

First axiomatisation of a variant of Strategy Logic

First-Order Coalition Logic

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Motivation *Strategy Logic* (SL) is believed to be not finitely axiomatisable. We present **the first axiomatisation** of a variant of SL, where we keep the arbitrary quantifier prefixes but consider only single-step strategies. We also **reopen** the problem of unaxiomatisability of SL.

A Well-Behaved Fragment of SL

$$\text{FOCL } \exists \varphi := p \mid \neg \varphi \mid (\varphi \wedge \varphi) \mid ((t_1, \dots, t_n)) \varphi \mid \forall x \varphi,$$

where term t_i is either a variable or a an explicit action.

Let $\mathfrak{G} = \langle n, \text{Ac}, \mathcal{D}, S, R, \mathcal{V} \rangle$ be a *Concurrent Game Structure* (CGS).

$$\begin{aligned} \mathfrak{G}, s \models ((a_1, \dots, a_n)) \psi &\text{ iff } \exists t \in S \text{ s.t. } \langle s, a_1, \dots, a_n, t \rangle \in R \text{ and } \mathfrak{G}, t \models \psi \\ \mathfrak{G}, s \models \forall x \psi &\text{ iff } \forall a \in \text{Ac} : \mathfrak{G}, s \models \psi[a/x] \end{aligned}$$

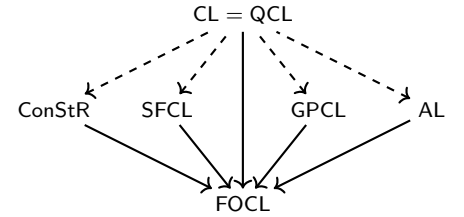
FOCL allows for arbitrary quantifier prefixes and strategy sharing. We can, for example, express Nash Equilibrium for deterministic strategies: $\exists x_1, \dots, x_n (\bigwedge_{i=1}^n \exists y_i ((x_1, \dots, y_i, \dots, x_n)) \psi_i \rightarrow ((x_1, \dots, x_i, \dots, x_n)) \psi_i)$.

Results

1. FOCL has a **sound and complete axiomatisation**.
2. FOCL is **more expressive** than other coalition logics.
3. **Model checking** is **PSPACE-complete**, **SAT** is **undecidable**.

Details

Expressivity FOCL allows for strategy sharing. E.g. formula $\exists x((x, x)) \neg p$ does not have an equivalent counterpart in many coalition logics, and hence FOCL is **strictly more expressive** than them.



Axiomatisation Compared to SL, we consider quantification over actions rather than strategies. This, together with explicit actions, allows us to provide the following **sound and complete** axiomatisation.

PC	Every propositional tautology
K	$((\langle \vec{t} \rangle) \varphi \wedge (\langle \vec{t} \rangle) \psi) \leftrightarrow (\langle \vec{t} \rangle) (\varphi \wedge \psi)$
N	$\neg(\langle \vec{t} \rangle) \varphi \leftrightarrow (\langle \vec{t} \rangle) \neg \varphi$
E	$\forall x \varphi \rightarrow \varphi[t/x]$
B	$\forall x(\langle \vec{t} \rangle) \varphi \rightarrow (\langle \vec{t} \rangle) \forall x \varphi$, s.t. $t_i \neq x$
MP	From $\varphi, \varphi \rightarrow \psi$, infer ψ
Nec	From φ , infer $(\langle \vec{t} \rangle) \varphi$
Gen	From $\varphi \rightarrow \psi[t/x]$, infer $\varphi \rightarrow \forall x \psi$, if $t \notin \varphi$

Undecidability We use a reduction from the tiling problem to show that the satisfiability problem for FOCL is **undecidable**. We have also discovered a **gap** in the Σ_1^1 -hardness proof for SL in the **original SL papers**. This implies that the question of whether SL is finitely, or even recursively, axiomatisable is **open again**. We conjecture that SL is indeed Σ_1^1 -hard, as quantification over strategies is essentially a second-order quantification over functions. However, the formal argument is **yet to be provided**.



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