What Anonymous Public Announcements Can Do

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Abstract. Is it possible to make a certain fact known through a public, but anonymous, announcement? Anonymous public announcements are everywhere – think of, for example, anonymous postings on a message board. It turns out that formalising anonymous public announcements involve complex reasoning about higher-order knowledge and belief as in "Ann knows that Bill knows that Ann knows ...", and recent work has shown that modal epistemic logic can shed considerable light on the phenomenon. In this paper we extend existing approaches to be able to reason about *quantification* over possible anonymous public announcements: is there an anonymous public announcement that can achieve a given epistemic goal, without revealing the identity of the announcer? This ranges over both direct anonymous announcements of a fact one wants to be known by everyone, to indirect announcements of something else that will enable someone to deduce the fact and someone else not. Examples are, e.g., found in whistleblowing scenarios. In this paper we introduce a logical operator that captures "there exists an anonymous public announcement such that ...", and study its properties. The main result is a sound and complete axiomatisation of the resulting logic.

1 Introduction

An anonymous public announcement is an announcement that is made anonymously, in public, i.e., everyone in a given group can observe it and knows that everyone else can observe it (and so on). Examples are found everywhere: anonymous postings on message boards, graffiti, pseudonymous books.

Yet, they have only been the target of formal analysis relatively recently [5, 4]. The study of anonymous public announcements differs somewhat from the study of other types of anonymity in that it requires careful study of often higher-order epistemic (e.g., "I know that you know that I know") effects. For example, if we know that the announcer *intended* to be anonymous, the anonymous announcement in fact reveals *more* information than otherwise [5]. It follows, for example, that k-anonymity [27] is not sufficient. For these reasons, analysing anonymous public announcements using (modal) *epistemic logic* [14, 20], the state-of-the art framework for reasoning about (higher-order) knowledge, has shown to be useful. In particular, [5, 4] introduce and study modal operators allowing us to express properties such as "after ϕ is anonymously announced, ψ is true".

In this paper we add *quantifiers* over possible anonymous announcements, enabling expressions such as "there exists a possible anonymous announcement that will make ψ true". Here, we assume that the announcer only makes an (anonymous) announcement if she knows that it will not reveal her identity. An

example is that a whistleblower makes an anonymous announcement with the goal of making someone know some fact. However, it might be that announcing that fact *directly* might reveal her identity and would thus not be considered a truly anonymous announcement, in which case we are interested to know whether there is some other, safe, *indirect* announcement, that can be used instead. Or it can be that goal is that *some* people gets to know the fact, but not *everyone*, in which case an indirect announcement also is relevant. It could also be that we are interested in knowing whether some anonymous announcement, perhaps inadvertently, could reveal some fact, even without the announcer knowing the fact herself. Or, finally, it could be that we are interested in whether or not our information is secure against anonymous public announcements: no matter what anyone announces anonymously, our information will not be revealed. All these scenarios are captured by the operators we introduce in this paper, which quantify over "safe" anonymous announcements that are guaranteed to not reveal the identity of the announcer, with respect to satisfying some goal specified in epistemic logic.

Formal methods in general, and formal logic in particular, of course have a long history in security and privacy [18, 29, 23], e.g., providing the foundations for verification techniques based on model checking or theorem proving, or for formalising different privacy guarantees. Most logical approaches are based on *modal logic* [11]: temporal logic enabling reasoning about properties along all (or some) possible execution paths, deontic logic enabling reasoning about obligations and permissions, and epistemic logic enabling reasoning about information and communications security [16]. Epistemic logic approaches for reasoning about *anonymity* and related concepts include [26, 28, 21, 19, 33, 32, 31, 30, 24, 10, 22, 5]. In this paper we are, like in [5], concerned with a particular type of *action anonymity*, the inability to identify who performed some action, as opposed to *data anonymity*, the inability to identify the identity of some record in a database [13] – the particular type of actions here being public announcements. We are not aware of any other works studying logical quantifiers over possible public announcements, which is what we do in this paper.

Adding quantifiers to the logic of (standard, not anonymous) public announcements [25, 12] is not new. [7] introduces an operator quantifying over all possible public announcements, while [2] introduces operators quantifying over all public announcements a given agent or group of agents can make. In this paper we combine these strands of research: adding quantifiers over anonymous announcements in the style of [7, 2] to the logic of anonymous public announcements in [5]. This will help us to understand precisely what anonymous public announcements can do, and to uncover the logical and computational principles behind quantifying over anonymous public announcements.

The remainder of the paper is organised as follows. In the next section we briefly review the technical background: epistemic logic and public announcement logic, existing approaches to quantifying over announcements, and, most importantly, the logic of anonymous public announcements. In Section 3 we define the language and semantics of two new logics with quantifiers over anony-

mous announcements, one with quantification over any anonymous announcement and one quantifying over possible anonymous announcements by a particular agent. We give an example in Section 4. The agent-based quantifiers can be seen as capturing a notion of *ability* – ability to make a certain state of affairs come about through an anonymous announcement. In Section 5 we discuss the relationship between knowledge and this kind of ability. (Infinitary) axiomatic proof systems for the new logics are introduced in Section 6, and shown to be sound. Section 7 contains the main results: proof that the systems are also complete. We conclude in Section 8.

2 Background

We briefly review the background theory we rely on in the following: epistemic logic and its extensions with public announcement operators and quantifiers over the latter, as well as anonymous public announcement logic. In the next section we add quantifiers to the latter.

2.1 Epistemic Logic and Public Announcement Logic

We give a brief overview of the main technical definitions and refer to, e.g., [12] for more details. Let N be a finite set of *agents*, and P be a countable set of *propositional variables*.

Definition 1. *An* (epistemic) model *is a triple $M = (S, \sim, V)$, where S is a non-empty set of states, $\sim\colon N \to 2^{S\times S}$ is an equivalence relation for each $i \in N$, and $V : P \to 2^S$ is the valuation function. For $s \in S$, a pair M, s is called a* pointed (epistemic) model.

The language $\mathcal{L}_K$ of epistemic logic is defined by the following grammar:

$$\phi ::= p \mid \neg\phi \mid (\phi \wedge \phi) \mid K_i\phi$$

where $p \in P$ and $i \in N$. Intuitively, $K_i\phi$ expresses the fact that agent i knows ϕ. We use the usual derived propositional connectives, in addition to $\hat{K}_a\phi$ for $\neg K_a\neg\phi$. When $G \subseteq N$, we write $E_G\phi$ for $\bigwedge_{i\in G} K_i\phi$ – everybody in G knows ϕ.

Truth of a formula ϕ in a pointed model M, s, denoted $M, s \models \phi$, is recursively defined as follows.

Definition 2. *Let $M = (S, \sim, V)$ be a model and $p \in P$, $i \in N$.*

$$\begin{array}{ll} M, s \models p & \textit{iff } s \in V(p) \\ M, s \models \neg\phi & \textit{iff } M, s \not\models \phi \\ M, s \models \phi \wedge \psi & \textit{iff } M, s \models \phi \textit{ and } M, s \models \psi \\ M, s \models K_i\phi & \textit{iff } M, t \models \phi \textit{ for all } t \in S \textit{ such that } s \sim_i t \end{array}$$

We call a formula ϕ *valid*, and write $\models \phi$, if for all M, s it holds that $M, s \models \phi$. Given M, we will write $[\![\phi]\!]_M$ for the set $\{s \in S \mid M, s \models \phi\}$.

The language $\mathcal{L}_!$ of public announcement logic (PAL) [25] adds public announcement operators $[\phi!]$ to epistemic logic:

$$\phi ::= p \mid \neg\phi \mid (\phi \wedge \phi) \mid K_i\phi \mid [\phi!]\phi$$

We write $\langle\phi!\rangle\psi$ for $\neg[\phi!]\neg\psi$. The idea is that $[\phi!]\psi$ means that if ϕ is true, then after it is publicly announced ψ becomes true, while $\langle\phi!\rangle\psi$ means that ϕ *is* true and that after it is announced ψ becomes true. That meaning is formalised by *updating* the model by removing states where ϕ is false. Formally, the interpretation in epistemic models is extended as follows for the new operators:

$$M, s \models [\phi!]\psi \text{ iff } M, s \models \phi \text{ implies } M^{\phi!}, s \models \psi$$

where *the updated model* $M^{\phi!}$ is $(S^{\phi!}, \sim^{\phi!}, V^{\phi!})$, where:

- $S^{\phi!} = [\![\phi]\!]_M$,
- $\sim_i^{\phi!} = \sim_i \cap (S^{\phi!} \times S^{\phi!})$ for all $i \in N$,
- and $V^{\phi!}(p) = V(p) \cap [\![\phi]\!]_M$ for all $p \in P$.

2.2 Quantifying over Announcements

We review two different ways public announcement logic has been extended with quantifiers over announcements.

Arbitrary public announcement logic (APAL) [7] adds a single arbitrary public announcement operator $\Diamond$, where the intended meaning of $\Diamond\phi$ is that ϕ is true after some public announcement. Group announcement logic (GAL) [2] adds a family of operators $\langle a\rangle$, one for each agent $a \in N$[1], such that $\langle a\rangle\phi$ is intended to mean that ϕ is true after some public announcement *by agent* a.

The new operators are interpreted in epistemic models as follows:

$$M, s \models \Diamond\phi \quad \text{iff } \exists\psi \in \mathcal{L}_K, M, s \models \langle\psi!\rangle\phi$$
$$M, s \models \langle a\rangle\phi \text{ iff } \exists\psi \in \mathcal{L}_K, M, s \models \langle K_a\psi!\rangle\phi$$

While $\Diamond\phi$ means that ϕ can be made true by some announcement, it doesn't assume that that announcement actually can be truthfully made by one of the agents in N. $\langle a\rangle\phi$ on the other hand, requires that one of the agents actually knows the formula and thus can truthfully announce it. We write $\Box\phi$ for $\neg\Diamond\neg\phi$, and $[a]\phi$ for $\neg\langle a\rangle\phi$.

2.3 Anonymous Public Announcement Logic

Anonymous Public Announcement Logic [5] extends epistemic logic with two new operators. Formally, the language $\mathcal{L}_{‡▲}$ is defined as follows.

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid [\phi‡]\phi \mid ▲\phi$$

[1] In fact for any *group* $G \subseteq N$. Here we look only at the single-agent operators because they are the most relevant for what follows.

The operator ▲ is called *safety*, and ▲ϕ is intended to mean that ϕ can be *safely announced*, in the sense that it can be announced by someone without anyone else learning who the announcer is. As shown in [5], it is necessary and sufficient for safety that there are three (different) agents who know that ϕ is true and that there are three (not necessarily the same) agents who know that ϕ is true and that *they* also know that there are three agents who know – ad infinitum. This is captured by letting ▲ϕ be true exactly in the greatest fixpoint of the function

$$f(S) = [\![\bigvee_{G \in N^3} E_G(\phi \wedge x)]\!]^M_{V[x=S]}$$

where

- x is a propositional variable not occurring in ϕ.
- $N^3 = \{\{a, b, c\} : a, b, c \in N; a \neq b, a \neq c, b \neq c\}$ is the set of all three-agent groups.
- $[\![\psi]\!]^M_{V[x=S]}$ is the extension of ψ (i.e., the states in which ψ is true) in the model that is the same as M, except that the valuation function has been changed so that the extension of the variable x is S.

Formally, we let:

$$M, s \models \blacktriangle\phi \Leftrightarrow s \in \bigcup \left\{ S : S \subseteq [\![\bigvee_{G \in N^3} E_G(\phi \wedge x)]\!]^M_{V[x=S]} \right\}$$

– i.e., ▲ϕ is true exactly in the union of all post-fixpoints which is equal to the greatest fixpoint.

The semantics of ▲ can alternatively and equivalently be given as follows [5]. Let:

$$\blacktriangle_n\phi = \phi \wedge \bigvee_{G_1 \in N^3} E_{G_1}(\phi \wedge \bigvee_{G_2 \in N^3} E_{G_2}(\phi \wedge \bigvee_{G_3 \in N^3} E_{G_3}(\phi \wedge \cdots \wedge \bigvee_{G_n \in N^3} E_{G_n}\phi)))$$

Thus, $\blacktriangle_0 = \phi$, $\blacktriangle_1 = \phi \wedge \bigvee_{G_1 \in N^3} E_{G_1}\phi$, and so on. Then:

$$M, s \models \blacktriangle\phi \text{ iff } M, s \models \blacktriangle_n\phi \text{ for all } n. \tag{1}$$

The operator $[\phi\ddagger]$ is the anonymous public announcement operator. The intended meaning is that $[\phi\ddagger]\psi$ holds in a state of an epistemic model if ψ is true after any anonymous announcement of ϕ, under the assumption that agents will only make announcements they know are safe. Formally:

$$M, s \models [\phi\ddagger]\psi \text{ iff } \forall a \in N : \big(M, s \models K_a\blacktriangle\phi \text{ implies } M^{\phi\ddagger}, (s, a) \models \psi\big)$$

where $M^{\phi\ddagger}$ is the *updated* model after an anonymous announcement of ϕ in M (defined below) and (s, a) is the situation where agent a announced ϕ in state s of M. In other words, $[\phi\ddagger]\psi$ holds if no matter who (safely) anonymously announced ϕ, ψ becomes true.

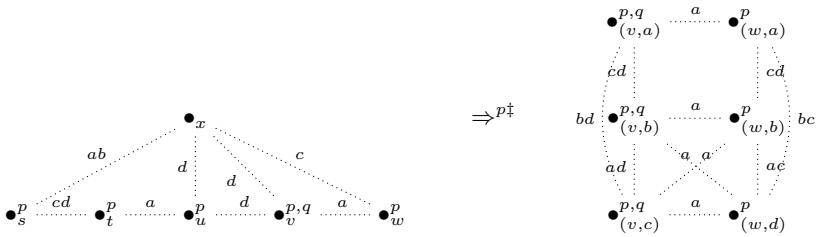


Fig. 1. Four-agent epistemic model M, with updated model $M^{p\ddagger}$. States are labelled by true atomic propositions.

Definition 3 ([5]). *The update of epistemic model $M = (S, \sim, V)$ by the safe anonymous announcement of ϕ is the epistemic model $M^{\phi\ddagger} = (S^{\phi\ddagger}, \sim^{\phi\ddagger}, V^{\phi\ddagger})$ where:*

- $S^{\phi\ddagger} = \{(s, a) : s \in S,\ M, s \models K_a \blacktriangle \phi\}$
- $(s, a) \sim_c^{\phi\ddagger} (t, b)$ *iff* $s \sim_c b$ *and* $(a = c$ *iff* $b = c)$
- $V^{\phi\ddagger}(s, a) = V(s)$

In the updated model, agent c can distinguish between the state (s, a) where agent a made the announcement of ϕ in state s of the original model and state (t, b) where b made it in state t, if either she can already distinguish between s and t in the original model, or she is either a or b but not both.

As an example, consider the epistemic model in Figure 1 (left) and its update $M^{p\ddagger}$ (right) after the anonymous public announcement of p. We have that:

- q cannot be safely announced in any state: $M, y \models \neg\blacktriangle q$, for $y \in \{s, t, u, v, w, x\}$. It is only true in v, where only a single agent (b) knows q.
- p can safely be announced only in v and w: $M, v \models \blacktriangle p$ and $M, w \models \blacktriangle p$, but $M, y \models \neg\blacktriangle p$ for all $y \notin \{v, w\}$.
- In state v agents a, b and c (but not d) know that p is safe, while in state w agents a, b and d (but not c) do: $M, v \models K_a\blacktriangle p \wedge K_b\blacktriangle p \wedge K_c\blacktriangle p \wedge \neg K_d\blacktriangle p$, and $M, w \models K_a\blacktriangle p \wedge K_b\blacktriangle p \wedge K_d\blacktriangle p \wedge \neg K_c\blacktriangle p$.
- $M, v \models [p\ddagger]K_cK_dq$: after the anonymous public announcement of p, no matter who made it, agent c knows that agent d knows that q is true.

As usual we define $\langle\phi\ddagger\rangle\psi$ as shorthand for $\neg[\phi\ddagger]\neg\psi$. We get that $M, s \models \langle\phi\ddagger\rangle\psi$ iff there is an agent $a \in N$ such that $M, s \models K_a\blacktriangle\phi$ and $M^{\phi\ddagger}, (s, a) \models \psi$.

It turns out that anonymous public announcements actually can be "simulated" by public announcements: they are equivalent to public announcement of safety:

Lemma 1 ([5], Lemma 3). *For any model M and state s and formulas ϕ and ψ, $M, s \models [\phi\ddagger]\psi$ iff $M, s \models \blacktriangle\phi$ implies $M^{\blacktriangle\phi!}, s \models \psi$.*

Thus, when it comes to checking whether a formula holds in an updated model $M^{\phi\ddagger}$ after a safe anonymous announcement of ϕ, it suffices to check the (typically simpler) updated model $M^{\blacktriangle\phi!}$. For example, the model $M^{p\ddagger}$ to the right in Figure 1 is equivalent with (in fact, is *bisimilar* to [5]) the model $M^{\blacktriangle p!}$ on the bottom right of Figure 2.

2.4 Agent-based Anonymous Public Announcement Operators

While $[\phi\ddagger]\psi$ say that ψ is true after *any* anonymous public announcement *no matter who makes it*, we can define a more fine-grained operator $[\phi\ddagger_a]\psi$ saying that ψ is true after an anonymous announcement *by agent a* [5] (note that this is different from a public announcement by $a!$):

$$M, s \models [\phi\ddagger_a]\psi \text{ iff } M, s \models K_a\blacktriangle\phi \text{ implies } M^{\phi\ddagger}, (s, a) \models \psi$$

As usual, we define $\langle\phi\ddagger_a\rangle\psi$ to stand for $\neg[\phi\ddagger]\neg\psi$. We get the following, similarly to Lemma 1:

Lemma 2. *For any model M and state s and formulas ϕ and ψ, $M, s \models [\phi\ddagger_a]\psi$ iff $M, s \models K_a\blacktriangle\phi \to [\blacktriangle\phi!]\psi$.*

3 Quantifying over Anonymous Public Announcements: Language and Semantics

We now introduce operators for quantifying over anonymous public announcements, that can be used to reason about what can be possibly be achieved with some anonymous public announcement. The resulting logics can be seen as variants of APAL/GAL (Section 2.2) with *anonymous* public announcements (Sections 2.3 and 2.4).

We define two different types of operators, similarly to the distinction between APAL and GAL. Intuitively, $\Diamond\!\!\!\!-\phi$ is intended to mean that ϕ can be made true with some anonymous announcement, while $\Diamond\!\!\!\!-_a\phi$ is intended to mean that ϕ can be made true with some anonymous announcement *by agent a*.

3.1 Arbitrary Anonymous Public Announcements

We extend public announcement logic with operators for safety, and for quantification over anonymous announcements. Recall that the anonymous announcement operators can be expressed by combining safety and public announcement operators. Formally, the language $\mathcal{L}_{!\blacktriangle\boxplus}$ is defined as follows:

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid [\phi!]\phi \mid \blacktriangle\phi \mid \boxplus\phi$$

and we write $\Diamond\!\!\!\!-\phi$ for $\neg\boxplus\neg\phi$. We write $[\phi\ddagger]\psi$ for $[\blacktriangle\phi!]\psi$ (see Lemma 1).

The language is interpreted in epistemic models as follows, extending the interpretation of the other clauses in the language definition given in the previous section:

$$M, s \models \boxplus\phi \text{ iff } \forall\psi \in \mathcal{L}_K M, s \models [\psi\ddagger]\phi.$$

In other words, $\boxplus\phi$ says that ϕ is true after *any* anonymous announcement.

Obviously, we get that

$$M, s \models \diamondplus\phi \text{ iff } \exists\psi \in \mathcal{L}_K M, s \models \langle\psi\ddagger\rangle\phi$$

– ϕ is true after *some* anonymous announcement.

3.2 Agent-based Arbitrary Anonymous Public Announcements

By making use of the more fine-grained agent-based operators $[\phi\ddagger_a]$ from Section 2.4, we can define GAL-style (see Section 2.2) operators $\diamondplus_a$ as well, where $\diamondplus_a\psi$ is intended to mean that agent a can make an anonymous public announcement such that ψ becomes true. Formally, let the language $\mathcal{L}_{!\blacktriangle N}$ be defined as follows:

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid [\phi!]\phi \mid \blacktriangle\phi \mid \boxplus_a\phi$$

where $a \in N$. Let $[\phi\ddagger]_a\psi$ stand for $K_a\blacktriangle\psi \rightarrow [\blacktriangle\phi!]\psi$ (see Lemma 2). Again, let $\diamondplus_a\phi$ stand for $\neg\boxplus_a\neg\phi$. Semantics for the new operators:

$$M, s \models \boxplus_a\phi \text{ iff } \forall\psi \in \mathcal{L}_K M, s \models [\psi\ddagger_a]\phi$$

and we easily get that:

$$M, s \models \diamondplus_a\phi \text{ iff } \exists\psi \in \mathcal{L}_K M, s \models \langle\psi\ddagger_a\rangle\phi$$

4 Example: Whistleblowing

> *A company has been evading tax. Is there a way for someone, a whistleblower, to (a) anonymously and publicly and safely alert Alice of this fact, possibly (b) without revealing it to Bob?*

The answer of course depends on the background knowledge of the involved agents. Consider the background knowledge in Figure 2 (middle) and let Alice be agent d, Bob agent a, and q be the proposition that the company has been evading tax. Let p be some related proposition, say "the company has had x amount of income". Let M be the model in the (middle of the) figure and let v be the actual, real, world. We can answer question (a):

$$M, v \models \diamondplus K_d q$$

and in fact also question (b):

$$M, v \models \diamondplus(K_d q \wedge \neg K_a q)$$

This isn't completely obvious: who is the whistleblower, and what is it the whistleblower can announce (it is actually not q!)? In detail:

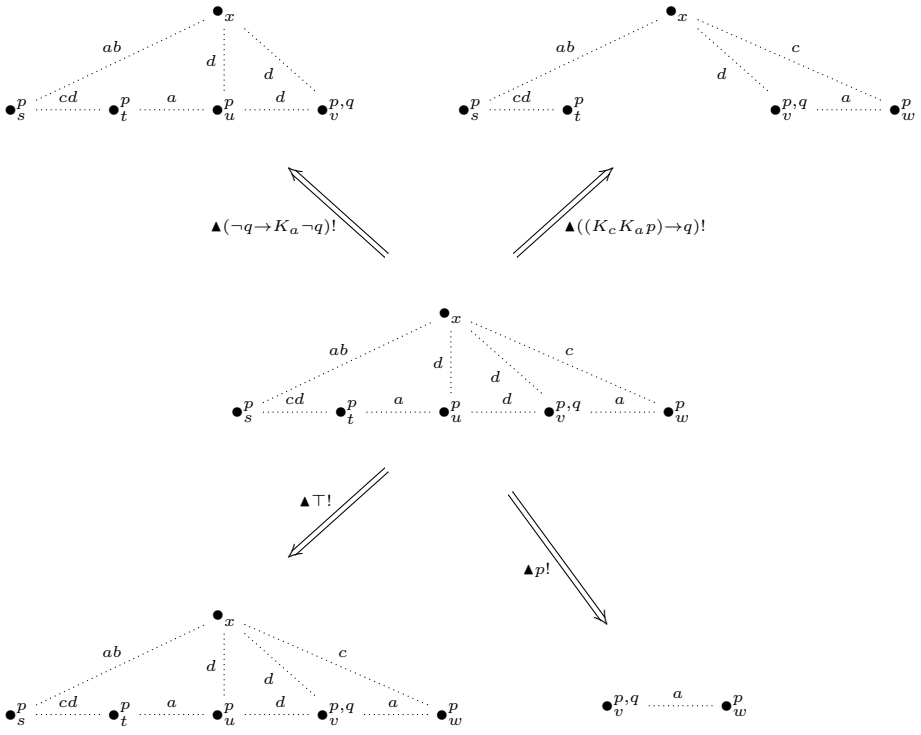


Fig. 2. Model from Figure 1, with all safe updates in state v.

- $M, v \models p \wedge q \wedge \neg K_d q \wedge \neg K_d p \wedge K_b p \wedge K_b q$: it is actually the case that the company has been evading tax, and that it has had x amount of income, b knows both of these facts, and Alice knows neither of them.
- $M, v \models \neg \blacktriangle q$: however, no-one can safely announce q without revealing their identity.
- There are, however, other safe announcements that can be made in state v – four to be precise, up to logical equivalence (in v). They are illustrated in Figure 2. Instead of showing the updated models $M^{\phi\ddagger}$ after safe announcement of ϕ explicitly, the result $M^{\blacktriangle\phi!}$ of the public announcement of $\blacktriangle\phi$ is shown, which we know from Lemma 1 satisfy exactly the same formulas. One of the four is the trivially safe announcement of a tautology, which does not change the model.
- Three of those four safe announcements do not achieve the goal, but one of them *does*: we have that $M, v \models \langle p\ddagger\rangle K_d q$ and in fact that $M, v \models \langle p\ddagger\rangle(K_d q \wedge \neg K_a q)$ (bottom right in the figure). This announcement can be safely made by a, b, and c: $M, v \models K_a \blacktriangle p \wedge K_b \blacktriangle p \wedge K_c \blacktriangle p$.
- Thus, $M, v \models \Diamond\!\!\!\!+(K_d q \wedge \neg K_a q)$.
- Other examples from the same model:

- $M, v \models \Diamond K_a q$, because $M, v \models \langle \neg q \rightarrow K_a \neg q ‡ \rangle K_a q$ (upper left in the figure). The announcement $\neg q \rightarrow K_a \neg q$ can be safely made in v by agents b, c and d.
- $M, v \models \Diamond K_d(q \vee \neg p)$, because $M, v \models \langle (K_c K_a p) \rightarrow q ‡ \rangle K_d(q \vee \neg p)$ (upper right in the figure). The announcement $(K_c K_a p) \rightarrow q$ can be safely made in v by agents a, b and c.
- While $M, v \models \Diamond K_d q \wedge \Diamond K_a q$, $M, v \models \neg \Diamond (K_a q \wedge K_d q)$ (or, said another way, $M, v \models \boxplus \neg (K_a q \wedge K_d q)$) – while there is an anonymous announcement after which a learns that q, and one after which d learns that q, there is none after which both of them learns that q.

Initially, neither d nor a know that p implies q, but after the anonymous announcement d *does* but a doesn't.

Using the agent-based operators we can be more specific about the identity of the whistleblower:

$$M, v \models \Diamond_b (K_d q \wedge \neg K_a q)$$

We in fact also have:

$$M, v \models \Diamond_a (K_d q \wedge \neg K_a q)$$

– Bob can be the whistleblower! This seems highly counter-intuitive, but it illustrates the difference between *being able to* and *knwowing that you are able to* – a well-known distinction in the logic of knowledge and ability [6]. Indeed, we have that:

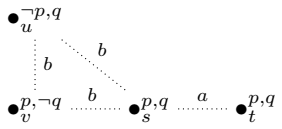
$$M, v \models K_b \Diamond_b (K_d q \wedge \neg K_a q) \quad \text{but} \quad M, v \models \neg K_a \Diamond_a (K_d q \wedge \neg K_a q)$$

– while both b and a can make some successful anonymous announcement, only b knows that he can. We now delve deeper into the relationship between knowledge and ability.

5 Knowing that vs. knowing how

We show that it is possible that a knows *that* she can make ϕ true with some anonymous public announcement ($K_a \Diamond_a \phi$) without a knowing *how*, i.e., which announcement to make – knowing *de dicto* but not knowing *de re*. This distinction is well-known in general [6], but it is not obvious that it applies in the case of anonymous announcements. The following example is adapted from an example in [2].

Consider the following model, where there are four agents (only reflexive access for c and d).



It is easy to see that $\blacktriangle p$ holds in (only) v, s, t, and $\blacktriangle q$ in (only) u, s, t. Let

$$\phi = K_b q \wedge (\neg K_b p \vee \hat{K}_a(K_b p \wedge \neg K_b q))$$

We argue that

$$M, s \models K_a \Diamond\!\!\!\!\!- _a \phi$$

but that the same announcement doesn't work in both s and t. We have that

$$M, s \models K_a \blacktriangle q \wedge \langle \blacktriangle q! \rangle \phi$$

and thus that

$$M, s \models \langle q!_a \rangle \phi \text{ and } M, s \models \Diamond\!\!\!\!\!- _a \phi$$

Similarly for t and p:

$$M, t \models K_a \blacktriangle p \wedge \langle \blacktriangle p! \rangle \phi \quad M, t \models \langle p!_a \rangle \phi \quad M, t \models \Diamond\!\!\!\!\!- _a \phi$$

Thus:

$$M, s \models K_a \Diamond\!\!\!\!\!- _a \phi$$

But:

$$M, s \not\models \langle \blacktriangle p! \rangle \phi \quad \text{and thus} \quad M, s \not\models \langle p!_a \rangle \phi$$

and similarly for t and q:

$$M, t \not\models \langle \blacktriangle q! \rangle \phi \quad M, t \not\models \langle q!_a \rangle \phi$$

It is thus possible to know *that* you can achieve some goal with some anonymous announcement, without knowing *how*, i.e., which announcement you can use.

While the "de dicto" property is expressed by the formula $K_a \Diamond\!\!\!\!\!- _a \phi$, a natural question then is whether the "de re" property, i.e.,

$$\exists \psi \forall t \sim_a s \; M, t \models \langle \psi!_a \rangle \phi$$

– agent a knows which announcement she should use – can be expressed. In standard combinations of knowledge modalities and general ability modalities it cannot [6], but when ability is defined in terms of (standard) public announcements it in fact can [2]. The same is in fact true in the case of anonymous announcements. The following can be shown in a very similar way to the proof of the corresponding property in [2].

Lemma 3. *$M, s \models \Diamond\!\!\!\!\!- _a K_a \phi$ iff $\exists \psi \forall t \sim_a s \; M, t \models \langle \psi!_a \rangle \phi$.*

6 Axiomatisation and Soundness

We define Hilbert-style axiomatic systems for the language $\mathcal{L}_{!\blacktriangle \boxplus}$ and $\mathcal{L}_{!\blacktriangle N}$ introduced in Sections 3.1 and 3.2 respectively, and argue that they are sound. In the next section we look at completeness. The systems are *infinitary*: they contain infinitary deduction rules; rules with possibly infinitely many premises. A proof in such a system is a tree, with possibly infinite branching but finite depth.

6.1 Arbitrary Anonymous Public Announcement Logic

We first consider the language $\mathcal{L}_{!\blacktriangle\boxplus}$.The system $\mathbf{S}_{\boxplus}$ is defined in Table 1. It has three parts. The first consists of standard axioms and rules for epistemic logic [14], as well as standard reduction axioms for public announcement logic [12]. The second consists of an axiom and an infinitary rule for safety[2]. The third consists of an axiom and an infinitary rule for the $\boxplus$ operator.

The Box1 axiom is a standard axiom going from an arbitrary anonymous announcement to a particular one. The Box2 rule takes us in the opposite direction. The latter makes use of a notion called a *necessity form*, originating in [17] and also used in completeness proofs for APAL and GAL [7, 2]. Formally, the set of necessity forms $\eta(\sharp)$ is defined recursively as follows, where $\varphi \in \mathcal{L}_{!\blacktriangle\boxplus}$, $a \in N$ and $\sharp \notin P$:

$$\eta(\sharp) ::= \sharp \mid \varphi \to \eta(\sharp) \mid K_a\varphi \mid \boxplus\eta(\sharp) \mid [\varphi!]\eta(\sharp)$$

The result of replacing the unique $\sharp$ with φ in a necessity form $\eta(\sharp)$ is denoted $\eta(\varphi)$.

All instances of propositional tautologies	Prop
From ϕ and $\phi \to \psi$, infer ψ	MP
$K_a(\phi \to \psi) \to (K_a\phi \to K_a\psi)$	K
$K_a\phi \to \phi$	T
$\neg K_a\phi \to K_a\neg K_a\phi$	5
$[\phi!]p \leftrightarrow (\phi \to p)$	Atoms
$[\phi!]\neg\psi \leftrightarrow (\phi \to \neg[\phi!]\psi$	Negation
$[\phi!](\psi_1 \wedge \psi_2) \leftrightarrow ([\phi!]\psi_1 \wedge [\phi!]\psi_2)$	Conjunction
$[\phi!]K_a\psi \leftrightarrow (\phi \to K_a[\phi!]\psi)$	Knowledge
$[\phi!][\psi!]\chi \leftrightarrow [\phi \wedge [\phi!]\psi!]\chi$	Composition
From ϕ, infer $K_a\phi$	NecK
$\blacktriangle\varphi \to \blacktriangle_n\varphi$	Safe1
From $\eta(\blacktriangle_n\varphi)$ for all $n \in \mathbb{N}$, infer $\eta(\blacktriangle\varphi)$	Safe2
$\boxplus\varphi \to [\blacktriangle\psi!]\varphi$ for any $\psi \in \mathcal{L}_K$	Box1
From $\eta([\blacktriangle\psi!]\varphi)$ for all ψ, infer $\eta(\boxplus\varphi)$	Box2

Table 1. Axiom schemata and inference rules of $\mathbf{S}_{\boxplus}$, over the language $\mathcal{L}_{!\blacktriangle\boxplus}$.

When S is an axiomatic system, we write $\vdash_S \phi$ to denote the fact that ϕ is derivable in S, and say that ϕ is a *theorem* of S. S is *sound* if for any formula ϕ, $\vdash_S \phi$ implies $\models \phi$; it is *complete* if for any formula ϕ, $\models \phi$ implies $\vdash_S \phi$.

[2] We note that there is in fact a finitary axiomatisation of the safety operator [5]. However, similarly to the axiomatisation of APAL with common knowledge in [3], since we already have an infinitary axiomatisation due to $\boxplus$ we use a simpler infinitary axiomatisation also of safety here.

Lemma 4. *The system* $\mathbf{S}_{\boxplus}$ *is sound.*

Proof. That the axioms and rules in the upper part of Table 1 are valid/preserve validity is well-known. Validity of Safe1 and Box1 is straightforward from the definition. That Safe2 and Box2 preserve validity follows from a straightforward induction on necessity forms.

6.2 Agent-based Arbitrary Anonymous Public Announcement Logic

We now consider the language $\mathcal{L}_{!\blacktriangle N}$ with the agent-based $\boxplus_a$ operators. The system $\mathbf{S}_{\boxplus^{\mathbf{N}}}$ over this language is defined in Table 2 – it is obtained by replacing Box1 and Box2 in $\mathbf{S}_{\boxplus}$ (Table 1) with the agent-based variants Box1' and Box2'. The forms of these are immediate from Lemma 2.

All instances of propositional tautologies	Prop
From ϕ and $\phi \to \psi$, infer ψ	MP
$K_a(\phi \to \psi) \to (K_a\phi \to K_a\psi)$	K
$K_a\phi \to \phi$	T
$\neg K_a\phi \to K_a\neg K_a\phi$	5
From ϕ, infer $K_a\phi$	NecK
$[\phi!]p \leftrightarrow (\phi \to p)$	Atoms
$[\phi!]\neg\psi \leftrightarrow (\phi \to \neg[\phi!]\psi$	Negation
$[\phi!](\psi_1 \wedge \psi_2) \leftrightarrow ([\phi!]\psi_1 \wedge [\phi!]\psi_2)$	Conjunction
$[\phi!]K_a\psi \leftrightarrow (\phi \to K_a[\phi]\psi)$	Knowledge
$[\phi][\psi]\chi \leftrightarrow [\phi \wedge [\phi]\psi]\chi$	Composition
$\blacktriangle\varphi \to \blacktriangle_n\varphi$	Safe1
From $\eta(\blacktriangle_n\varphi)$ for all $n \in \mathbb{N}$, infer $\eta(\blacktriangle\varphi)$	Safe2
$\boxplus_a\varphi \to (K_a\blacktriangle\psi \to [\blacktriangle\psi!]\varphi)$ for any $\psi \in \mathcal{L}_K$	Box1'
From $\eta(K_a\blacktriangle\psi \to [\blacktriangle\psi!]\varphi)$ for all ψ, infer $\eta(\boxplus_a\varphi)$	Box2'

Table 2. Axiom schemata and inference rules of $\mathbf{S}_{\boxplus^{\mathbf{N}}}$, over the language $\mathcal{L}_{!\blacktriangle N}$.

The following can be shown in the same way as Lemma 4.

Lemma 5. *The system* $\mathbf{S}_{\boxplus^{\mathbf{N}}}$ *is sound.*

7 Completeness

7.1 Arbitrary Anonymous Public Announcement Logic

In this section, we prove the completeness of $\mathbf{S}_{\boxplus}$, the infinitary axiomatisation for $\mathcal{L}_{!\blacktriangle\boxplus}$. The proof strategy is based on the completeness proof for APAL [9] and for APAL with common knowledge [3]. The latter is especially relevant here as common knowledge, like the safety operator, has a fixpoint characterisation.

We first define theories, that are infinite sets with a closure condition, that will be used as states in our canonical model construction.

Definition 4. *A* theory *x is a set that contains all theorems and is closed under MP, NecK, Safe2, and Box2. Thus, the smallest theory is the set of all theorems of $\mathbf{S}_{\boxplus}$. We call theory x* consistent *if there is no $\phi \in \mathcal{L}_{!\blacktriangle\boxplus}$ such that $\phi \in x$ and $\phi \notin x$. We also call theory x* maximal *if any theory y such that $x \subset y$ is inconsistent.*

For theories we in particular have that theory x is maximal if and only if for all $\phi \in \mathcal{L}_{!\blacktriangle\boxplus}$ it holds that either $\phi \in x$ or $\neg\phi \in x$ (see [3, Lemma 9] for a proof).

The next lemma is proved similarly to [3, Lemma 10] using Safe2, Box1, and Box2 for the cases of safety and arbitrary anonymous announcements.

Lemma 6. *Let x be a theory, $a \in N$, and let $\phi \in \mathcal{L}_{!\blacktriangle\boxplus}$. Then $x + \phi = \{\chi \mid \phi \to \chi \in x\}$, $K_a x = \{\chi \mid K_a\chi \in x\}$, and $[\phi!]x = \{\chi \mid [\phi!]\chi \in x\}$ are theories as well. Moreover, $x + \phi$ is consistent if and only if $\neg\phi \notin x$.*

The final step before defining the canonical model is to state the Lindenbaum lemma that states that every consistent theory can be extended to a maximal consistent theory. Its proof is standard following the proofs for [9] and [3], and we omit it for brevity.

Lemma 7. *If x is a consistent theory, then it can be extended to a maximal consistent theory y such that $x \subseteq y$.*

With all the ingredients in place, we define the canonical model.

Definition 5. *The model $\mathcal{M} = (\mathcal{S}, \mathcal{R}, \mathcal{V})$, where $\mathcal{S} = \{x \mid x$ is a maximal consistent theory$\}$, $\mathcal{R}(a) = \{(x, y) \mid K_a x \subseteq y\}$, and $\mathcal{V}(p) = \{x \mid p \in x\}$, is called* the canonical model.

The most important part in our completeness proof is the following Truth Lemma that ties together the formulas in maximal consistent theories with the truth of those formulas in the corresponding state of the canonical model.

For the induction hypothesis in the proof of the Truth Lemma, we need to define an appropriate size measure over formulas of $\mathcal{L}_{!\blacktriangle\boxplus}$ that will assign the greatest weight to arbitrary anonymous announcement operators, then to safety modalities, and then, finally, to all other operators and subformulas. Luckily for us, we can use relation $<^{\forall}_{\blacktriangle}$, which is exactly like the corresponding relation $<^{\forall}_{\blacksquare}$ in [3, Definition 5.3], substituting $\blacksquare$ with $\blacktriangle$, $\Box^n_G$ with $\blacktriangle_n$, and $[G]$ with $\boxplus$. Note that such a relation $<^{\forall}_{\blacktriangle}$ is a well-founded partial order.

Lemma 8. *For all maximal consistent theories x and formulas $\phi \in \mathcal{L}_{!\blacktriangle\boxplus}$, $\phi \in x$ if and only if $\mathcal{M}, x \models \phi$.*

Proof. Boolean, epistemic, and public announcement cases follow from the proof of Lemma 11 in [9]. Here, we focus on the cases involving safety and arbitrary anonymous announcements.

Induction Hypothesis (IH): for all maximal consistent theories y and formulas $\phi \in \mathcal{L}_{!\blacktriangle\boxplus}$, if $\psi <^{\forall}_{\blacktriangle} \phi$, then $\psi \in y$ iff $\mathcal{M}, y \models \psi$.

Case $\phi = [\psi!]\blacktriangle\chi$. *From left to right.* Assume that $[\psi!]\blacktriangle\chi \in x$. Since x is a theory, and hence contains all the theorems of $\mathbf{S}_{\boxplus}$, we have $[\psi!](\blacktriangle\chi \to \blacktriangle_n\chi)$ for all $n \in \mathbb{N}$ due to Safe1 and the standard necessitation rule for public announcements. Moreover, it is a theorem of PAL [12, Proposition 4.46], and hence of our logic, that $[\psi!](\blacktriangle\chi \to \blacktriangle_n\chi) \to ([\psi!]\blacktriangle\chi \to [\psi!]\blacktriangle_n\chi)$, and hence $[\psi!](\blacktriangle\chi \to \blacktriangle_n\chi) \to ([\psi!]\blacktriangle\chi \to [\psi!]\blacktriangle_n\chi) \in x$ for all $n \in \mathbb{N}$. Since x is closed under MP, we then have that $[\psi!]\blacktriangle\chi \to [\psi!]\blacktriangle_n\chi \in x$. From the latter, using the assumption that $[\psi!]\blacktriangle\chi \in x$ and MP, we conclude that $[\psi!]\blacktriangle_n\chi \in x$ for all $n \in \mathbb{N}$. By **IH**, this is equivalent to the fact that for all $n \in \mathbb{N}$, it holds that $\mathcal{M}, x \models [\psi!]\blacktriangle_n\chi$. By the semantics of public announcements, the latter is equivalent to the fact that $\mathcal{M}, x \models \psi$ implies $\mathcal{M}^{\psi!}, x \models \blacktriangle_n\chi$ for all $n \in \mathbb{N}$, which, in turn, is equivalent to $\mathcal{M}, x \models \psi$ implies $\mathcal{M}^{\psi!}, x \models \blacktriangle\chi$ by the semantics of safety. Applying the definition of the semantics of public announcements one more time, we get $\mathcal{M}, x \models [\psi!]\blacktriangle\chi$.

From right to left. Let $\mathcal{M}, x \models [\psi!]\blacktriangle\chi$. By the definition of the semantics, it is equivalent to the fact that $\mathcal{M}, x \models \psi$ implies $\mathcal{M}^{\psi!}, x \models \blacktriangle\chi$. Using the definition of the semantics for safety, we obtain the equivalent statement that for all $n \in \mathbb{N}$, $\mathcal{M}, x \models \psi$ implies $\mathcal{M}^{\psi!}, x \models \blacktriangle_n\chi$. Now, applying the definition of the semantics of public announcements again, we get that for all $n \in \mathbb{N}$, $\mathcal{M}, x \models [\psi!]\blacktriangle_n\chi$. By **IH**, the latter is equivalent to the fact that for all $n \in \mathbb{N}$, $[\psi!]\blacktriangle_n\chi \in x$. Since $[\psi!]\blacktriangle_n\chi$ is in the necessity form, and since x is a theory, and thus closed under Safe2, we conclude that $[\psi!]\blacktriangle\chi \in x$.

Case $\phi = [\psi!] \boxplus \chi$. *From left to right.* Let $[\psi!] \boxplus \chi \in x$. Similarly to the previous case, we have that for all $\tau \in \mathcal{L}_K$, $[\psi!] \boxplus \chi \to [\psi!][\blacktriangle\tau!]\chi \in x$. Using the assumption that $[\psi!] \boxplus \chi \in x$ we get $[\psi!][\blacktriangle\tau!]\chi \in x$ for all $\tau \in \mathcal{L}_K$. By **IH**, we have the equivalent statement that for all $\tau \in \mathcal{L}_K$, $\mathcal{M}, x \models [\psi!][\blacktriangle\tau!]\chi$. Similarly to the previous case, we can conclude, using the definition of the semantics of arbitrary anonymous announcements, that $\mathcal{M}, x \models [\psi!] \boxplus \chi$.

From right to left. Assume that $\mathcal{M}, x \models [\psi!] \boxplus \chi$. Yet again, similarly to the previous case, we can use the definition of semantics to obtain $\mathcal{M}, x \models [\psi!][\blacktriangle\tau!]\chi$ for all $\tau \in \mathcal{L}_K$, which is equivalent to $[\psi!][\blacktriangle\tau!]\chi \in x$ for all $\tau \in \mathcal{L}_K$ by **IH**. Since $[\psi!][\blacktriangle\tau!]\chi$ is in the necessity form, and due to the fact that x is closed under Box2, we conclude $[\psi!] \boxplus \chi \in x$.

Case $\phi = \blacktriangle\psi$. *From left to right.* Suppose that $\blacktriangle\psi \in x$. Since x is a theory, and hence contains all the theorems of $\mathbf{S}_{\boxplus}$, we have that for all $n \in \mathbb{N}$, $\blacktriangle\psi \to \blacktriangle_n\psi \in x$. By MP, we get $\blacktriangle_n\psi \in x$ for all $n \in \mathbb{N}$. By **IH**, this is equivalent to $\mathcal{M}, x \models \blacktriangle_n\psi$ for every $n \in \mathbb{N}$, which, in turn in equivalent to $\mathcal{M}, x \models \blacktriangle\psi$ by the semantics.

From right to left. Let $\mathcal{M}, x \models \blacktriangle\psi$. By the semantics, this is equivalent to $\forall n \in \mathbb{N}$, $\mathcal{M}, x \models \blacktriangle_n\psi$. By **IH** we have $\blacktriangle_n\psi \in x$ for all $n \in \mathbb{N}$. Since x is a theory and thus closed under Safe2, we infer that $\blacktriangle\psi \in x$.

Case $\phi = \boxplus\psi$. Similar as the previous case, using Box1 and Box2.

Finally, we are ready to prove the completeness of the axiom system $\mathbf{S}_{\boxplus}$.

Theorem 1. *For all $\phi \in \mathcal{L}_{!\blacktriangle\boxplus}$, if ϕ is valid, then ϕ is derivable in $\mathbf{S}_{\boxplus}$.*

Proof. Assume towards a contradiction that φ is valid and ϕ is not a theorem of $\mathbf{S}_{\boxplus}$. Since the set of all theorems of $\mathbf{S}_{\boxplus}$ constitutes a consistent theory x, $x + \neg\phi$ is also a consistent theory. By the Lindenbaum lemma, we can extend $x + \neg\phi$ to a maximal consistent theory y such that $x + \neg\phi \subseteq y$. Note that $\neg\phi \in x + \neg\phi$, which implies $\neg\phi \in y$. By the Truth Lemma, this means that $\mathcal{M}, y \models \neg\phi$, which is equivalent to $\mathcal{M}, y \not\models \phi$ by the definition of the semantics, thus contradicting the assumption that φ is valid.

7.2 Agent-based Arbitrary Anonymous Public Announcement Logic

In this section, we prove the completeness of $\mathbf{S}_{\boxplus\mathbf{N}}$, the infinitary axiomatisation for $\mathcal{L}_{!\blacktriangle N}$.

The completeness proof for $\mathbf{S}_{\boxplus\mathbf{N}}$ follows exactly the same steps as the proof for $\mathbf{S}_{\boxplus}$, where we substitute $\boxplus$ for $\boxplus_a$, and use Box1' and Box2' in the proof of the Truth Lemma. Hence, we obtain the following result.

Theorem 2. *The system $\mathbf{S}_{\boxplus\mathbf{N}}$ over the language $\mathcal{L}_{!\blacktriangle N}$ (Section 3.2) is complete.*

8 Conclusions

We introduced operators for quantifying over anonymous public announcements that don't reveal the identity of the announcer. We saw that like for GAL [2], there is a distinction between knowing that you can achieve something with an anonymous public announcement, and knowing which announcement will be effective. Our main result is a completeness result, showing that the axiomatisation captures the most basic principles of such announcements. As for most other known axiomatisations of logics quantifying over announcements [7, 9, 8, 2], our axiomatisation is infinitary. The existence of a finitary axiomatisation for APAL or GAL is still an open problem; we can add the logics in this paper to that list. It is known that APAL extended with common knowledge is not finitarily axiomatisable [15]. Since the safety operator is somewhat similar to the common knowledge operator, the same might be true for our logic, although it does not seem to follow in a straightforward way from the proof in [15]. Another open problem is decidability of the satisfiability problem: we conjecture that it is undecidable, like, e.g., APAL and GAL [1].

There is plenty of other opportunities for future work. There are many combinations of operators that deserve attention: while we exploited the relationship between anonymous public announcements and announcements of safety and included standard public announcement operators in the language, of prime interest would be an axiomatisation of the language with anonymous public announcement operators and the safety operator but without standard public announcement operators, giving us the most basic and fundamental principles of

that operator. Finally, it can be mentioned that for APAL extended with common knowledge [3], the *scope* of quantification matters: if the scope is extended to include common knowledge, we get a different logic. We have left it for future work to find out whether this also holds for our quantifiers.

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