

GROUP KNOWING-HOW

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THE CONCEPT

Knowledge how / Abilities:

The ability of an agent to achieve a given goal.

IN THE LITERATURE

The prominent formal approaches.

- **Abilities** (e.g., Pauly 2002; Alur et al. 2002; Ågotnes et al. 2009; Mogavero et al. 2010):
 - joint actions (hence groups),
 - one-step,
 - $\langle C \rangle \varphi$ iff C has a collective action that guarantees φ .
 - Variations: e.g., Catta et al. (2025) and Belardinelli et al. (2019).
- **Knowledge how** (e.g., Wang 2018; Li and Wang 2019; 2021; Liu and Wang 2025):
 - individual agents,
 - ‘complex’ plans (finite sequence of basic actions),
 - $\mathbf{Kh}(\varphi, \psi)$ iff there is an action leading to ψ whenever φ holds.
 - Variations: e.g., Li (2017) and Li and Wang (2017).

THE GENERAL GOAL

*Study how the plans available to individual agents (and thus **an agent's individual abilities/knowledge-how**) can be combined to give raise to the abilities/knowledge-how of a group.*

OUR FOCUS

*We focus on the **sequential** forms of group know-how (i.e., an **'assembly/production line'** approach): a group can use sequences of actions whose components can be performed by its members/subgroups.*

Let

- **Prop** be a countable set of propositional symbols,
- **Act** be a countable set of symbols for basic actions,
- **Agt** $\neq \emptyset$ a finite set of agents.

PLANS

Definition (Plans)

- **Act*** is the set of *plans*.
- $|\sigma|$ is the *length* of $\sigma \in \mathbf{Act}^*$.
- $\sigma^{\vdash k}$ is σ 's initial segment up to (and including) the k -th position ($0 \leq k \leq |\sigma|$).
- $\sigma[k]$ is the basic action in σ 's k -th position ($0 < k \leq |\sigma|$).

THE STRUCTURE

Definition (Multi-agent labelled transition system)

A *multi-agent labelled transition system* (MLTS) is a tuple

$$\mathfrak{M} = \langle \mathbf{S}, \mathbf{R}, \mathbf{A}, \mathbf{V} \rangle$$

where

- $\mathbf{S} \neq \emptyset$,
- $\mathbf{R} = \{\mathbf{R}_a \subseteq \mathbf{S} \times \mathbf{S} \mid a \in \mathbf{Act}\}$,
- $\mathbf{V} : \mathbf{S} \rightarrow \wp(\mathbf{Prop})$,
- $\mathbf{A} : \mathbf{Agt} \rightarrow \wp(\mathbf{Act}^*)$ satisfies $\epsilon \in \mathbf{A}(i)$ for every $i \in \mathbf{Agt}$.

The pair $(\mathfrak{M}, s)$ is called a *pointed MLTS*.

$\mathbf{R}$ is extended to elements of $\mathbf{Act}^*$ in the standard way.

THE BASIC LANGUAGE

A very simple language.

Definition (Language L_{Kh_i})

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathbf{Kh}_i(\varphi, \varphi).$$

with $p \in \mathbf{Prop}$ and $i \in \mathbf{Agt}$.

$\mathbf{Kh}_i(\psi, \varphi)$: “when ψ is the case, agent i knows how to achieve φ ”.

SEMANTIC INTERPRETATION

Let $\mathfrak{M} = \langle \mathbf{S}, \mathbf{R}, \mathbf{A}, \mathbf{V} \rangle$ be an MLTS.

Definition (Strong executability)

A plan $\sigma \in \mathbf{Act}^*$ is *strongly executable* (SE) at $s \in \mathbf{S}$ iff, for all $k \in [0 .. |\sigma|-1]$,
 $t \in \mathbf{R}_{\neg k}(s)$ implies $\mathbf{R}_{\sigma[k+1]}(t) \neq \emptyset$.

Define $\text{SE}(\sigma) := \{s \in \mathbf{S} \mid \sigma \text{ is SE at } s\}$.

A ‘complex’ (global!) modality.

Definition (Semantic interpretation)

Given $\llbracket \chi \rrbracket^{\mathfrak{M}} := \{s \in \mathbf{S} \mid \mathfrak{M}, s \models \chi\}$, define

$\mathfrak{M}, s \models \mathbf{Kh}_i(\psi, \varphi)$ iff there is a $\sigma \in \mathbf{A}(i)$ such that:

(i) $\llbracket \psi \rrbracket^{\mathfrak{M}} \subseteq \text{SE}(\sigma)$, & (ii) $\mathbf{R}_\sigma(\llbracket \psi \rrbracket^{\mathfrak{M}}) \subseteq \llbracket \varphi \rrbracket^{\mathfrak{M}}$.

EXAMPLE

Example 1.

THE IDEA

The language L_{Kh_G} extends L_{Kh_i} with the following ($\emptyset \subset G \subseteq \mathbf{Agt}$).

$\mathfrak{M}, s \models \mathbf{Kh}_G(\psi, \varphi)$ iff there is $\sigma \in \mathbf{A}(G)$ such that:

- (i) $\llbracket \psi \rrbracket^{\mathfrak{M}} \subseteq \text{SE}(\sigma)$, &
- (ii) $\mathbf{R}_\sigma(\llbracket \psi \rrbracket^{\mathfrak{M}}) \subseteq \llbracket \varphi \rrbracket^{\mathfrak{M}}$.

Two ideal properties (common when relating individuals and groups):

(R1) For every $i \in \mathbf{Agt}$,

$$\mathbf{A}(\{i\}) = \mathbf{A}(i).$$

(R2) For every $G, H \subseteq \mathbf{Agt}$,

$$\text{if } G \subseteq H \text{ then } \mathbf{A}(G) \subseteq \mathbf{A}(H).$$

THE ASSEMBLY-LINE APPROACH

The idea: *each* G 's member gets to participate *at most once*.

Definition (The assembly line approach)

$\mathbf{ALP}(G) := \{\sigma_1 \cdots \sigma_{|G|} \in \mathbf{Act}^* \mid \sigma_t \in \mathbf{A}(\alpha'_G[t]) \text{ with } \alpha'_G \text{ a permutation of } \alpha_G\}.$

$\mathbf{L}_{\text{Kh}_G}^{\text{AL}}$ is the language $\mathbf{L}_{\text{Kh}_G}$ with $\mathbf{A}(G) := \mathbf{ALP}(G).$

Example. Take $G = \{1, 2\}$ with $\mathbf{A}(1) = \{\epsilon, a, b\}$ and $\mathbf{A}(2) = \{\epsilon, ab\}$. From $\mathbf{ALPS}(1.2) = \{\epsilon, ab, a, aab, b, bab\}$ and $\mathbf{ALPS}(2.1) = \{\epsilon, a, b, ab, aba, abb\}$, we get

$\mathbf{ALP}(\{1, 2\}) = \{\epsilon, ab, a, aab, b, bab, aba, abb\}.$

Note: $\mathbf{ALP}(G)$ satisfies both **(R1)** and **(R2)**.

THREE APPROACHES

EXAMPLE

Example 3.

THE FLEXIBLE ASSEMBLY-LINE APPROACH

The idea: *assembly-line* with turns *used as G sees fit*.

Definition (The flexible assembly line approach)

$$\mathbf{FALP}(G) := \{\sigma_1 \cdots \sigma_{|G|} \in \mathbf{Act}^* \mid \sigma_i \in \sqcup \{\mathbf{A}(i) \mid i \in G\}\}.$$

$\mathbf{L}_{\text{Kh}_G}^{\text{FAL}}$ is the language $\mathbf{L}_{\text{Kh}_G}$ with $\mathbf{A}(G) := \mathbf{FALP}(G)$.

Example. Take $G = \{1, 2\}$ with $\mathbf{A}(1) = \{\epsilon, a, b\}$ and $\mathbf{A}(2) = \{\epsilon, ab\}$. From $\mathbf{A}(1) \sqcup \mathbf{A}(2) = [\epsilon, a, b, \epsilon, ab]$, we get

$$\mathbf{FALP}(\{1, 2\}) = \{\epsilon, a, b, ab, aa, aab, ba, bb, bab, aba, abb, abab\}.$$

Note: $\mathbf{FALP}(G)$ satisfies both **(R1)** and **(R2)**.

THREE APPROACHES

EXAMPLE

Example 5.

ITERATED APPROACH

The idea: the *Kleene closure* of plans available to G 's members.

Definition (Iterated approach)

$$\mathbf{I}(G) := \left(\bigcup_{i \in G} \mathbf{A}(i) \right)^*.$$

$\mathbf{L}_{\text{Kh}_G}^I$ is the language $\mathbf{L}_{\text{Kh}_G}$ with $\mathbf{A}(G) := \mathbf{I}(G)$.

Example. Take $G = \{1, 2\}$ with $\mathbf{A}(1) = \{\epsilon, a, b\}$ and $\mathbf{A}(2) = \{\epsilon, ab\}$. Then,

$$\mathbf{I}(\{1, 2\}) = (\{\epsilon, a, b\} \cup \{\epsilon, ab\})^* = \{\epsilon, a, ab, bab, \dots\}.$$

Note: $\mathbf{I}(G)$ satisfies **(R2)** but does not satisfy **(R1)**. To get the latter, this approach is discussed only when $\mathbf{A}(i) = (\mathbf{A}(i))^*$.

THREE APPROACHES

EXAMPLE

Example 7

THE ASSEMBLY-LINE APPROACH

Proposition (Model checking for $L_{Kh_G}^{AL}$)

The model-checking problem for $L_{Kh_G}^{AL}$ is in P^{NP} .

Proof (sketch). An algorithm that, to evaluate $Kh_G(\psi, \varphi)$, uses an NP-oracle which guesses and verifies a group plan (which, note, is linear in the size of the group).

THE FLEXIBLE ASSEMBLY-LINE APPROACH

Proposition (Model checking for $L_{Kh_G}^{FAL}$)

The model-checking problem for $L_{Kh_G}^{FAL}$ is in P^{NP} .

Proof (sketch). A small adaptation of the previous case.

THE ITERATED APPROACH

Proposition (Model checking for $L_{Kh_G}^I$)

The model-checking problem for $L_{Kh_G}^I$ is **PSpace-complete**.

Proof (sketch). Witnesses for $Kh_G(\psi, \varphi)$ can be of exponential length.

RELATIVE EXPRESSIVITY

Proposition (Expressivity results)

- $L_{Kh_i} < L_{Kh_G}^{AL}$
- $L_{Kh_i} < L_{Kh_G}^{FAL}$
- $L_{Kh_i} < L_{Kh_G}^I$ (over models satisfying $\mathbf{A}(i) = (\mathbf{A}(i))^*$)
- $L_{Kh_G}^{AL}$ and $L_{Kh_G}^{FAL}$ are incomparable.

SUMMARY & WORK TO DO

So far.

- *Knowledge how for groups* based on *sequential composition*.
- *Three* forms of doing so.
- *Model checking* & some *relative expressivity* results.

To do.

- *Semantic characterisation of expressivity*.
- *Complexity* of the *satisfiability* problems.
- *Axiomatisations*.



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