# First-Order Coalition Logic

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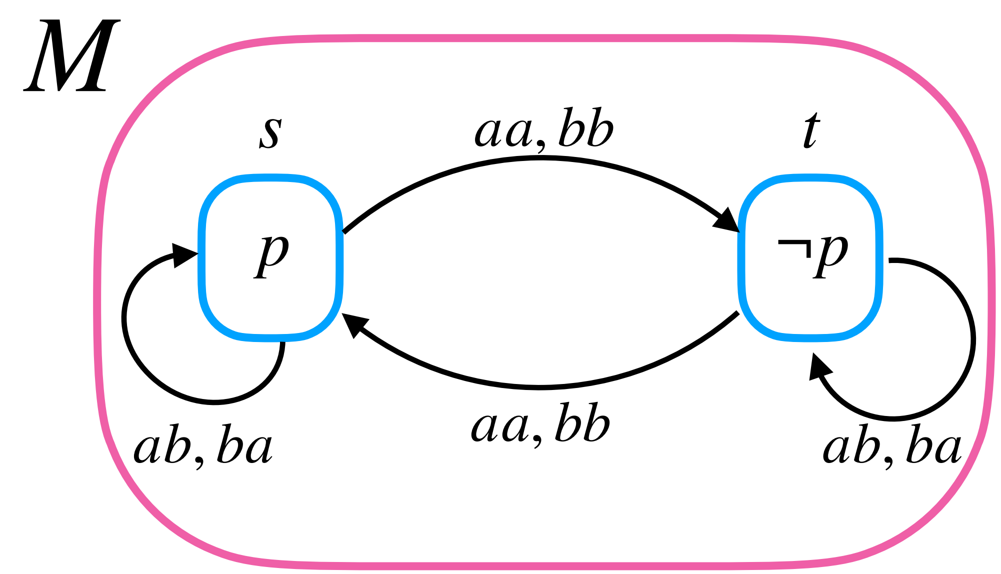
# Concurrent Game Models

A CGM $M$ is $\langle n, \mathtt{Ac}, \mathscr{D}, S, R, \mathscr{V} \rangle$, where $n \geqslant 1$ is the number of agents, $\mathtt{Ac} \neq \varnothing$ is a set of action, $\mathscr{D} = \mathtt{Act}^n$ is a set of decision, $S \neq \varnothing$ is a set of states, $R : S \times \mathscr{D} \to S$ is a transition function, $\mathscr{V} : \mathtt{Ap} \to 2^S$ is a valuation function




Logics interpreted on CGMs are used for specification and verification of such MAS as voting protocols, autonomous submarines, manufacturing robots, etc.

Alur, Henzinger, Kupferman *Alternating-time temporal logic*, 2002

# Logics for Reasoning About Strategic Abilities

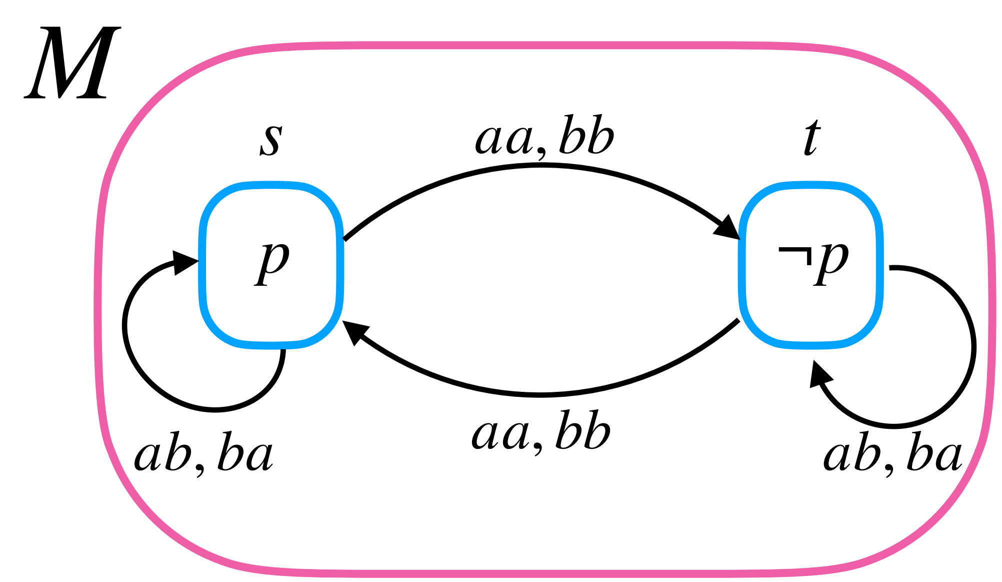
$$\text{ATL} \ni \varphi := p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \langle\langle C \rangle\rangle \mathsf{X}\varphi \mid \langle\langle C \rangle\rangle \varphi \mathsf{U} \psi \mid \langle\langle C \rangle\rangle \varphi \mathsf{R} \psi$$

$$\text{CL} \ni \varphi := p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \langle\langle C \rangle\rangle \mathsf{X}\varphi$$

$\langle\langle C \rangle\rangle \varphi$: coalition $C$ has a strategy to ensure $\varphi$ no matter what agents outside of the coalition do

$$M, s \models \langle\langle \{1,2\} \rangle\rangle \mathsf{X} \neg p$$

$$M, s \models \neg \langle\langle \{1\} \rangle\rangle \mathsf{X} \neg p$$





Alur, Henzinger, Kupferman *Alternating-time Temporal Logic*, 2002

Pauly *A Modal Logic for Coalitional Power in Games*, 2002

# Logics for Reasoning About Strategic Abilities

$\text{ATL} \ni \varphi := p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \langle\langle C \rangle\rangle \mathsf{X}\varphi \mid \langle\langle C \rangle\rangle \varphi \mathsf{U} \psi \mid \langle\langle C \rangle\rangle \varphi \mathsf{R} \psi$

$\text{CL} \ni \varphi := p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \langle\langle C \rangle\rangle \mathsf{X}\varphi$

$\langle\langle C \rangle\rangle \varphi$: coalition $C$ **has** a strategy to ensure $\varphi$ **no matter what** agents outside of the coalition do

$\exists\forall$

$[[C]]\varphi$: **whatever** coalition $C$ does, agents outside of the coalition **have** a strategy to ensure $\varphi$

$\forall\exists$

Fixed quantification and no way to reference strategies (and hence no NE)

# Strategy Logic

$$\mathsf{SL} \ni \varphi := p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathsf{X}\varphi \mid \varphi \mathsf{U} \varphi \mid \varphi \mathsf{R} \varphi \mid \forall x \varphi \mid \exists x \varphi \mid (i, x)\varphi$$

$\forall x\varphi$: for all strategies $x$, $\varphi$ holds

$\exists x\varphi$: there exists strategy $x$ such that $\varphi$ holds

$(i, x)\varphi$: after assigning strategy $x$ to agent $i$,$\varphi$ holds

**Temporal goal Nash Equilibrium**

$$\exists x_1 \ldots \exists x_n (1, x_1) \ldots (n, x_n) \left( \bigwedge_{i=1}^{n} \exists y (i, y) \psi_i \rightarrow \psi_i \right)$$

Mogavero, Murano, Vardi *Reasoning About Strategies*, 2010

# Strategy Logic

$$\mathsf{SL} \ni \varphi := p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathsf{X}\varphi \mid \varphi\mathsf{U}\varphi \mid \varphi\mathsf{R}\varphi \mid \forall x\varphi \mid \exists x\varphi \mid (i,x)\varphi$$

**Strategy Sharing**

$$\exists x(1,x)(2,x)\mathsf{X}\neg p$$

# Strategy Logic

$\mathsf{SL} \ni \varphi := p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathsf{X}\varphi \mid \varphi \mathsf{U} \varphi \mid \varphi \mathsf{R} \varphi \mid \forall x \varphi \mid \exists x \varphi \mid (i, x)\varphi$

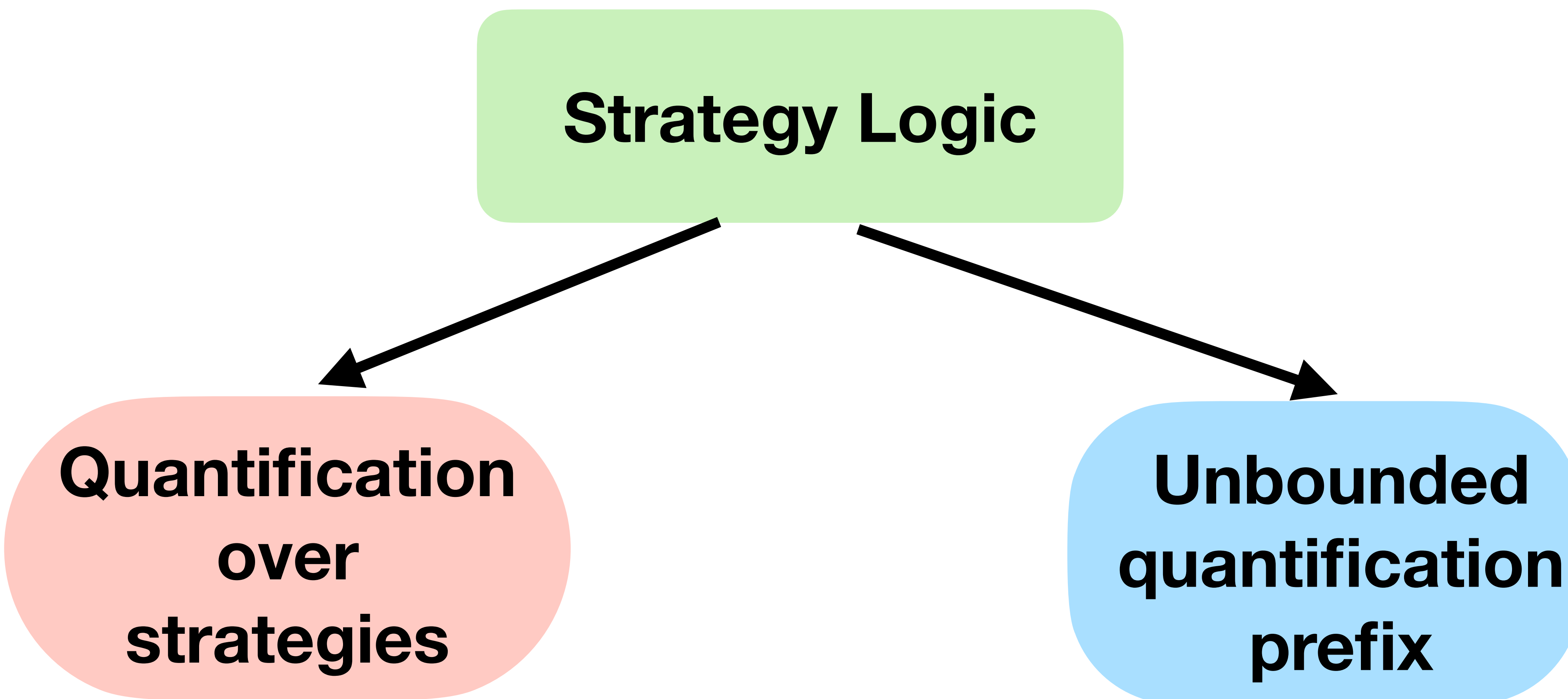
**Very expressive:** more expressive than CL, ATL, and ATL*

**Model checking:** decidable. NonElementarySpace-hard for the full language; from NonElementrayTime to PTime for fragments

**Satisfiability:** highly undecidable for the full language

**Axiomatisations:** non-axiomatisable for the full language; nothing on fragments *

# Why axiomatising (fragments of) SL is hard




We focus on the unbounded quantification prefix and consider only next-time strategies

# First-Order Coalition Logic

$$\text{FOCL} \ni \varphi := p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid (\!(t_1, \ldots, t_n)\!)\varphi \mid \forall x \varphi$$

$(\!(t_1, \ldots, t_n)\!)\varphi$: after agents execute actions assigned to $t_1, \ldots, t_n$, $\varphi$ holds

Each $t_i$ is either a **variable** or an **explicit action** from $\mathtt{Ac}$

**Temporal goal Nash Equilibrium**

$$\exists x_1 \ldots \exists x_n \left( \bigwedge_{i=1}^{n} \exists y_i (\!(x_1, \ldots, y_i, \ldots, x_n)\!)\psi_i \rightarrow (\!(x_1, \ldots, x_i, \ldots, x_n)\!)\psi_i \right)$$

# First-Order Coalition Logic

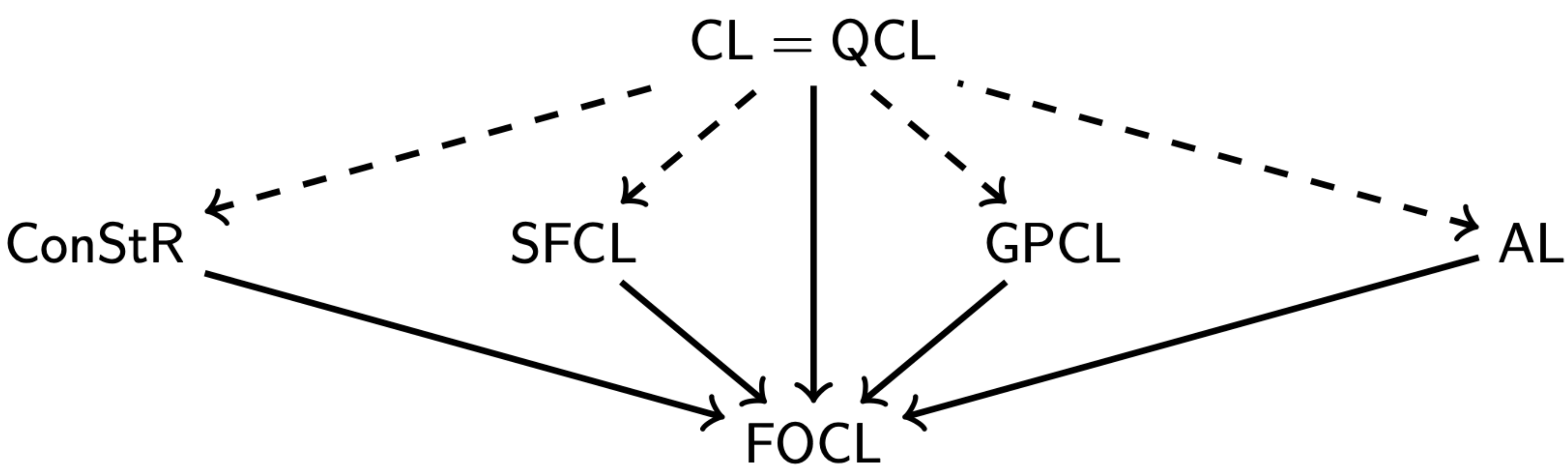
$$\mathsf{FOCL} \ni \varphi := p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid (\!(t_1, \ldots, t_n)\!)\varphi \mid \forall x \varphi$$

**Strategy Sharing**

$$\exists x (\!(x, x)\!) \neg p$$

# First-Order Coalition Logic

**Expressivity:** strictly more expressive than coalition logics in the literature




Goranko *Logics for Strategic Reasoning of Socially Interacting Rational Agents: An Overview and Perspectives*, 2023
Borgo *Coalitions in Action Logic*, 2007

# First-Order Coalition Logic

**Expressivity:** strictly more expressive than coalition logics in the literature

**Model checking:** PSPACE-complete

**Axiomatisation:** a sound and complete finitary axiomatisation. Akin to the one of FOML but on serial and functional frames

**Satisfiability:** undecidable via tiling

# Road Ahead

**First axiomatisation** of any variant of SL, a basis for future axiomatisations of more expressive fragments

While proving the undecidability of SAT, we uncovered a **gap in the proof** of the high undecidability of SAT for SL

**(Re)Open(ed) question 1:** is SL indeed not finitely axiomatisable?

**Open question 2:** axiomatisations of more expressive variants of SL based on the one for FOCL