Groups Vs. Coaltions

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Side by Side

GAL

$M_s \models [G]\varphi$ iff $\forall \psi_G \in \mathcal{PAL} : M_s \models [\psi_G]\varphi$

$M_s \models \langle G \rangle\varphi$ iff $\exists \psi_G \in \mathcal{PAL} : M_s \models \langle \psi_G \rangle\varphi$

CAL

$M_s \models [\langle G \rangle]\varphi$ iff $\forall \psi_G \exists \chi_{A \setminus G} : M_s \models \psi_G \to \langle \psi_G \wedge \chi_{A \setminus G} \rangle\varphi$

$M_s \models \langle [G] \rangle\varphi$ iff $\exists \psi_G \forall \chi_{A \setminus G} : M_s \models \psi_G \wedge [\psi_G \wedge \chi_{A \setminus G}]\varphi$

Truthful part

$\varphi_a := \Box_a \varphi$

Simultaneous part

$$\varphi_G := \bigwedge_{a \in G} \varphi_a$$

Side by Side

GAL

$$M_s \models [G]\varphi \text{ iff } \forall \psi_G \in \mathscr{PAL} : M_s \models [\psi_G]\varphi$$

$$M_s \models \langle G \rangle\varphi \text{ iff } \exists \psi_G \in \mathscr{PAL} : M_s \models \langle \psi_G \rangle\varphi$$

CAL

$$M_s \models [\langle G \rangle]\varphi \text{ iff } \forall \psi_G \exists \chi_{A \setminus G} : M_s \models \psi_G \rightarrow \langle \psi_G \land \chi_{A \setminus G} \rangle\varphi$$

$$M_s \models \langle [G] \rangle\varphi \text{ iff } \exists \psi_G \forall \chi_{A \setminus G} : M_s \models \psi_G \land [\psi_G \land \chi_{A \setminus G}]\varphi$$

Is it just me, or it looks like CAL modalities can be expressed with GAL modalities?

Side by Side

GAL

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$$M_s \models \langle G\rangle\varphi \text{ iff } \exists \psi_G \in \mathscr{PAL} : M_s \models \langle \psi_G\rangle\varphi$$

CAL

$$M_s \models [\langle G\rangle]\varphi \text{ iff } \forall \psi_G \exists \chi_{A\setminus G} : M_s \models \psi_G \to \langle \psi_G \wedge \chi_{A\setminus G}\rangle\varphi$$

$$M_s \models \langle [G]\rangle\varphi \text{ iff } \exists \psi_G \forall \chi_{A\setminus G} : M_s \models \psi_G \wedge [\psi_G \wedge \chi_{A\setminus G}]\varphi$$

$\langle [A]\rangle\varphi \leftrightarrow \langle A\rangle\varphi$: CAL and GAL modalities coincide for the grand coalition

$\langle [G]\rangle\varphi \to \langle G\rangle\varphi$: if a group can force φ in the presence of opponents, it can also force φ alone

Side by Side

GAL

$$M_s \models [G]\varphi \text{ iff } \forall \psi_G \in \mathscr{PAL} : M_s \models [\psi_G]\varphi$$

$$M_s \models \langle G \rangle\varphi \text{ iff } \exists \psi_G \in \mathscr{PAL} : M_s \models \langle \psi_G \rangle\varphi$$

CAL

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$$M_s \models \langle [G] \rangle\varphi \text{ iff } \exists \psi_G \forall \chi_{A\setminus G} : M_s \models \psi_G \wedge [\psi_G \wedge \chi_{A\setminus G}]\varphi$$

What about the following definition?

$\langle [G] \rangle\varphi \leftrightarrow \langle G \rangle [A \setminus G]\varphi$: we can decompose a coalition announcement into two group announcements

Side by Side

$$\langle[G]\rangle\varphi \to \langle G\rangle[A\backslash G]\varphi$$

Both G and $A\backslash G$ make their announcements simultaneously

$A\backslash G$ makes their announcement after G, and they may have learnt new epistemic formulas

We quantify over all announcements by $A\backslash G$, including *We know that after announcement ψ_G, we will learn $\chi_{A\backslash G}$*

Proposition. $\langle[G]\rangle\varphi \to \langle G\rangle[A\backslash G]\varphi$ is valid

Forgetting How To Play

$$\langle G\rangle[A\setminus G]\varphi \rightarrow \langle\!\langle G\rangle\!\rangle\varphi$$

Can we apply a similar reasoning to this direction?

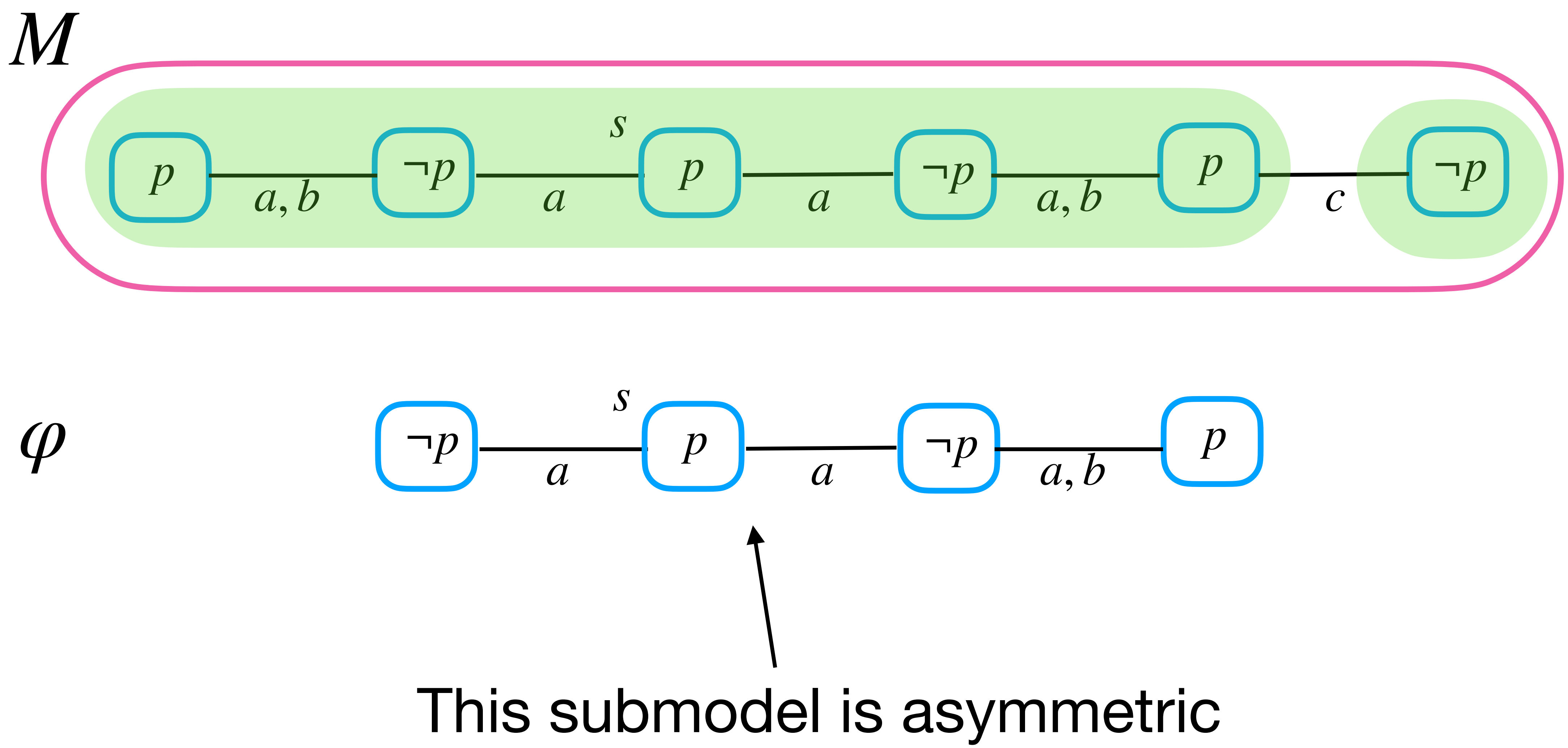
Forgetting How To Play

$$\langle G \rangle [A \backslash G] \varphi \rightarrow \langle\!\langle [G] \rangle\!\rangle \varphi$$

Can we apply a similar reasoning to this direction? No!

Forgetting How To Play

$\langle a \rangle [b, c] \neg \varphi \to \langle\!\![a]\!\!\rangle \neg \varphi$



Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

Forgetting How To Play

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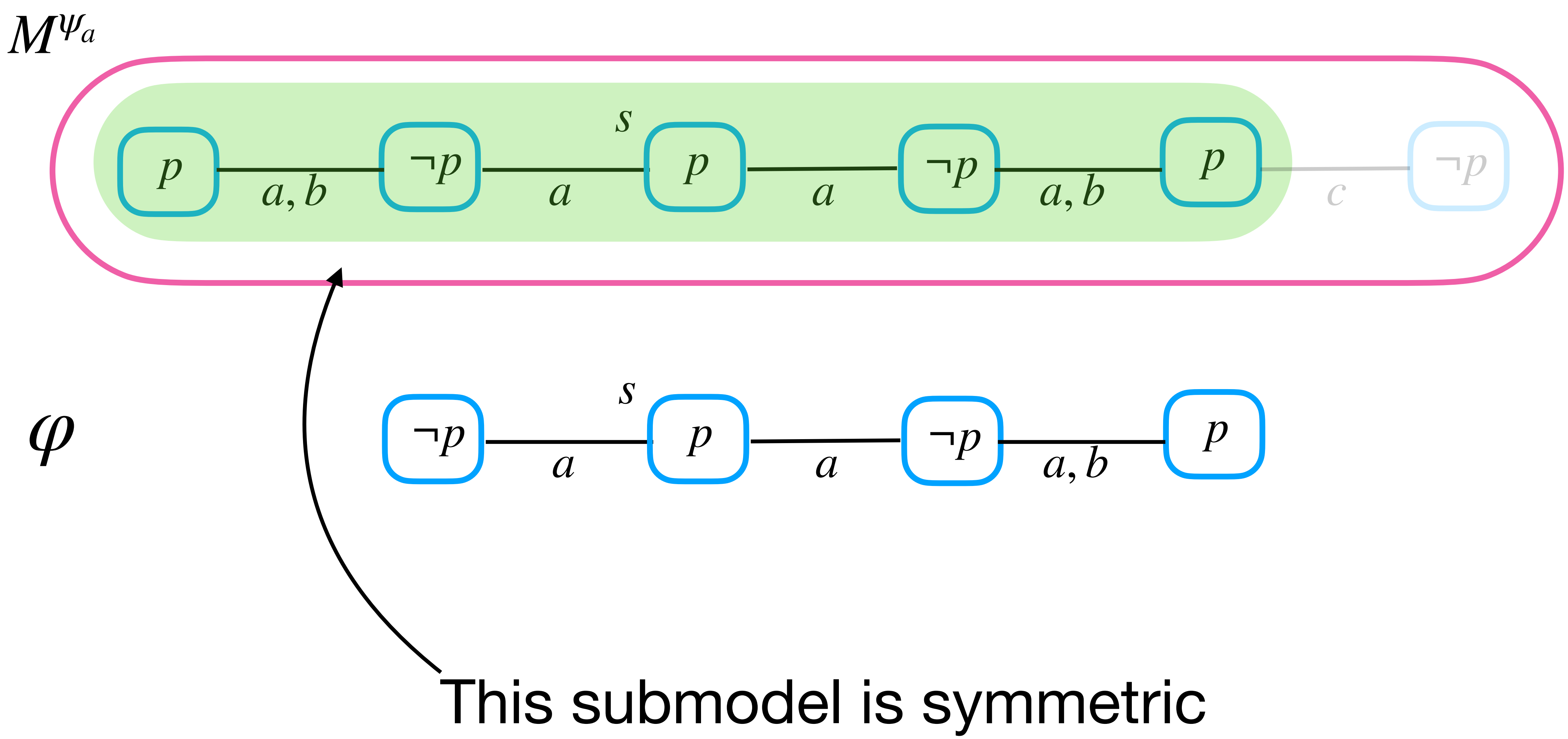

$\psi_a := \Box_a (\neg p \rightarrow \Diamond_b p)$

Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

Forgetting How To Play

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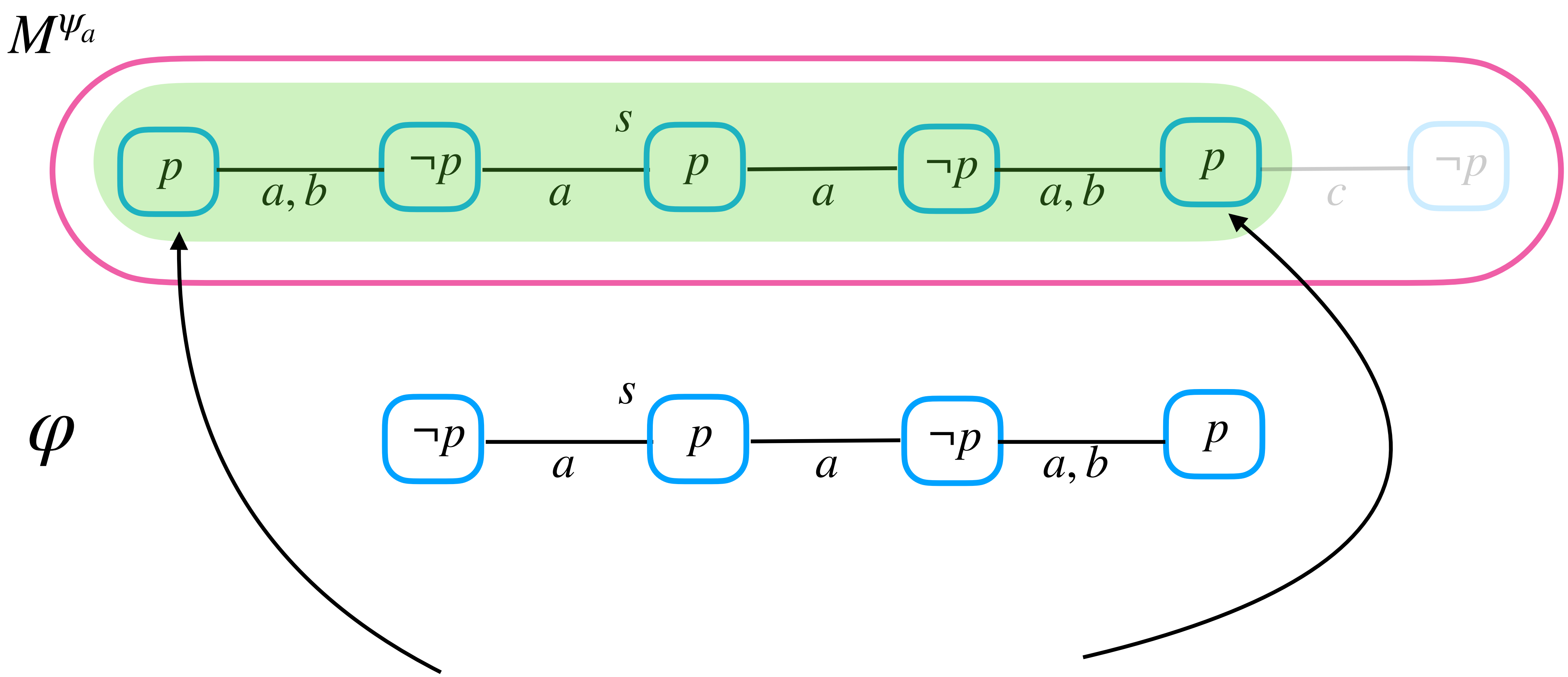

These states are identical

Any announcement that removes one, removes the other

$$\psi_a := \Box_a(\neg p \rightarrow \Diamond_b p)$$

Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

Forgetting How To Play

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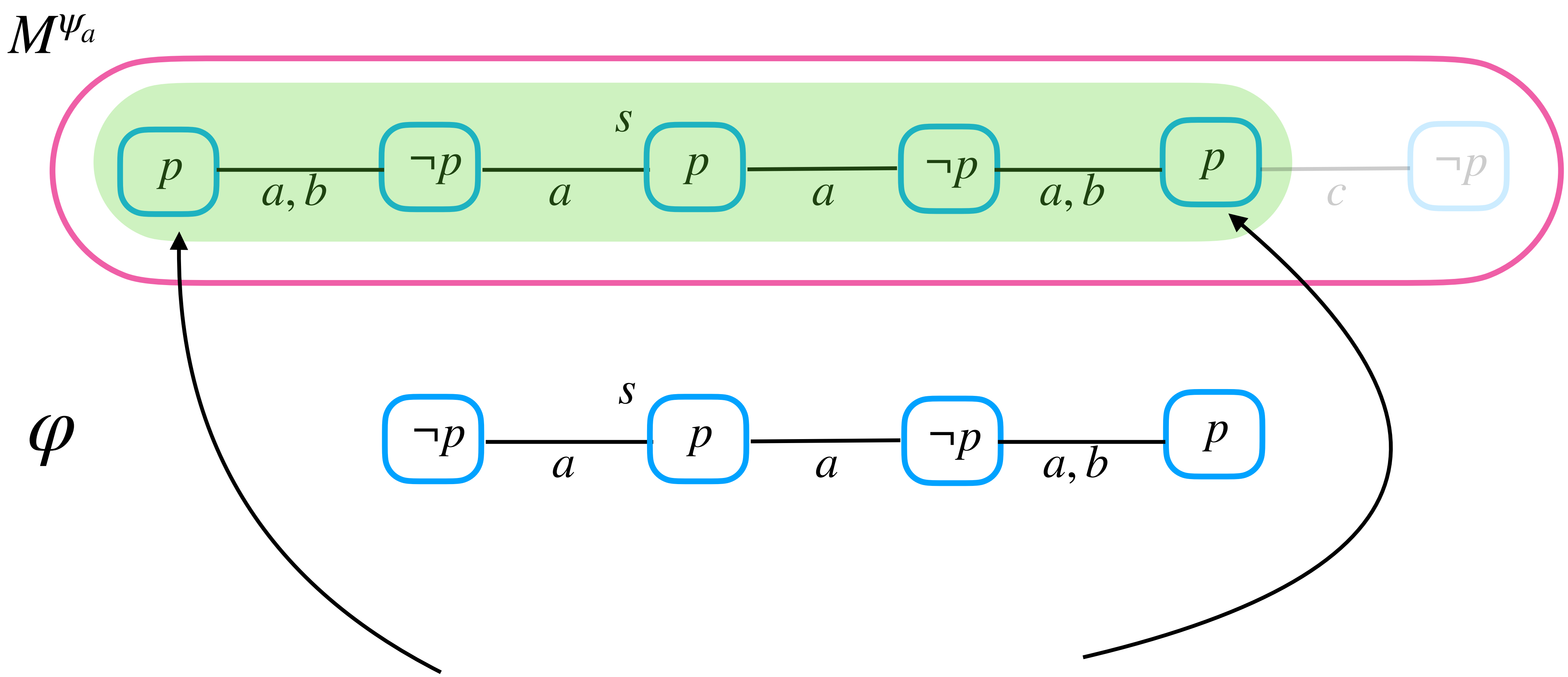

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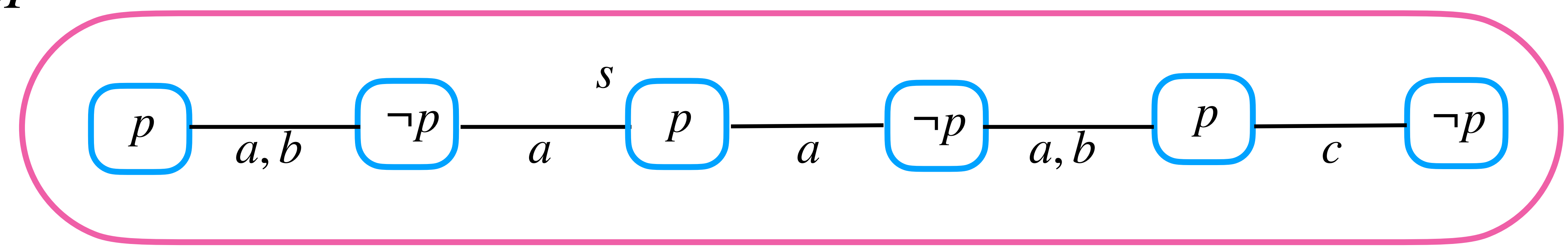
There is no way to make $M * \psi_a, s$ satisfy φ

Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

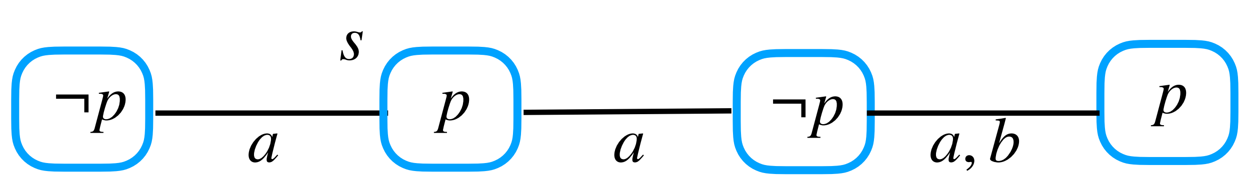
Forgetting How To Play

$$\langle a \rangle [b, c] \neg \varphi \rightarrow \langle [a] \rangle \neg \varphi$$

M^{ψ_a}



φ



We have that $M, s \models \langle a \rangle [b, c] \neg \varphi$

Left to show that $M, s \not\models \langle [a] \rangle \neg \varphi$, or, equivalently, $M, s \models [\langle a \rangle] \varphi$

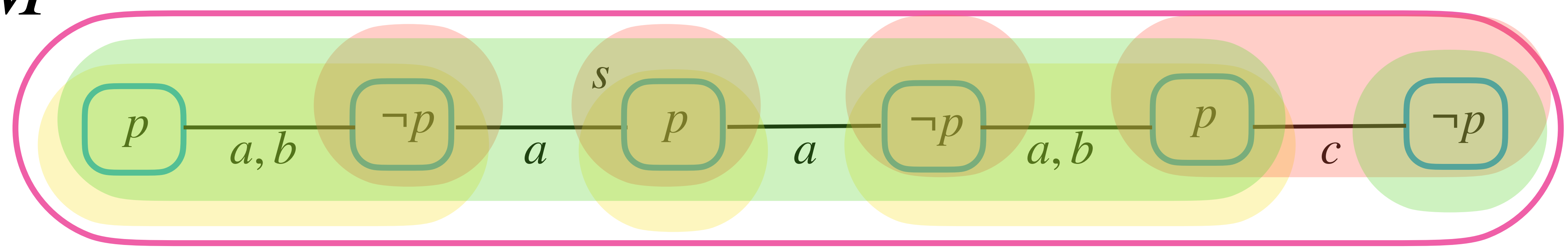
$M, s \models [\langle a \rangle] \varphi$: agents b and c can force φ no matter what a announces at the same time

Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

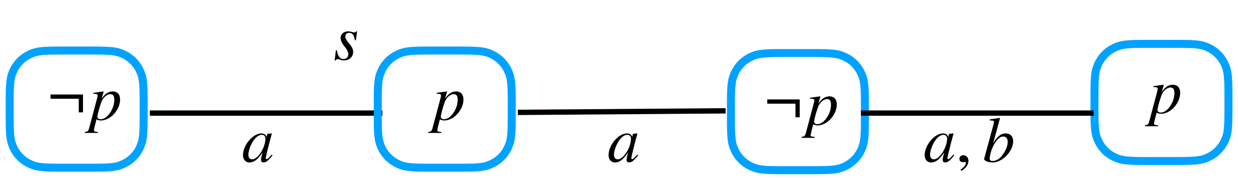
Forgetting How To Play

$$\langle a\rangle[b,c]\neg\varphi \rightarrow \langle\!\![a]\!\!\rangle\neg\varphi$$

M



φ



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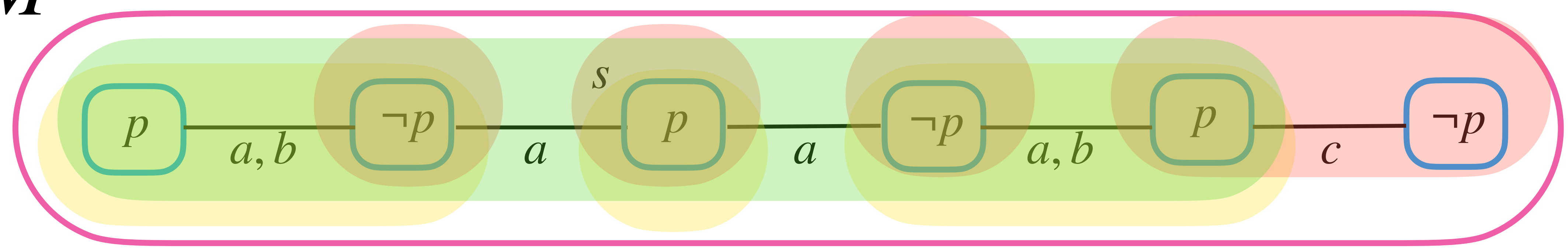
Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

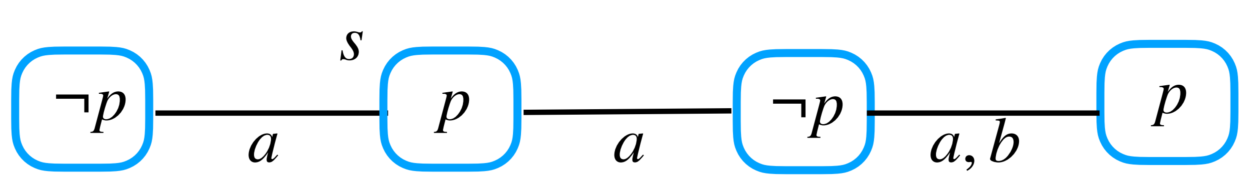
Forgetting How To Play

$$\langle a \rangle [b, c] \neg \varphi \rightarrow \langle [a] \rangle \neg \varphi$$

M



φ



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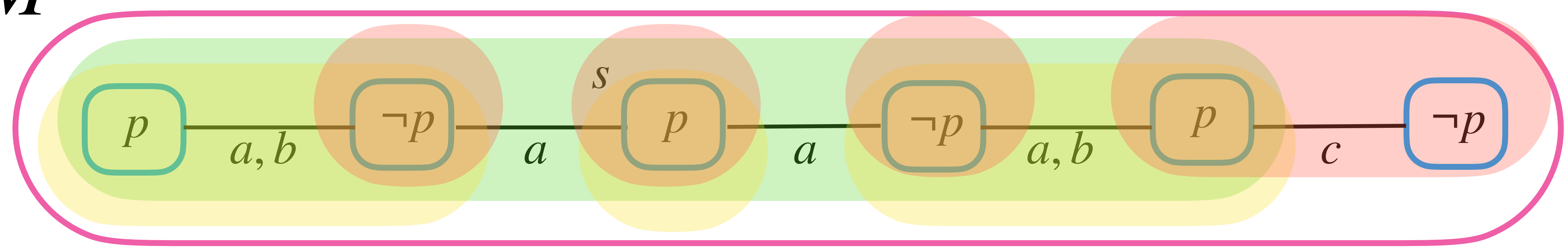
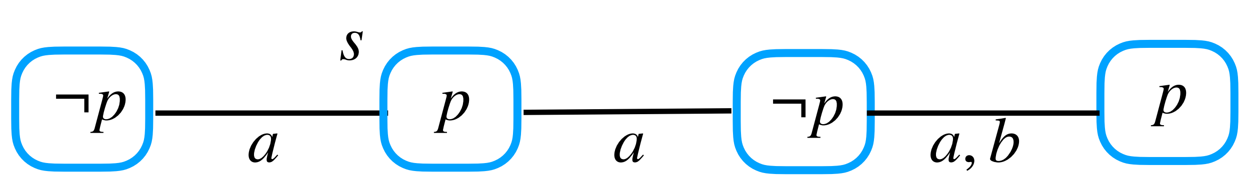
Forgetting How To Play

$\langle a\rangle[b, c]\neg\varphi$ ✗ $\langle\![a]\!\rangle\neg\varphi$

M



φ



This was but one possible translation of CAL modalities into GAL modalities

Maybe there is a translation that works?

We don’t know!

Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

Logics of Quantified Announcements

APAL is incomparable to GAL

There are some classes of models that GAL can distinguish and CAL cannot

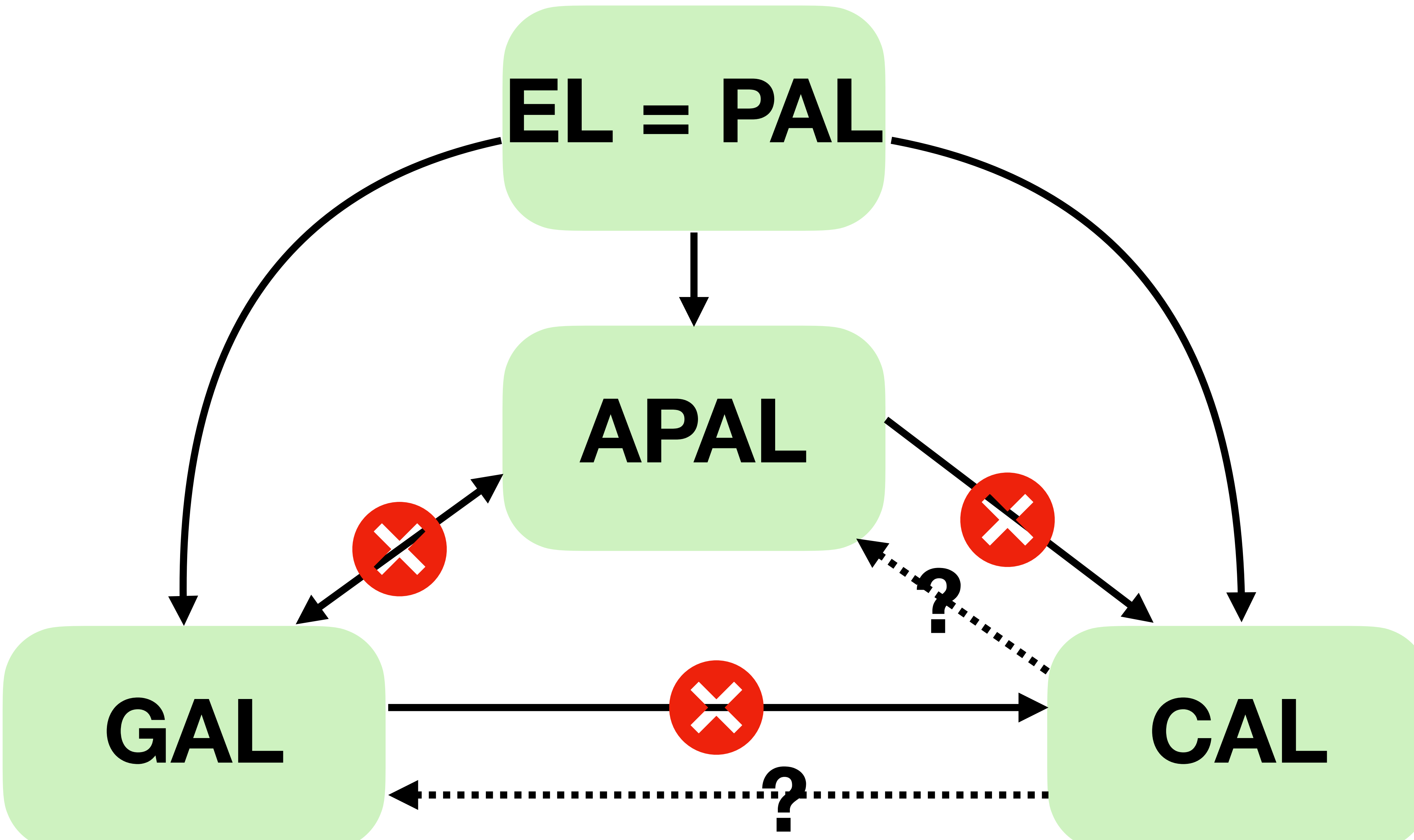
There are some classes of models that APAL can distinguish and CAL cannot

Open Problem. Full expressivity characterisation of APAL, GAL, and CAL

Conjecture. APAL, GAL, and CAL are mutually incomparable

Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

Logics of Quantified Announcements



Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

Forgetting How To Play

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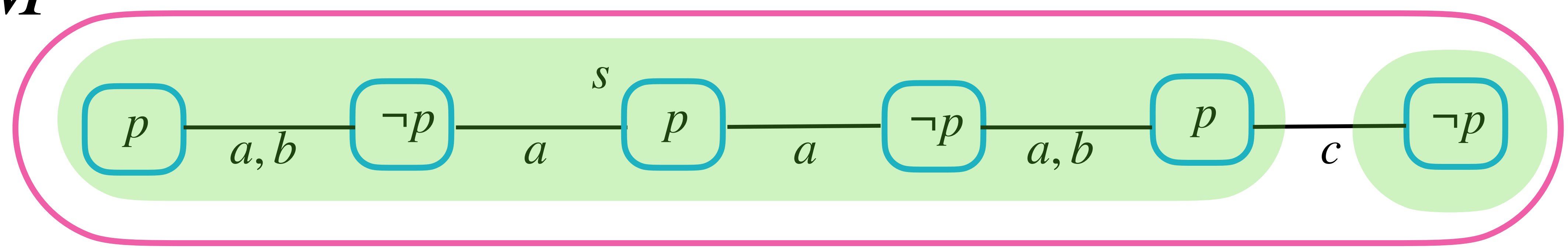
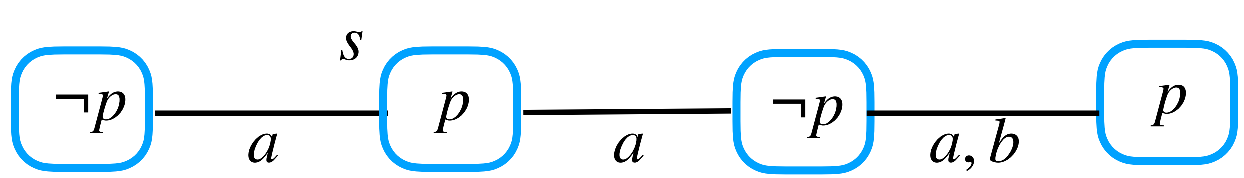


φ



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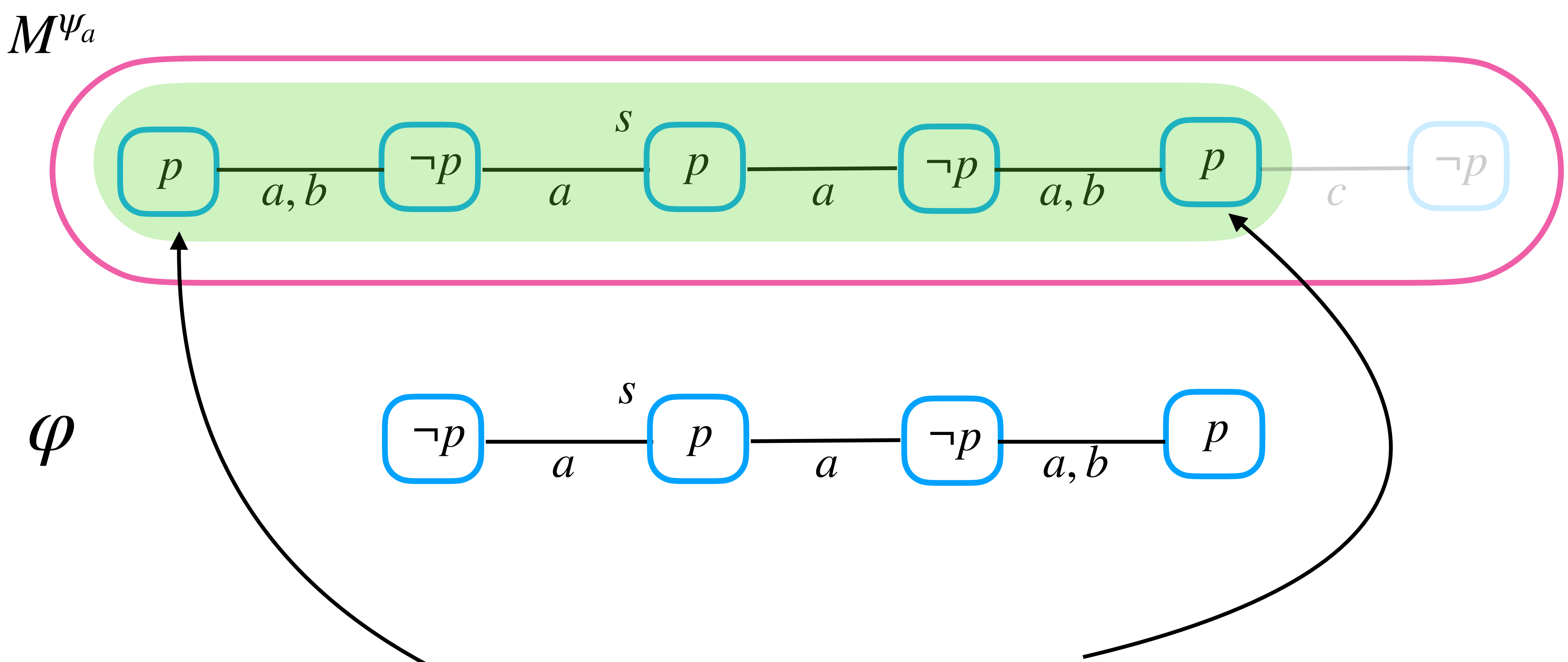

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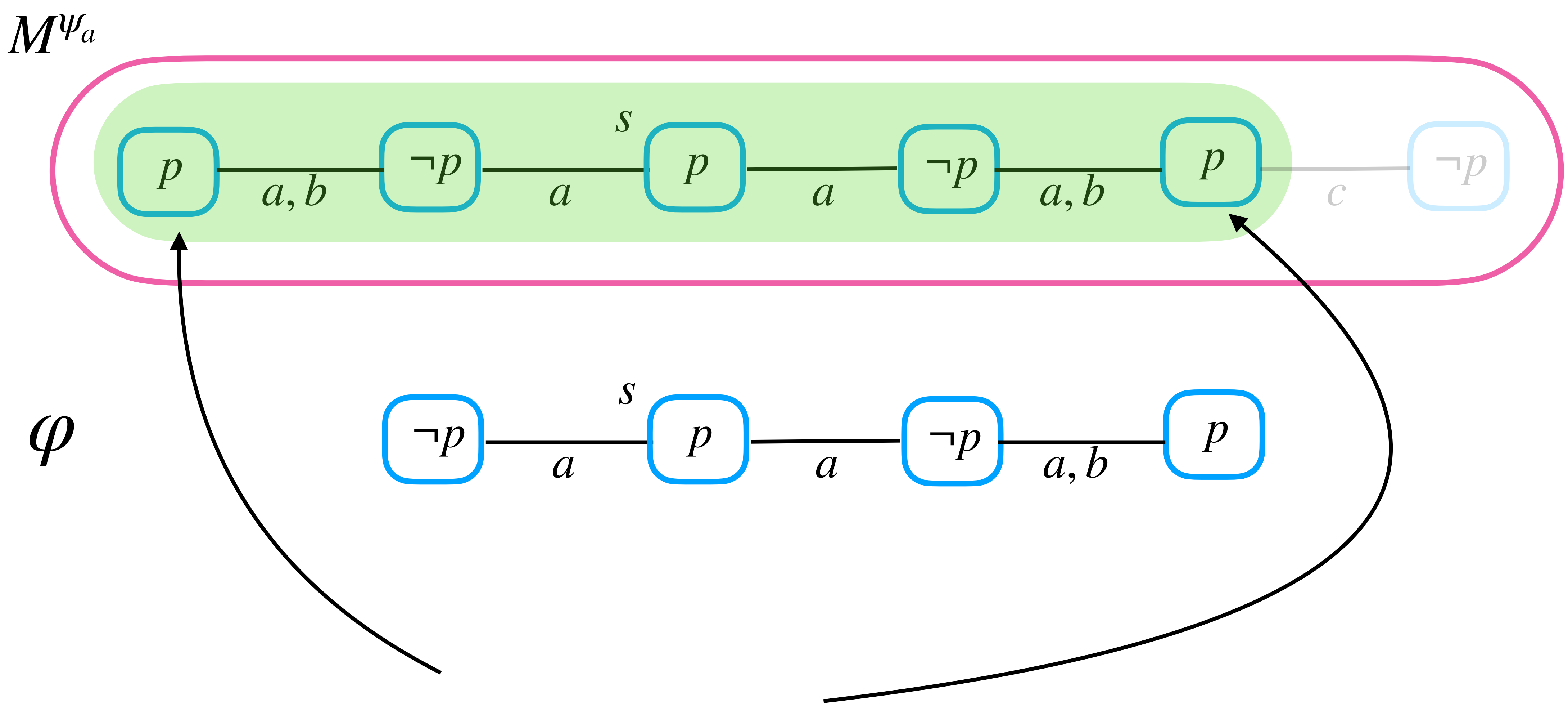


Agents b and c ‘forgot’ the difference between them

And thus they lost their distinguishing powers

$$\psi_a := \Box_a(\neg p \rightarrow \Diamond_b p)$$

Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

APAL with Memory

$$\mathscr{APALM} \ni \varphi ::= \top \mid p \mid 0 \mid \varphi^0 \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid U\varphi \mid [\varphi]\varphi \mid [!]\varphi$$

$$\mathscr{GALM} \ni \varphi ::= \top \mid p \mid 0 \mid \varphi^0 \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid U\varphi \mid [\varphi]\varphi \mid [G]\varphi$$

$$\mathscr{CALM} \ni \varphi ::= \top \mid p \mid 0 \mid \varphi^0 \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid U\varphi \mid [\varphi]\varphi \mid [\langle G \rangle]\varphi$$

An epistemic model with memory $M = (S, S^0, \sim, V)$ is an epistemic model, where S^0 is the initial domain, and $S = S^0 * \psi$ for some quantifer-free ψ

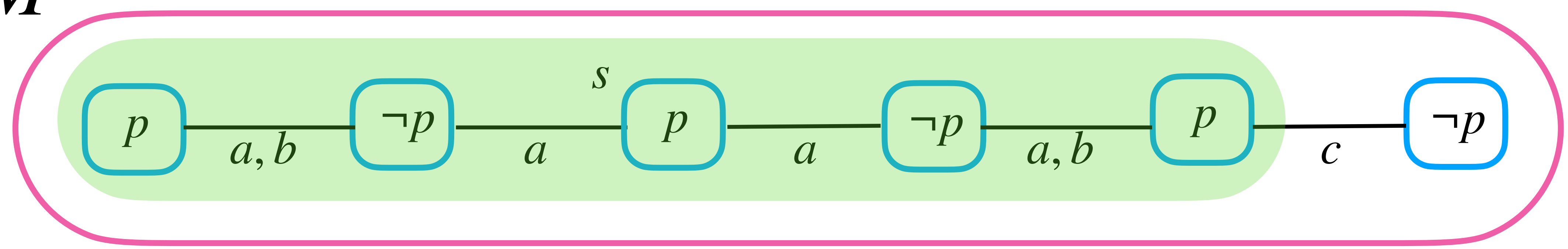
Agents have memory only of the initial configuration

Baltag et al. *Arbitrary Public Announcement Logic with Memory*, 2023.

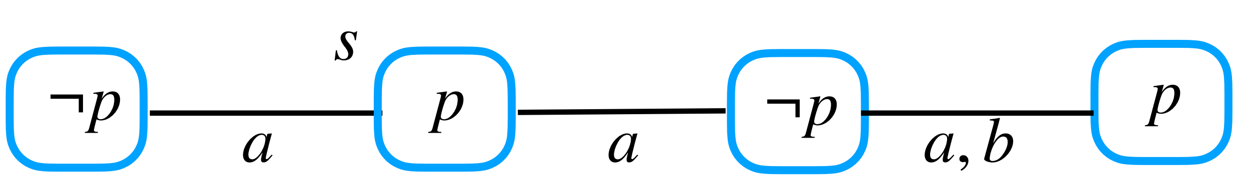
Remembering How To Play

$$\langle a \rangle [b, c] \neg \varphi \to \langle\!\langle [a] \rangle\!\rangle \neg \varphi$$

M



φ



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Remembering How To Play

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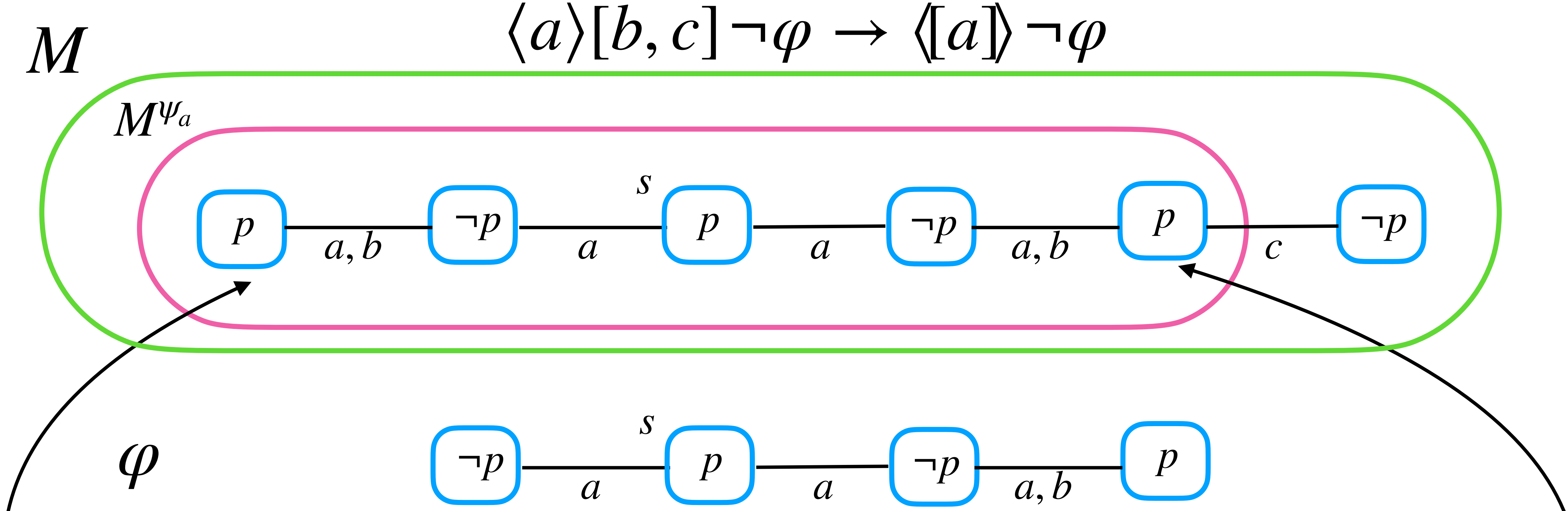


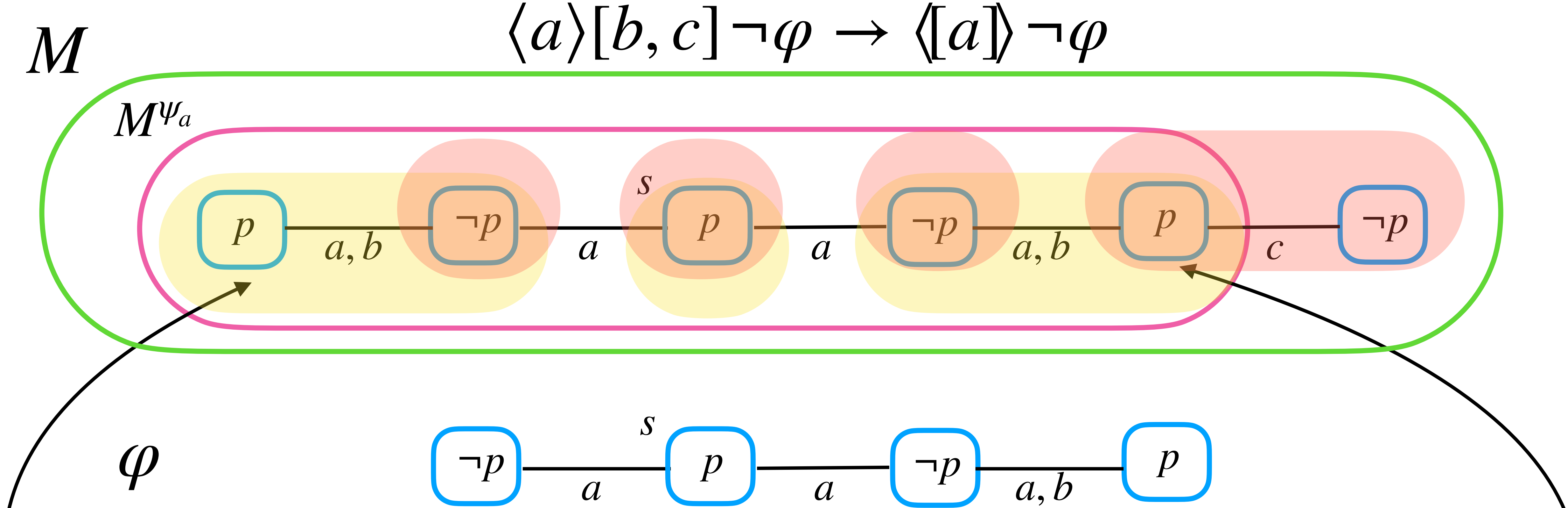
M is the initial model, and $M * \psi_a$ is the current model

Agents has access to the initial model, and thus these states are still distinguishable

Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

Remembering How To Play

$$\langle a \rangle [b, c] \neg \varphi \rightarrow \langle\!\![a]\!\!\rangle \neg \varphi$$



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Remembering How To Play

Proposition. $\langle G \rangle [A \setminus G] \varphi \leftrightarrow \langle\!\langle [G] \rangle\!\rangle \varphi$ is valid for GALM and CALM

Corollary. CALM can be translated to GALM

Open Problem. Is GALM translatable to CALM?

Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

Take-home message

- Group announcement logic (GAL) and Coalition announcement logic (CAL) are more agent-centric versions of APAL
- CAL is game-theoretic in its nature
- Most probably, APAL, GAL, and CAL are all different expressivity-wise

Take-home message

Open Problem. Is there a finitary axiomatisation of GAL?

Open Problem. Is there an axiomatisation, finitary or infinitary, of CAL (without additional modalities)?

Open Problem. Full expressivity characterisation of APAL, GAL and CAL

Open Problem. Is GALM translatable to CALM?