called *maximal* if for all φ, either $\varphi \in x$ or $\neg\varphi \in x$. **GALD** is the smallest theory, and $\mathcal{L}_{\text{GALD}}$ is the largest theory.

Theories are not required to be closed under $R1$ and $R2$ since these rules of inference, unlike $R0$ and $R3$, preserve only validity and not truth.

Lemma 2. Let x be a theory, and $\varphi, \psi \in \mathcal{L}_{\text{GALD}}$. The following are theories: $x + \varphi = \{\psi \mid \varphi \to \psi \in x\}$, $K_a x = \{\varphi \mid K_a\varphi \in x\}$, $D_G x = \{\varphi \mid D_G\varphi \in x\}$, and $[\varphi]x = \{\psi \mid [\varphi]\psi \in x\}$.

Proof. Cases for $x + \varphi$, $K_a x$, $[\varphi]x$ are proved in [7, Lemma 4.11]. Here we argue that $D_G x$ is a theory.

We need to show that $D_G x$ contains **GALD** and is closed under $R0$ and $R3$. Let $\varphi \in$ **GALD**. Then we also have that $D_G\varphi \in$ **GALD** by the necessitation of D_G, which is derivable in PALD [32]. Since x is a theory, and hence **GALD** $\subseteq x$, we have that $D_G\varphi \in x$, and $\varphi \in D_G x$. This establishes that **GALD** $\subseteq D_G x$.

Assume that $\varphi \to \psi, \varphi \in D_G x$. By $A5$ and $R0$ this implies that $D_G\psi \in x$, or, equivalently, $\psi \in D_G x$.

Suppose that $\eta([\psi_H]\varphi) \in D_G x$ for all $\psi_H \in \mathcal{L}^H_{\text{EL}}$. This means that $D_G\eta([\psi_H]\varphi) \in x$ for all ψ_H, and from the fact that $D_G\eta(\sharp)$ is a necessity form, we conclude by $R3$ that $D_G\eta([H]\varphi) \in x$. Finally, by the definition of $D_G x$ we yield $\eta([H]\varphi) \in D_G x$. □

Lemma 3. For all consistent theories x, $\neg\varphi \notin x$ if and only if $x + \varphi$ is consistent.

Lemma 4 (Theorem 2.5.2 of [23])**.** Every consistent theory can be extended to a maximal consistent theory.

Definition 12 (Canonical pseudo model)**.** The *canonical pseudo model* is the tuple $\mathfrak{M}^C = (S^C, \sim^C, V^C)$, where $S^C = \{x \mid x \text{ is maximal consistent theory}\}$, $x \sim^C_a y$ if and only if $K_a x \subseteq y$, $x \sim^C_G y$ if and only if for all $H \subseteq G$ it holds that $D_H x \subseteq y$, and $V^C(p) = \{x \in S^C \mid p \in x\}$.

For the rest of the section, we employ the following strategy. First, we prove the truth lemma for the canonical pseudo model. Next, we unravel $\mathfrak{M}^C$ into the tree-like pre-model $\mathcal{M}^C$, which satisfies the same GALD formulas as $\mathfrak{M}^C$. After that, we fold $\mathcal{M}^C$ into the model M^C. Folding is a truth-preserving operation, and hence we will be able to conclude the completeness of GALD.

Definition 13 (Size Relation)**.** The $|\cdot|$-*size* and d-*depth* of $\varphi \in \mathcal{L}_{\text{GALD}}$ are defined as follows:

$$
\begin{array}{ll}
|p| = 1 & d(p) = 0 \\
|\neg\varphi| = |K_a\varphi| = |D_G\varphi| = & d(\neg\varphi) = d(K_a\varphi) = d(D_G\varphi) = d(\varphi) \\
\quad = |[G]\varphi| = |\varphi| + 1 & d(\varphi \wedge \psi) = \max\{d(\varphi), d(\psi)\} \\
|\varphi \wedge \psi| = \max\{|\varphi|, |\psi|\} + 1 & d([\psi]\varphi) = d(\psi) + d(\varphi) \\
|[\psi]\varphi| = |\psi| + 3 \cdot |\varphi| & d([G]\varphi) = d(\varphi) + 1
\end{array}
$$

The binary relation $<_d^{|\cdot|}$ between $\varphi, \psi \in \mathcal{L}_{\text{GALD}}$ is defined as follows:

$\varphi <_d^{|\cdot|} \psi$ iff $d(\varphi) < d(\psi)$, or $d(\varphi) = d(\psi)$ and $|\varphi| < |\psi|$. The relation is a well-founded strict partial order between formulas. Note that for all $\psi \in \mathcal{L}_{\text{PALD}}$ we have that $d(\psi) = 0$.

Lemma 5. Let $\varphi, \chi \in \mathcal{L}_{\text{GALD}}$.

1. $\varphi <_d^{|\cdot|} \neg\varphi$,
2. $\varphi <_d^{|\cdot|} \varphi \wedge \psi$,
3. $\varphi <_d^{|\cdot|} K_a\varphi$,
4. $\varphi <_d^{|\cdot|} D_G\varphi$,
5. $(\varphi \to p) <_d^{|\cdot|} [\varphi]p$,
6. $(\varphi \to \neg[\varphi]\psi) <_d^{|\cdot|} [\varphi]\neg\psi$,
7. $([\varphi]\psi \wedge [\varphi]\chi) <_d^{|\cdot|} [\varphi](\psi \wedge \chi)$,
8. $[\varphi \wedge [\varphi]\chi]\psi <_d^{|\cdot|} [\varphi][\chi]\psi$,
9. $(\varphi \to K_a[\varphi]\psi) <_d^{|\cdot|} [\varphi]K_a\psi$,
10. $(\varphi \to D_G[\varphi]\psi) <_d^{|\cdot|} [\varphi]D_G\psi$,
11. $[\psi_G]\varphi <_d^{|\cdot|} [G]\varphi$,
12. $[\chi][\psi_G]\varphi <_d^{|\cdot|} [\chi][G]\varphi$.

Lemma 6. Let x be a theory. If $D_G\varphi \notin x$, then there is a maximal consistent theory y such that $D_Gx \subseteq y$ and $\varphi \notin y$.

Proof. Assume that $D_G\varphi \notin x$. This means that $\varphi \notin D_Gx$, and hence $D_Gx + \neg\varphi$ is a consistent theory by Lemma 3. By Lemma 4, $D_Gx + \neg\varphi$ can be extended to a maximal consistent theory y. Since $\neg\varphi \in y$, by consistency we have that $\varphi \notin y$. □

Lemma 7. Let x be a theory. If $K_a\varphi \notin x$, then there is a maximal consistent theory y such that $K_ax \subseteq y$ and $\varphi \notin y$.

Proof. Similar to the proof of Lemma 6. □

In the following, we use satisfaction with respect to pre- and pseudo models. The definition of pre- and pseudo semantics is exactly like the definition of normal semantics, where group relations $\sim_G$ are treated as primitive relations. We will use the same symbol, $\models$, for all three versions of satisfaction, since it is clear which one is employed from the font used for models (M, $\mathcal{M}$, or $\mathfrak{M}$).

Lemma 8. For all formulas φ and maximal consistent theories x it holds that $\mathfrak{M}^C_x \models \varphi$ if and only if $\varphi \in x$.

Proof. The proof is by induction on the size of φ. Boolean cases are straightforward, and cases with public announcements are dealt with using $A11$–$A16$. Here we show only cases with distributed knowledge and group announcements.

Case $D_G\varphi$. ($\Rightarrow$): Let $\mathfrak{M}^C_x \models D_G\varphi$. By the semantics we have that for all $y \in S^C$: $x \sim^C_G y$ implies $\mathfrak{M}^C_y \models \varphi$. By the definition of the canonical pseudo model, axiom $A10$, Lemma 5, and the induction hypothesis, the latter is equivalent to the fact that for all $y \in S^C$ and all $H \subseteq G$: $D_H x \subseteq y$ implies $\varphi \in y$. In particular, for all $y \in S^C$: $D_G x \subseteq y$ implies $\varphi \in y$. By the contraposition of Lemma 6 this implies that $D_G\varphi \in x$.

($\Leftarrow$): Assume that $D_G\varphi \in x$ and $x \sim^C_G y$ for some maximal consistent theory y. By $A7$ and $R0$ it holds that $D_G D_G\varphi \in x$. By the definition of the canonical model, we have that $D_G\varphi \in y$. Since y is a maximal consistent theory and thus contains $D_G\varphi \to \varphi$, it holds that $\varphi \in y$. Next, by the induction hypothesis we have that $\mathfrak{M}^C_y \models \varphi$. Since y was arbitrary, we have that $\mathfrak{M}^C_y \models \varphi$ for all y such that $x \sim^C_G y$. The latter is equivalent to $\mathfrak{M}^C_x \models D_G\varphi$ by the semantics.

Case $[\varphi]D_G\psi$. $\mathfrak{M}^C_x \models [\varphi]D_G\psi$ if and only if $\mathfrak{M}^C_x \models \varphi \to D_G[\varphi]\psi$ by the validity of $A15$. By Lemma 5 and the induction hypothesis, $\mathfrak{M}^C_x \models \varphi \to D_G[\varphi]\psi$ if and only if $\varphi \to D_G[\varphi]\psi \in x$ if and only if $[\varphi]D_G\psi \in x$ by $A15$.

Case $[\varphi][G]\psi$. ($\Rightarrow$): Let $\mathfrak{M}^C_x \models [\varphi][G]\psi$. By the semantics, $\mathfrak{M}^C_x \models [\varphi][\psi_G]\psi$ for all ψ_G. By Lemma 5 and the induction hypothesis, $[\varphi][\psi_G]\psi \in x$ for all ψ_G. Note that $[\varphi](\sharp)$ is a necessity form, hence, by $R3$, we have that $[\varphi][G]\psi \in x$.

($\Leftarrow$): Let $[\varphi][G]\psi \in x$. The distributivity rule for public announcements is derivable in PAL [13, Proposition 4.46]. Hence, by $A17$ and $R0$ it holds that $[\varphi][\psi_G]\psi \in x$. By Lemma 5 and the induction hypothesis we have that $\mathfrak{M}^C_x \models [\varphi][\psi_G]\psi$ for all ψ_G. By the semantics, $\mathfrak{M}^C_x \models \varphi$ implies $(\mathfrak{M}^C_x)^\varphi \models [\psi_G]\psi$, for every ψ_G. The latter is equivalent to the fact that $\mathfrak{M}^C_x \models \varphi$ implies $(\mathfrak{M}^C_x)^\varphi \models [G]\psi$, and thus $\mathfrak{M}^C_x \models [\varphi][G]\psi$.

Case $[G]\varphi$. ($\Rightarrow$): Let $\mathfrak{M}^C_x \models [G]\varphi$. By the semantics, this is equivalent to $\mathfrak{M}^C_x \models [\psi_G]\varphi$ for all ψ_G. By Lemma 5 and the induction hypothesis, for every ψ_G we have that $[\psi_G]\varphi \in x$, and by $R3$, $[G]\varphi \in x$.

($\Leftarrow$): Let $[G]\varphi \in x$. By the validity of $A17$, $[\psi_G]\varphi \in x$ for all ψ_G. By Lemma 5 and the induction hypothesis, $\mathfrak{M}^C_x \models [\psi_G]\varphi$ for every ψ_G, which is equivalent to $\mathfrak{M}^C_x \models [G]\varphi$ by the semantics. □

For the rest of the proof, we closely follow [32]. Since most of the remaining part involves transformation of the canonical model, group announcement operators do not play a role here. Hence, we just present main points of the transformation, and particular details can be found in the cited literature.

The canonical pseudo model $\mathfrak{M}^C$ can be unravelled into the treelike canonical pre-model $\mathcal{M}^C$. Such an operation preserves collective bisimulation.

Definition 14 (Folding)**.** Let $\mathcal{M} = (S, \sim, V)$ be a pre-model. The *folding* of $\mathcal{M}$ is the tuple $(S, \sim^*, V)$, where for all $a \in A$, $\sim^*_a$ is the transitive closure of $\sim^\cup_a = \sim_a \cup \bigcup\{\sim_G \mid a \in G\}$.

Folding of an unravelled tree-like pre-model yields an epistemic model.

Definition 15 (Trans-bisimulation)**.** Let $M = (S^M, \sim^M, V^M)$ be a model and $\mathcal{N} = (S^{\mathcal{N}}, \sim^{\mathcal{N}}, V^{\mathcal{N}})$ be a pre-model. A non-empty binary relation $Z \subseteq S^M \times S^{\mathcal{N}}$ is called a *trans-bisimulation* if and only if for all $s \in S^M$ and $u \in S^{\mathcal{N}}$ with $(s, u) \in Z$:

- for all $p \in P$, $s \in V^M(p)$ if and only if $u \in V^{\mathcal{N}}(p)$;
- for all $a \in A$ and all $t \in S^M$: if $s \sim_a^M t$ or $s \sim_{\{a\}}^M t$, then there is a $v \in S^{\mathcal{N}}$ such that $u \sim_{\nu_0} \ldots \sim_{\nu_n} v$, where ν_i is either a or G such that $a \in G$, and $(t, v) \in Z$;
- for all $G \subseteq A$ such that $|G| \geqslant 2$ and all $t \in S^M$: if $s \sim_G^M t$, then there is a $v \in S^{\mathcal{N}}$ such that $u \sim_H \ldots \sim_I v$ with $G \subseteq H \cap \ldots \cap I$, and $(t, v) \in Z$;
- for all ν among a and G and all $v \in S^{\mathcal{N}}$: if $u \sim_\nu^{\mathcal{N}} v$, then there is a $t \in S^M$ such that $s \sim_\nu^M t$, and $(t, v) \in Z$.

If there is a trans-bisimulation between model M and pre-model $\mathcal{N}$ linking states s and u, we say that M_s and $\mathcal{N}_u$ are trans-bisimilar, and write $M_s \rightleftarrows^T \mathcal{N}_u$.

Folding preserves trans-bisimulation. Before stating the completeness, we need one more result.

Lemma 9. Given M_s, $\mathcal{M}_t$, and $\mathfrak{M}_u$, if $M_s \leftrightarrows^T \mathcal{M}_t \leftrightarrows^C \mathfrak{M}_u$, then for all $\varphi \in \mathcal{L}_{\text{GALD}}$: $M_s \models \varphi$ if and only if $\mathcal{M}_t \models \varphi$.

Proof. The proof is by induction on φ. Boolean cases, cases for knowledge and distributed knowledge, and the case for public announcements are proved in [32, Lemma 26]. We show the case of $[G]\psi$.

Assume that $M_s \models [G]\psi$. By the semantics this is equivalent to the fact that $M_s \models [\psi_G]\psi$ for all ψ_G. By the induction hypothesis we have that $\mathcal{M}_t \models [\psi_G]\psi$ for every ψ_G, which is equivalent to $\mathcal{M}_t \models [G]\psi$ by the semantics. □

Finally, we have everything we need to prove the completeness of GALD.

Theorem 2. For all $\varphi \in \mathcal{L}_{\text{GALD}}$, if φ is valid, then $\varphi \in \mathbf{GALD}$.

Proof. Suppose towards a contradiction that φ is valid and $\varphi \notin \mathbf{GALD}$. Since **GALD** is a consistent theory, by Lemma 3 $\mathbf{GALD} + \neg\varphi$ is a consistent theory. By Lemma 4, $\mathbf{GALD}+\neg\varphi$ can be extended to a maximal consistent theory x such that $\mathbf{GALD}+\neg\varphi \subseteq x$, and $\neg\varphi \in x$. By Lemma 8, we have that $\mathfrak{M}_x^C \not\models \varphi$. Next, the canonical pseudo model $\mathfrak{M}_x^C$ can be unravelled into the collectively bisimilar canonical pre-model $\mathcal{M}_y^C$, and the latter can be folded into the trans-bisimilar canonical model M_z^C. So, we have that $\mathfrak{M}_x^C \leftrightarrows^C \mathcal{M}_y^C$ and $\mathcal{M}_y^C \leftrightarrows^T M_z^C$. By Lemma 9, $\mathfrak{M}_x^C \leftrightarrows^C \mathcal{M}_y^C \leftrightarrows^T M_z^C$ imply the modal equivalence of M_z^C and $\mathfrak{M}_x^C$, and from $\mathfrak{M}_y^C \not\models \varphi$ we can infer that $M_z^C \not\models \varphi$, which contradicts φ being a validity. □

We can obtain corresponding proofs for GALD^{ga+D} and GALD$^{pa\text{-}D}$ in the same way. For GALD^{ga+D}, it is enough to substitute $\mathcal{L}^G_{\mathrm{EL}}$ with $\mathcal{L}^G_{\mathrm{ELD}}$ in $A17$ and $R3$, and treat every ψ_G in the proof as a formula from $\mathcal{L}^G_{\mathrm{ELD}}$. For GALD$^{pa\text{-}D}$, each public announcement in the axiomatisation, apart from cases $A17$ and $R3$, becomes some $\psi \in \mathcal{L}_{\mathrm{GAL}}$, and the proof follows.

Theorem 3. GALD$^{pa\text{-}D}$ and GALD^{ga+D} are sound and complete.

5 Expressivity

Having three different versions of group announcement logic with distributed knowledge, it is very intriguing to analyse how they stand against each other in terms of expressivity.

Definition 16 (Expressivity)**.** Let $\mathcal{L}_1$ and $\mathcal{L}_2$ be two languages. We say that $\mathcal{L}_1$ *is at least as expressive as* $\mathcal{L}_2$ ($\mathcal{L}_2 \leqslant \mathcal{L}_1$) if and only if for all $\varphi \in \mathcal{L}_2$ there is an equivalent $\psi \in \mathcal{L}_1$. If $\mathcal{L}_1$ is *not* at least as expressive as $\mathcal{L}_2$, we write $\mathcal{L}_2 \nleqslant \mathcal{L}_1$. If $\mathcal{L}_2 \leqslant \mathcal{L}_1$ and $\mathcal{L}_1 \nleqslant \mathcal{L}_2$, we write $\mathcal{L}_2 < \mathcal{L}_1$, and we also write $\mathcal{L}_2 \equiv \mathcal{L}_1$ if $\mathcal{L}_2 \leqslant \mathcal{L}_1$ and $\mathcal{L}_1 \leqslant \mathcal{L}_2$. We will also abuse the notation and write L instead of $\mathcal{L}_{\mathrm{L}}$.

It is known that PALD $\equiv$ ELD [32] since each formula of PALD can be translated into an equivalent formula of ELD using the reduction axioms of PALD.

That all logics of group announcements with distributed knowledge are strictly more expressive than PALD comes as no surprise.

Proposition 4. PALD $<$ GALD$^{pa\text{-}D}$, PALD $<$ GALD, and PALD $<$ GALD^{ga+D}

Proof. The proof is similar to the one for PAL $<$ GAL [2, Theorem 19]. □

As $\mathcal{L}_{\mathrm{GALD}^{pa\text{-}D}}$ is a syntactic fragment of $\mathcal{L}_{\mathrm{GALD}}$, it is immediate that GALD is at least as expressive as GALD$^{pa\text{-}D}$.

Proposition 5. GALD$^{pa\text{-}D} \leqslant$ GALD

In the proof of the next proposition we exploit the fact that GALD$^{pa\text{-}D}$ can only witness a difference between two bisimilar (but not necessarily collectively bisimilar) models via K and D 'steps', which may be futile if models are large enough, i.e. if they exceed the modal depth of a formula. GALD and GALD^{ga+D}, on the other hand, can witness the difference via a public announcement of a PALD formula.

Proposition 6. GALD $\nleqslant$ GALD$^{pa\text{-}D}$ and GALD$^{ga+D} \nleqslant$ GALD$^{pa\text{-}D}$

Proof. Let $\varphi := p \land \widehat{K}_a(K_b\neg p \land K_c\neg p) \land \widehat{K}_a(\widehat{K}_b(p \land K_a p) \land \widehat{K}_c(p \land K_a p))$, $\chi := \neg K_a D_{\{b,c\}} p$, and consider a GALD formula $[\chi]\langle\{a,b,c\}\rangle\varphi$. Assume towards a contradiction that there is an equivalent GALD$^{pa\text{-}D}$ formula ψ, and $|\psi| = n$.

Consider models M_s and N_t (Figures 5 and 6). For both models, the size of the lower chain is $n+1$ and the size of the upper chain is either $n+2$, in the case of M_s, or $n+4$, in the case of N_t.

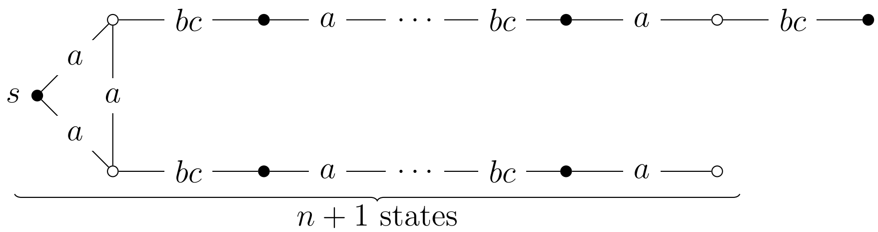

Figure 5: Model M_s. Propositional variable p holds in black states.

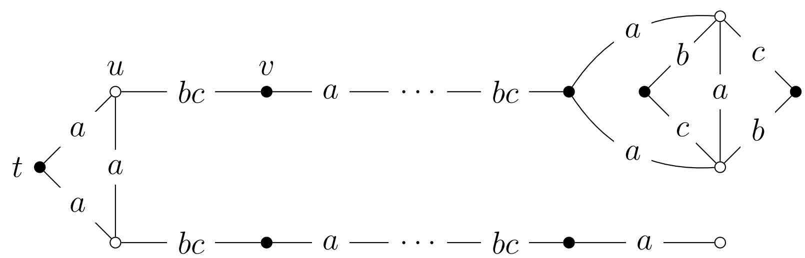

Figure 6: Model N_t. Propositional variable p holds in black states.

These models are bisimilar, and hence they agree on formulas of GAL. Structurally, every model is almost symmetric, and the only difference are bits on the right: the corresponding states are not collectively bisimilar. Formula φ describes the configuration depicted in Figure 7, i.e. it is false in M_s and N_t.

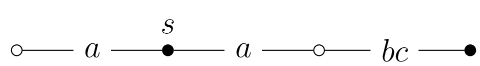

Figure 7: An $\{a, b, c\}$-definable submodel of M_s satisfying φ.

Let us argue that $M_s \models [\chi]\langle\{a, b, c\}\rangle\varphi$. Formula χ is true in every state of the model, and hence the announcement of it has no effect, and the agents can make φ true (note that the intersection of agents' relations is the identity). The existence of the corresponding group announcement follows from the fact that each state in M_s can be uniquely specified by an epistemic formula. The upper rightmost state is the only one satisfying $K_a p$. The one next to it would be the only one satisfying $\neg p \wedge \widehat{K}_b K_a p \wedge \widehat{K}_c K_a p$, and so on.

On the other hand, we have that $N_t \not\models [\chi]\langle\{a, b, c\}\rangle\varphi$. The announcement of χ removes two rightmost upper black states in the model, and the resulting updated model, N_t^χ, is fully symmetric and upper and lower halves of the model become bisimilar. In order to get a further update of N_t^χ that will be bisimilar to the model in Figure 7, the agents should preserve states u and v in the upper half, and delete the corresponding 'mirror' states in the lower half. However, since the upper and lower halves of N_t^χ are bisimilar, there is no announcement that will be true in u and v, and false in the 'mirror' states. Hence, the configuration depicted in Figure 7 is unattainable.

To see that $M_s \models \psi$ if and only if $N_t \models \psi$, it is enough to notice the following two things. First, since M_s and N_t are bisimilar and distributed knowledge operators do not occur in public announcements, denotations of $[G]\psi'$ and $[\chi]\psi'$ coincide on both models. Second, since the models are sufficiently large (the lengths of upper and lower chains are $n+2$, or $n+4$ in the case of N_t, and $n+1$ correspondingly) and $|\psi| = n$, no sequence of nested K and D modalities occurring in ψ can reach the states that are not collectively bisimilar (upper rightmost states).

Note that formula $[\chi]\langle\{a,b,c\}\rangle^{\triangle}\varphi$ is a formula of GALD^{ga+D}, and upper and lower halves of N_t^{χ} are collectively bisimilar. Hence the same proof applies. □

Interestingly, GALD^{ga+D} is not more expressive than GALD and even GALD$^{pa\text{-}D}$. In the proof of this result, we use the fact that two models that are bisimilar except for some propositional variable q, can be distinguished by GALD and GALD$^{pa\text{-}D}$ as group announcements would implicitly quantify over formulas with q. At the same time, this new q is only true in the states that GALD^{ga+D} could already distinguish, and thus the set of all possible submodels a GALD^{ga+D} formula can enforce would coincide for both models.

Proposition 7. GALD $\not\leq$ GALD^{ga+D} and GALD$^{pa\text{-}D}$ $\not\leq$ GALD^{ga+D}.

Proof. Let us consider formula $[\{c\}]\neg\langle\{a,b,c\}\rangle\varphi \in \mathcal{L}_{\text{GALD}}$, where $\varphi := \widehat{K}_c(K_a\neg p \wedge K_b\neg p) \wedge \widehat{K}_c(\widehat{K}_a K_c p \wedge \widehat{K}_b K_c p)$. Assume that there is an equivalent formula $\psi \in \mathcal{L}_{\text{GALD}^{ga+D}}$. Since the size of ψ is finite, we may assume that there is a propositional variable q that does not appear in ψ. Also, consider two models M_s and N_t depicted in Figure 8. Models M_s and N_t are almost identical with the only difference that q is false in all states of M_s and true in the crossed-out states of N_t.

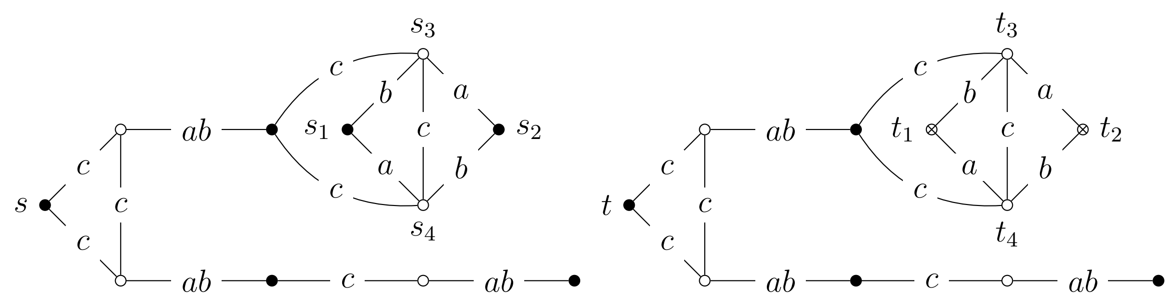

Figure 8: Models M_s (left) and N_t (right). Propositional variable p is true in black and crossed-out states, and q is true in crossed-out states.

A model that satisfies φ is depicted in Figure 9. At the same time, neither M_s nor N_t satisfy φ.

To see that $M_s \models [\{c\}]\neg\langle\{a,b,c\}\rangle\varphi$ it is enough to notice that in M_s all states in the upper part of the model are bisimilar to the corresponding states in the lower part (with s_1 and s_2 being bisimilar to the rightmost lower black state, and s_3 and s_4 being bisimilar to the penultimate lower white state). Hence, all the updates of M_s by announcements of agent c and consecutive announcements of $\{a,b,c\}$ would remove states symmetrically

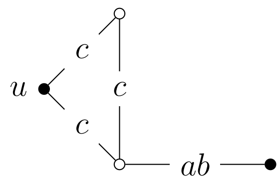

Figure 9: Model O_u satisfying φ.

from the upper and lower parts of the model, thus precluding obtaining a model bisimilar to O_u and satisfying φ.

On the other hand, even though q does not occur in $[\{c\}]\neg\langle\{a,b,c\}\rangle\varphi$, we still implicitly quantify over c- and $\{a,b,c\}$-announcements that may contain q. In particular, $N_t^{K_c\neg q} \models \langle\{a,b,d\}\rangle\varphi$ as $N_t^{K_c\neg q}$ (which is N_t without the crossed-out states) has two states uniquely distinguishable by $\neg p \land K_a\neg p \land K_b\neg p$ (these states are t_3 and t_4). The state to the left (the upper black state) can be distinguished by $p \land \widehat{K}_c(\neg p \land K_a\neg p \land K_b\neg p)$. In such a fashion we can assign a unique formula to each state of $N_t^{K_c\neg q}$, and, as the intersection of $a-$, $b-$, and $c-$relations is the identity, $\{a,b,c\}$ can force any submodel of $N_t^{K_c\neg q}$ including the one isomorphic to O_u. Finally, from $N_t^{K_c\neg q} \models \langle\{a,b,c\}\rangle\varphi$ and $N_t \models K_c\neg q$ we infer that $N_t \models \langle\{c\}\rangle\langle\{a,b,c\}\rangle\varphi$.

That $M_s \models \psi$ if and only if $N_t \models \psi$ can be shown by a straightforward induction on the complexity of ψ. Since M and N are collectively bisimilar except for q, propositional, epistemic, and public announcement cases are trivial. For $[G]^\triangle\psi'$ we need to show that for all ψ_G such that $M_{s'} \models \psi_G$ there is a formula ψ'_G such that $N_{t'} \models \psi'_G$, $M_{s'}^{\psi_G}$ and $N_{t'}^{\psi'_G}$ are collectively bisimilar except for q, and vice versa. Notice that the only case when the denotation of some ψ_G does not coincide on M and N is when ψ_G contains q. So let us assume that ψ_G contains q. If $M_{s'} \models \psi_G$, then there is an equivalent formula ψ'_G with the same denotation such that $N_{t'} \models \psi'_G$. This formula is exactly like ψ_G with all q's substituted with $(p \land \neg p)$ (since q has the empty denotation in M). On the other hand, if $N_{t'} \models \psi_G$, then there is ψ'_G where all q's are replaced with $D_{\{a,b\}}p \land K_c p$ (true only in s_1 and s_2, and t_1 and t_2), and $M_{s'} \models \psi'_G$. Since denotations of ψ_G and ψ'_G coincide in both cases, the resulting updates are collectively bisimilar except for q. $\square$

The overview of the relative expressivity of group announcement logics with distributed knowledge is presented in Figure 10.

We leave as an open problem whether $\text{GALD}^{ga+D} \not\leq \text{GALD}$, and conjecture that it is indeed the case.

6 Resolving Distributed Knowledge

In Section 3 we mentioned that distributed knowledge does not capture the property of a group of agents 'pooling their knowledge together'. The notion of resolved distributed knowledge [5] was introduced as a better formalisation of this intuition.

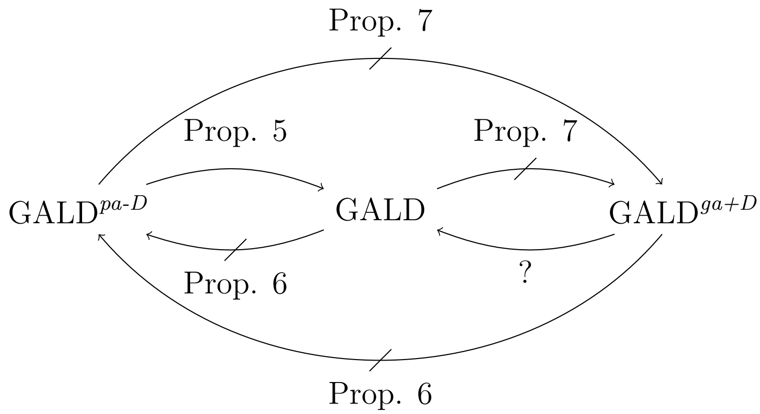

Figure 10: Relative expressivity of GALD$^{pa\text{-}D}$, GALD, and GALD^{ga+D}. An arrow from one logic to another signifies that the latter is at least as expressive as the former, striked out arrow — not at least as expressive, and the arrow with the question mark represents an open problem.

Resolution modalities are dynamic modalities that are used to express what is true after a group has actually shared what they know with each other. More precisely, they model the result of the publicly observable event that they privately share all their knowledge with each other. This is in contrast to standard distributed knowledge modalities which are static.

Taking into account both the group and dynamic aspects of resolved distributed knowledge, we discuss the extension of GAL with resolution modalities.

Definition 17 (Resolution)**.** Let $M = (S, \sim, V)$ be an epistemic model. A *global G-resolved update of M* is the model $M^G = (S^G, \sim^G, V^G)$, where $S^G = S$, $V^G = V$, and

$$\sim_a^G = \begin{cases} \bigcap_{b \in G} \sim_b & \text{if } a \in G, \\ \sim_a & \text{otherwise.} \end{cases}$$

Observe that according to the definition, $\sim_a^{\{a\}} = \sim_a$, and thus $M^{\{a\}}$ is the same as M.

For an example, consider model M (Figure 11) and the result of resolution relative to group $\{a, b\}$ (model $M^{\{a,b\}}$). Informally, if two states are distinguished by any agent (meaning that there is no corresponding arrow between the states) from a group, then they will be distinguished by all agents from the group after the resolved update.

Definition 18 (Language)**.** The *language of group announcement logic with resolved distributed knowledge* is defined by the following grammar:

$$\mathcal{L}_{\text{GALR}} \quad \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid R_G\varphi \mid [\varphi]\varphi \mid [G]\varphi$$

where $p \in P$, $a \in A$, and $G \subseteq A$.

The semantics is defined exactly like for GAL, with the following additional clause for the resolution modalities:

$$M_s \models R_G\varphi \quad \text{iff} \quad M_s^G \models \varphi$$

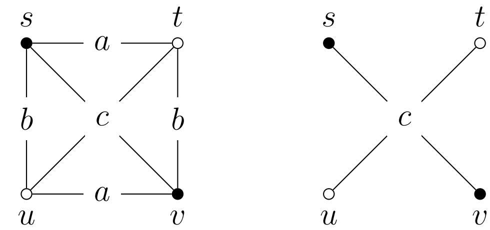

Figure 11: Models M (left) and $M^{\{a,b\}}$ (right). Propositional variable p is true in black states.

Since resolved distributed knowledge models private communication, it does not coincide with group announcements, which are public. Indeed, a group of agents may have a goal to inform not only the members of the group but other agents as well of some fact. Such a goal can be achieved via public announcements, but not necessarily via private communication. And vice versa, a group's epistemic goal may include not only informing each other of some fact, but also leaving the outsiders unaware of the truthfulness of the fact. In such scenario, group announcements fall short, while private communication allows the group to achieve their goal. We demonstrate this in Fact 4.

Fact 4. $\langle G\rangle\varphi \to R_G\varphi$ and $R_G\varphi \to \langle G\rangle\varphi$ are not valid.

Proof. For the first formula, consider $\varphi := K_a p$ and models M_s and $M_s^{K_b p}$ in Figure 12. From the fact that $M_s^{K_b p} \models \varphi$ it follows that $M_s \models \langle\{b\}\rangle\varphi$. At the same time, the b-resolved update of M_s leaves the model intact, i.e. $M_s^{\{b\}}$ is exactly the same as M_s. Hence, from the fact that $M_s^{\{b\}} \not\models K_a p$ it follows that $M_s \not\models R_{\{b\}}K_a p$.

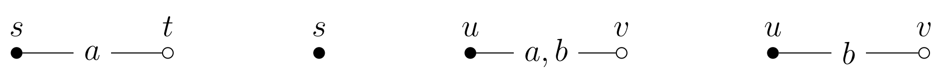

Figure 12: Models, from left to right, M_s, $M_s^{K_b p}$, N_u, and $N_u^{\{a,c\}}$. Propositional variable p is true in black states.

For the second formula, consider $\varphi := K_a p \wedge \neg K_b p$ and models N_u and $N_u^{\{a,c\}}$ in Figure 12. From the fact that $N_u^{\{a,c\}} \models \varphi$ it follows that $N_u \models R_{\{a,c\}}\varphi$. On the other hand, due to the fact that public announcements remove states from a model, there is no truthful update of N_u that would satisfy φ. $\square$

Even if we require the target formula to be positive, neither resolution implies ability, nor ability implies resolution. In the proof of the previous proposition, the counterexample for $\langle G\rangle\varphi \to R_G\varphi$ used positive formula $K_a p$.

Fact 5. $R_G\varphi \to \langle G\rangle\varphi$ with $\varphi \in \mathcal{L}^+$ is not valid.

Proof. Let $\varphi := K_a p$, and consider model M_s from Figure 2 . The $\{a, b\}$-resolved update of M_s, $M_s^{\{a,b\}}$, is model M_s without any non-reflexive arrows between its states. It is clear that $M_s^{\{a,b\}} \models \varphi$. On the other hand, $M_s \not\models \langle\{a, b\}\rangle K_a p$ due to the fact that $M_s \leftrightarrows M_v$, $M_t \leftrightarrows M_u$, and by the argument similar to the one in the proof of Fact 3. $\square$

A perhaps surprising consequence of the proof of this proposition, where $G = A$, is that semi-private communication between *all* agents does not imply the possibility of equivalent public communication between *all* agents. Formally, $R_A\varphi \to \langle A\rangle\varphi$ is not valid even for positive φ.

Some results on the expressivity of logics with distributed knowledge and resolution are shown in [5]. Relative expressivity of GALR and versions of GALD is an open question. Here we present some preliminary results.

Proposition 8. GALR $\not\leq$ GALD$^{pa\text{-}D}$

Proof. The proof is similar to the one for Proposition 6 with formula $[\chi]\langle\{a, b, c\}\rangle\varphi$ being substituted with $R_{\{b,c\}}\langle\{a, b, c\}\rangle\varphi$. Note that $R_{\{b,c\}}$ has no effect on M_s since b and c relations always occur together. On the other hand, N_t update with $R_{\{b,c\}}$ makes the rightmost upper black states disconnected from the rest of the model, and thus in $N_t^{\{b,c\}}$ the upper and lower halves are bisimilar. The remaining proof is the same as for Proposition 6. $\square$

Proposition 9. GALR $\not\leq$ GALD^{ga+D}

Proof. Exactly like the one of Proposition 7 with $[\{c\}]\neg\langle\{a, b, c\}\rangle\varphi \in \mathcal{L}_{\text{GALR}}$. $\square$

7 Coalition Announcement Logic

Group announcement modalities $\langle G\rangle$ intuitively refer to *coalitional ability*: $\langle G\rangle\varphi$ holds if the group, or coalition, G has the ability to make φ true by making some joint public announcement. General coalitional ability modalities have been extensively studied. Two prime examples of logics with such modalities are coalition logic (CL) [28] and alternating-time temporal logic [6]. However, these logics typically formalise a stronger notion of coalitional ability, well-known from game theory, namely that the coalition can perform some joint action such that *no matter what the other agents do*, φ will be true.

Coalition announcement logic (CAL) [3, 18] can be considered as either a restriction of the set of actions in CL to public announcements, or as a variant of GAL, where the agents outside of a group also participate in the joint announcement. CAL extends PAL with formulas $\langle\!\langle G\rangle\!\rangle\varphi$ meaning that G can make a joint announcement such that no matter what the remaining agents outside of G announce at the same time, φ will be true in the resulting updated model. Thus, in CAL agents outside of the group G may prevent the group from reaching its epistemic goal.

Formally, the language of CAL is the same as GAL except that the $\langle G\rangle$ modalities are replaced by $\langle\!\langle G\rangle\!\rangle$ (the dual diamond is taken as primary here instead of the box), with the following semantics:

$M_s \models \langle\!\langle G \rangle\!\rangle\varphi$ iff $M_s \models \psi_G \wedge [\psi_G \wedge \psi_{A\setminus G}]\varphi$ for some $\psi_G \in \mathcal{L}_{\mathrm{EL}}^{G}$ and all $\psi_{A\setminus G} \in \mathcal{L}_{\mathrm{EL}}^{A\setminus G}$

We can now extend CAL with distributed knowledge operators into the logic CALD, in the same way as for GAL and GALD.

It turns out that the CALD counterparts of most of the observations we made about GALD still hold. This is not immediately obvious, since the semantics is significantly different and, moreover, since the the expressive power of CAL and GAL is different [17]. In particular, we have the following (most of these points can be shown in the same way as for GALD, and hence we omit the proofs):

- $\langle\!\langle G \rangle\!\rangle\varphi \to D_G\langle\!\langle G \rangle\!\rangle\varphi$ is not valid.
- $D_G\langle\!\langle G \rangle\!\rangle\varphi \to \langle\!\langle G \rangle\!\rangle D_G\varphi$ is valid.
- $D_G\langle\!\langle G \rangle\!\rangle\varphi \to \langle\!\langle G \rangle\!\rangle\varphi$ is valid.
- $\langle\!\langle G \rangle\!\rangle D_G\varphi \to D_G\langle\!\langle G \rangle\!\rangle\varphi$ is valid.
- $D_G\varphi \to \langle\!\langle G \rangle\!\rangle E_G\varphi$ is not valid.
- $\langle\!\langle G \rangle\!\rangle E_G\varphi \to D_G\varphi$ is not valid.
- $D_G\varphi \to \langle\!\langle G \rangle\!\rangle E_G\varphi$ with $\varphi \in \mathcal{L}^+$ is valid on finite bisimulation contracted models.
- $D_G\varphi \to \langle\!\langle G \rangle\!\rangle E_G\varphi$ with $\varphi \in \mathcal{L}^+$ is not valid in general.

We can consider variants of CALD similarly to what we did for GALD, i.e. by allowing quantification over distributed knowledge formulas, or disallowing formulas with distributed knowledge in public announcement operators. Let the resulting logics be CALD^{ga+D} and CALD$^{pa\text{-}D}$, respectively. We get the same relative expressivity results as for GALD:

Proposition 10.
- CALD$^{pa\text{-}D}$ $<$ CALD
- CALD^{ga+D} $\not\leq$ CALD$^{pa\text{-}D}$
- CALD $\not\leq$ CALD^{ga+D}
- CALD$^{pa\text{-}D}$ $\not\leq$ CALD^{ga+D}

Sketch. For the two first points the proof is exactly like for Proposition 6. Note that G is either the grand or the empty coalition here, so the semantics of the group/coalition announcement operators coincide. For the two latter points, the proof is similar to that of Proposition 7, except that we use $\neg\langle\!\langle \{a, b, c\} \rangle\!\rangle\varphi$ as the distinguishing formula. In the first model, the upper and the lower halves are bisimilar, and the agents cannot do anything to force an interesting submodel. In the second model, we have special states with q and can construct an announcement by the grand coalition to force the interesting submodel. The reasoning that no CALD^{ga+D} formula can distinguish the models is the same as for GALD^{ga+D}, i.e., for each announcement involving q there is an equivalent announcement without q. $\square$

We leave a more in-depth study of CALD to future work.

8 Conclusions and Future Work

In this paper we studied the interaction between group announcements and distributed knowledge. In particular, we considered extensions of GAL with distributed knowledge modalities. We looked at the following three semantic variants of the language, ordered by the relative closeness to GAL:

- GALD$^{pa\text{-}D}$: semantics of group announcement operators is identical to GAL, the same set of public announcement operators as in GAL;
- GALD: semantics of group announcement operators is identical to GAL, public announcement operators include formulas with distributed knowledge;
- GALD^{ga+D}: group announcement operators quantify over formulas that may contain distributed knowledge, public announcement operators include formulas with distributed knowledge.

While the three languages have different expressive power, they, perhaps surprisingly, have similar sound and complete axiomatisations. The corresponding proof systems are obtained by combining the axioms and rules of GAL and PALD, showing that there are no non-trivial interaction axioms. We find this result interesting as it is contrary to the intuitions about distributed knowledge and the ability of groups.

The relationship between the scope of quantification and expressive power turned out to be not obvious or trivial. In particular, broadening the scope of quantification does not necessarily lead to the increase in expressive power: there are properties that GALD^{ga+D} cannot express, and GALD and GALD$^{pa\text{-}D}$ can. On the other hand, GALD is more expressive than GALD$^{pa\text{-}D}$, which is as expected. Whether there are some properties expressible in GALD^{ga+D} and not expressible in GALD is an open question.

In special cases (the positive fragments of the languages, bisimulation contracted models) the operators interact more in line with the intuition (see Fact 2). Note that the formula in Fact 2 is valid on the class of finite bisimulation contracted models, but not valid on the class of all models or even on the class of finite models (see Fact 3). This means that the logic has a different axiomatisation on the class of finite bisimulation contracted models. We find that interesting, because this is typically not the case for other epistemic logics, with or without distributed knowledge. We leave a complete axiomatisation of a logic for this class of models for future research.

We also briefly studied two related logics. The first is GAL extended with the closely related dynamic version of distributed knowledge, namely resolved distributed knowledge. We showed some expressivity results relating GAL with resolution and versions of GALD, however the full expressivity picture is yet unclear. Moreover, a complete axiomatisation of the logic is an open problem. Finally, there are also variants of the logic which are left to future work to explore, such as GALR$^{pa\text{-}R}$ and GALR^{ga+R} (with the obvious meaning).

The second logic related closely to GALD is CAL with distributed knowledge. Recall that in CAL agents outside of a group also make a simultaneous joint announcement that can preclude the group from reaching its epistemic goals. We argued that the expressivity landscape for CALD is similar to that of GALD. Finding a complete axiomatisation of CALD seems hard, since there are no known axiomatisations of CAL (although there is an axiomatisation of an extended version of CAL [19]).

GALD and the related logics we have studied here are the first step towards enriching logics of quantified actions (like [7, 25, 10, 14]) with distributed knowledge modalities. Particularly, we believe that the completeness and expressivity results for APAL with distributed knowledge can be obtained via a straightforward adaptation of the corresponding proofs presented in this paper.

Another avenue of further research is investigating logics of quantified actions in the presence of other types of group knowledge like common knowledge [16, 31] and relativised common knowledge [9]. So far, only APAL and GAL with common knowledge were studied [20].

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