Group Announcement Logic

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We are dealing with S5 models (agents' relation is equivalence)

Overview of APAL

Axioms of EL and PAL

$[!]\varphi \rightarrow [\psi]\varphi$ with $\psi \in \mathscr{PAL}$

From $\{\eta([\psi]\varphi) \mid \psi \in \mathscr{PAL}\}$ infer $\eta([!]\varphi)$

Infinite number of premises

Open Problem. Is there a finitary axiomatisation of APAL?

Theorem. APAL is more expressive than PAL

Theorem. APAL is sound and complete

Theorem. SAT-APAL is undecidable

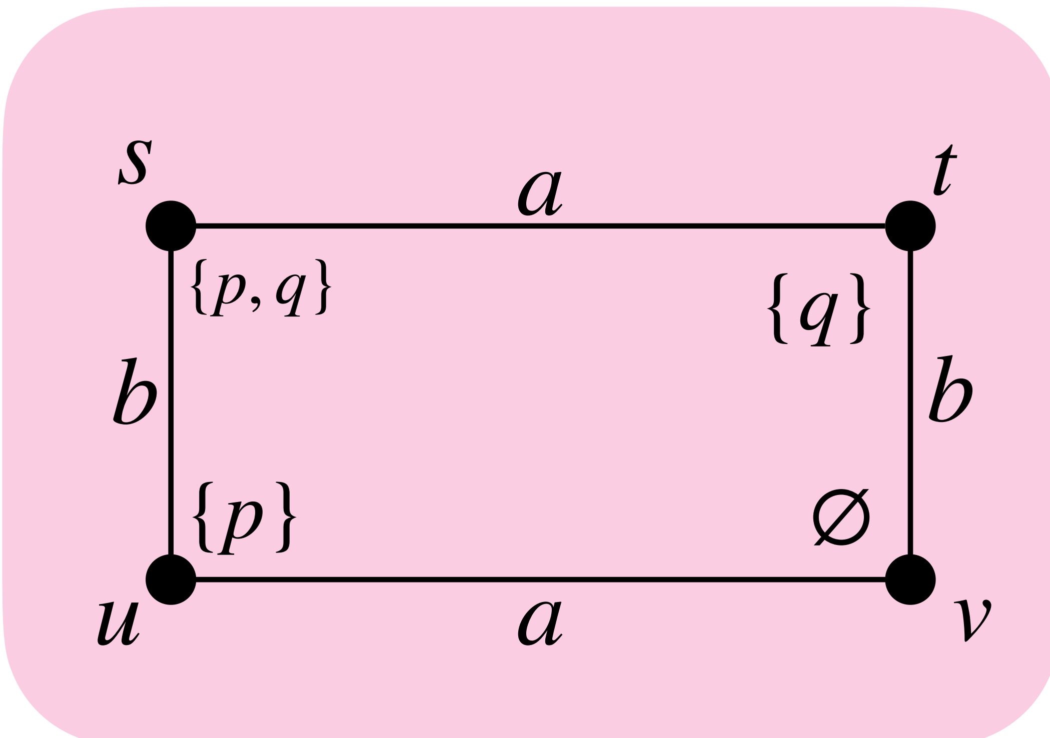
Theorem. Complexity of MC-APAL is PSPACE-complete

French, Van Ditmarsch. *Undecidability for arbitrary public announcement logic*, 2008.

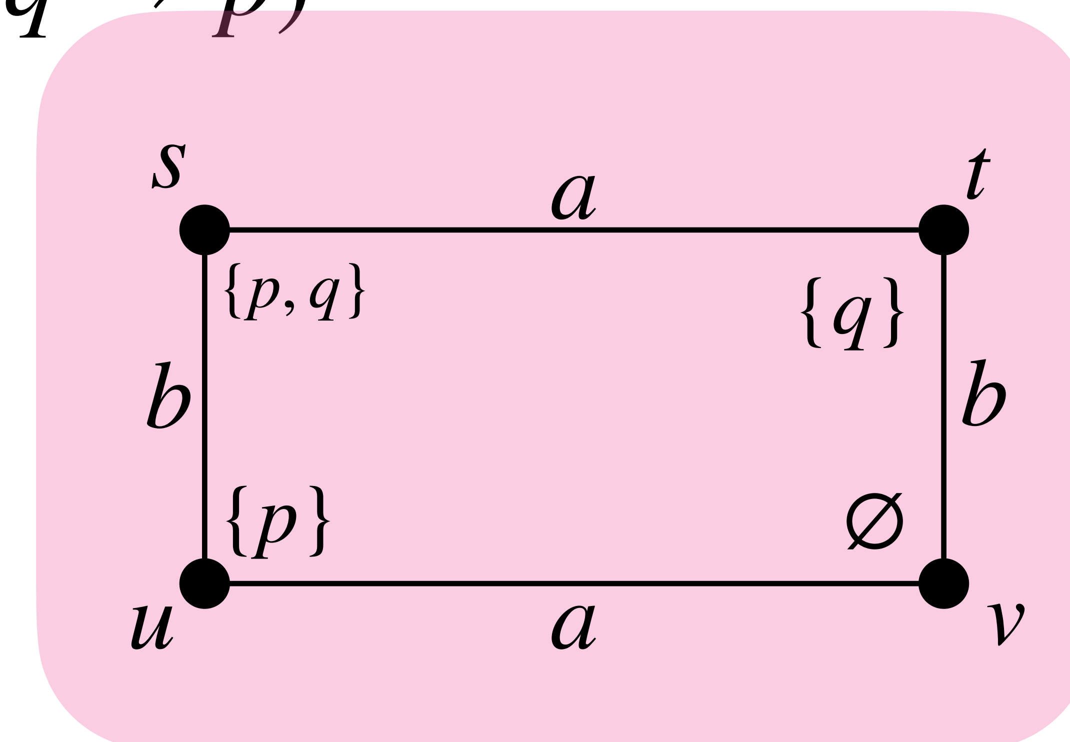
Balbiani, Van Ditmarsch. *A simple proof of the completeness of APAL*, 2015.

Letting agents do the work

M



$M * (q \to p)$



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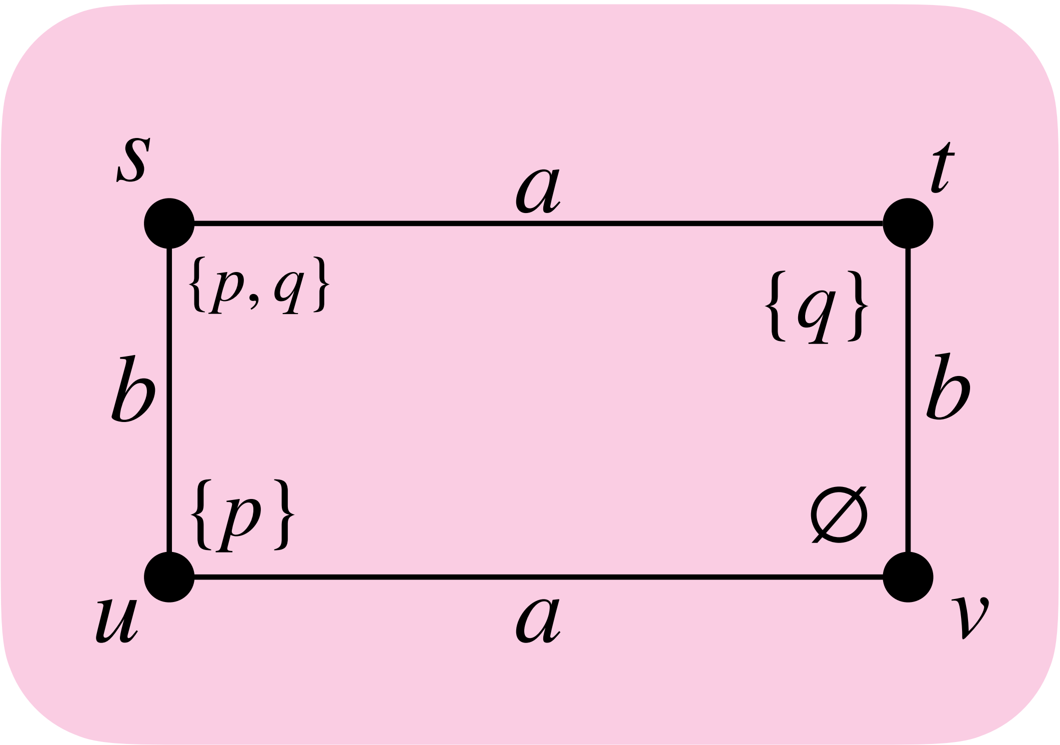

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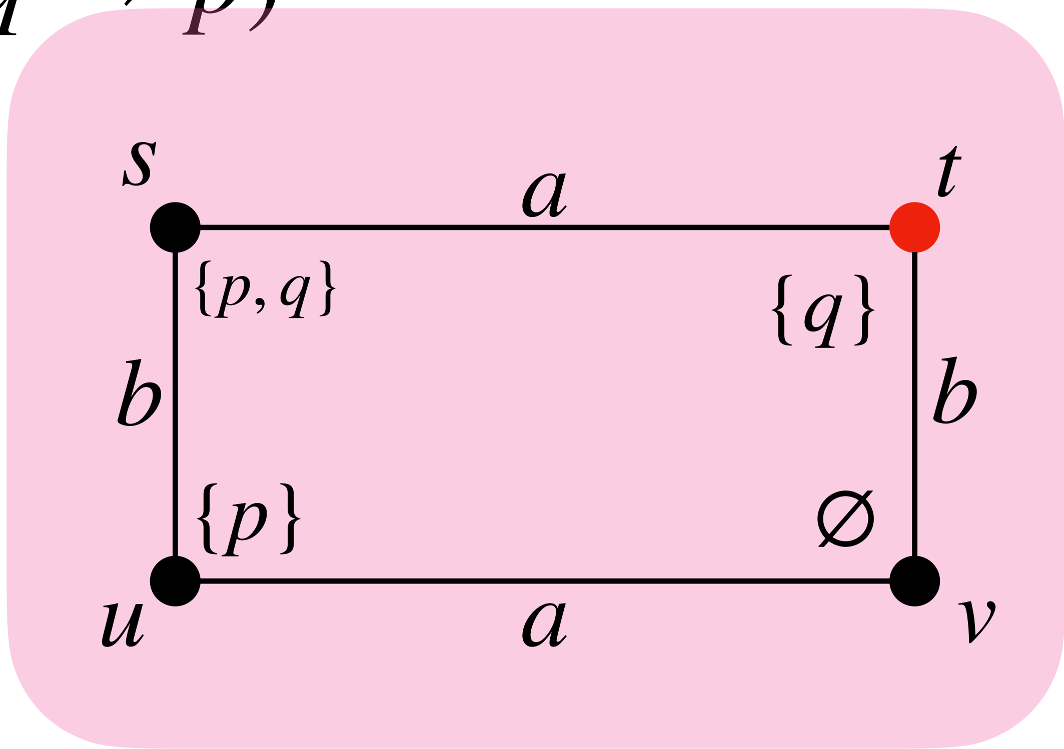

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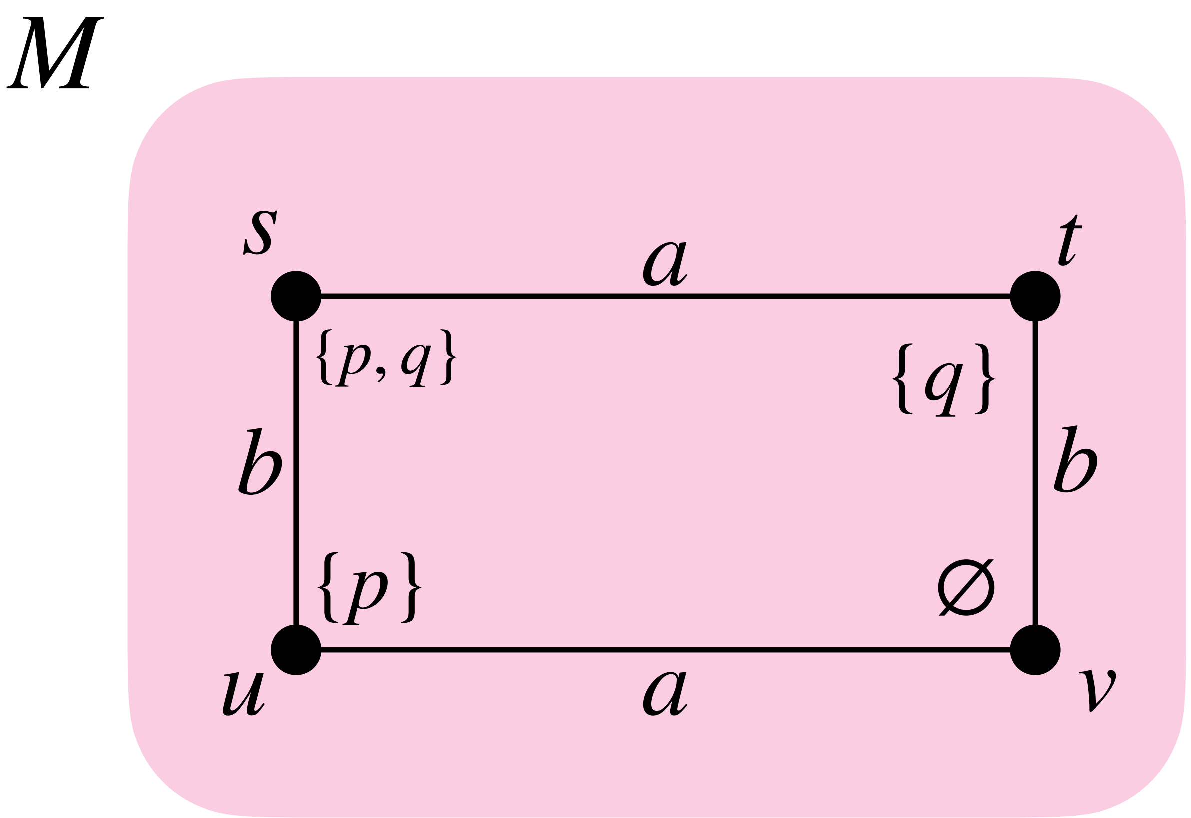
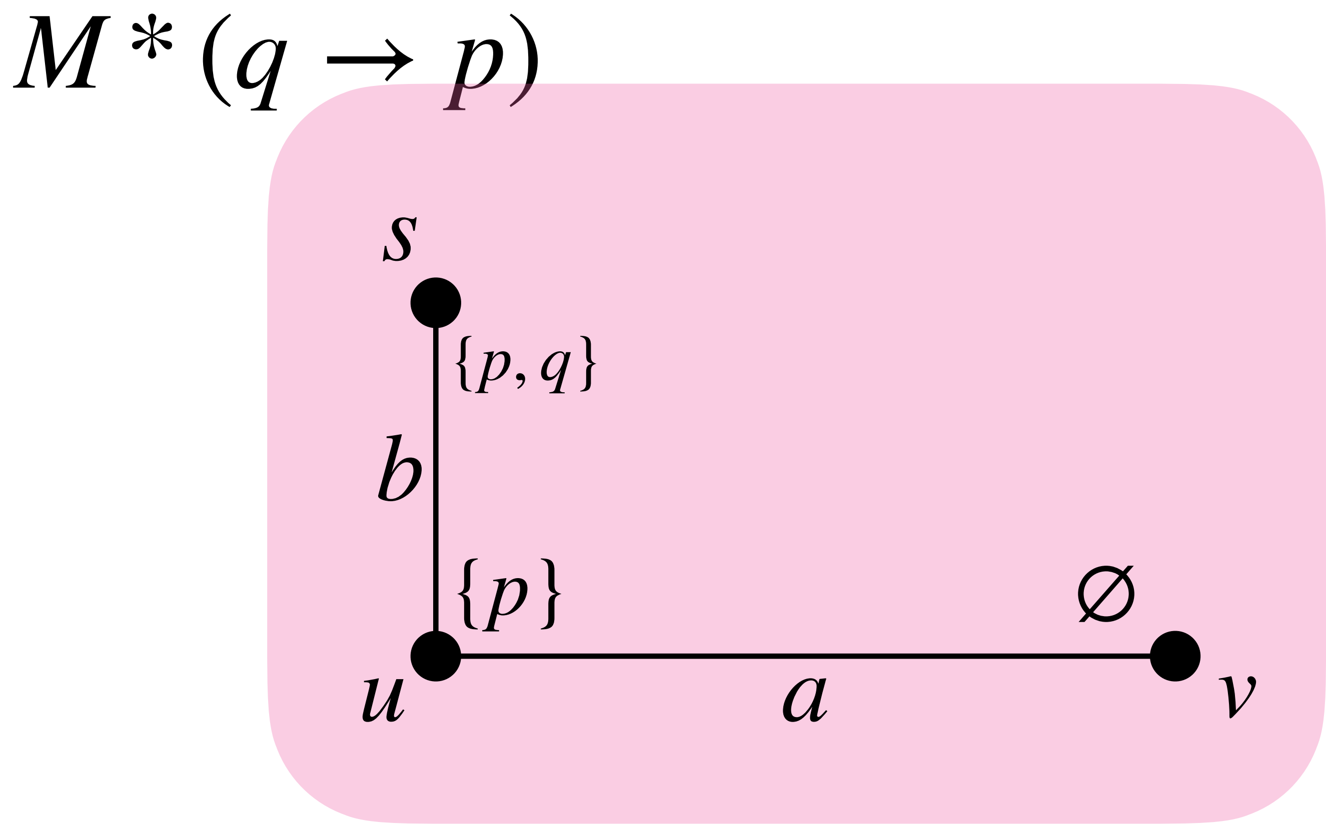
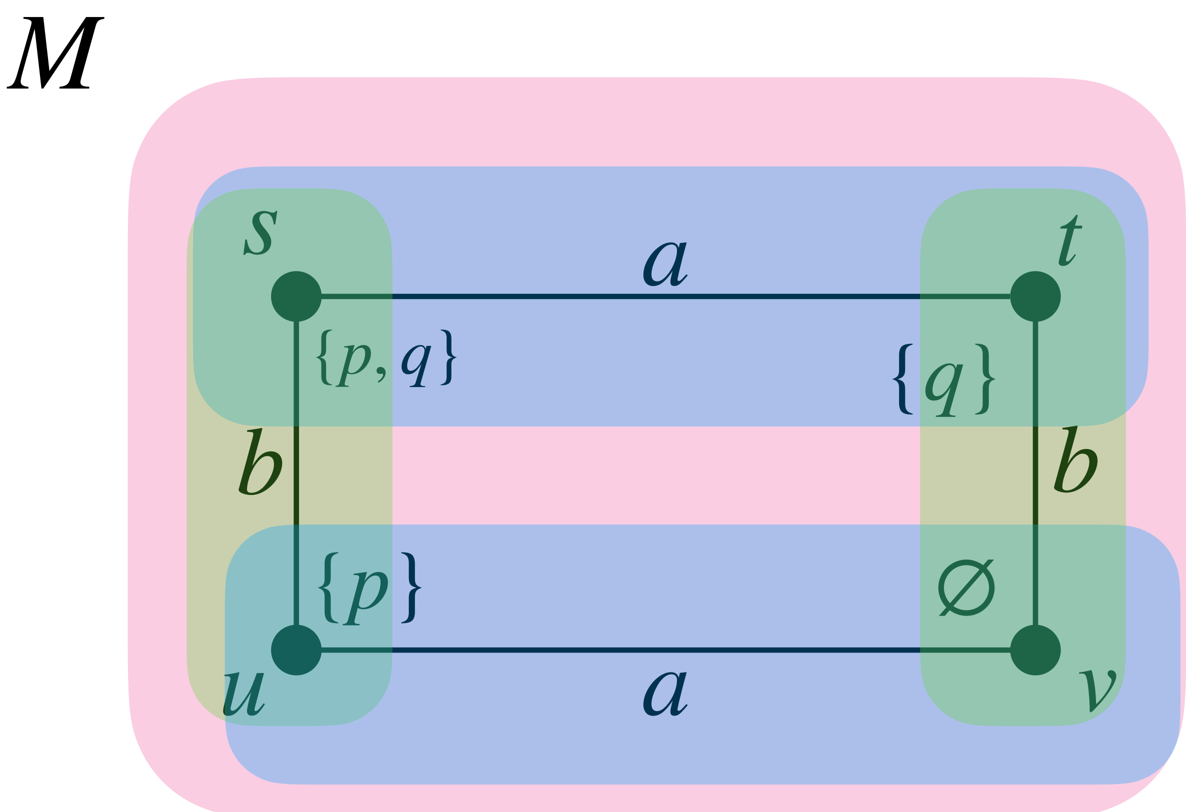


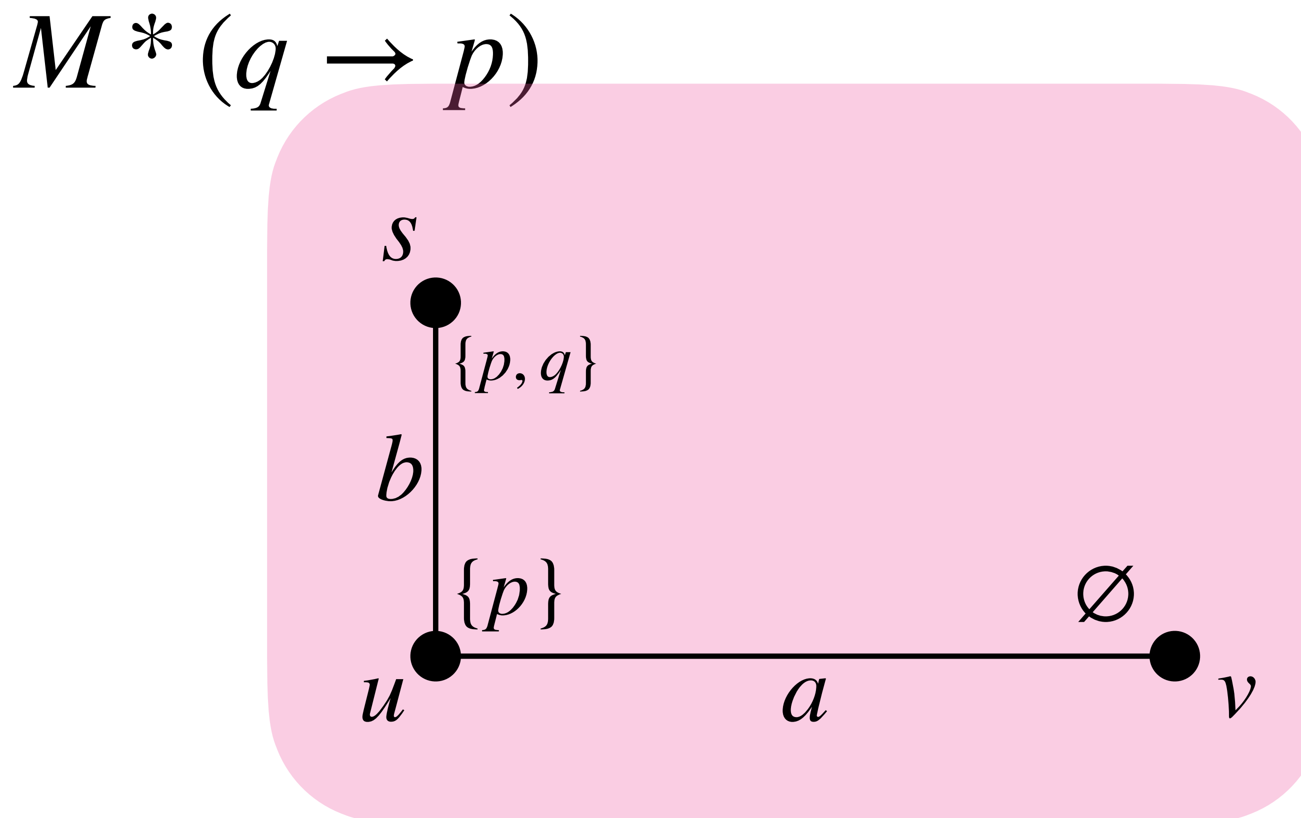


Announcement of $q \to p$ comes from an external source

Can agents a and b truthfully transform M the same way?

Letting agents do the work





Announcement of $q \to p$ comes from an external source

Can agents a and b truthfully transform M the same way?

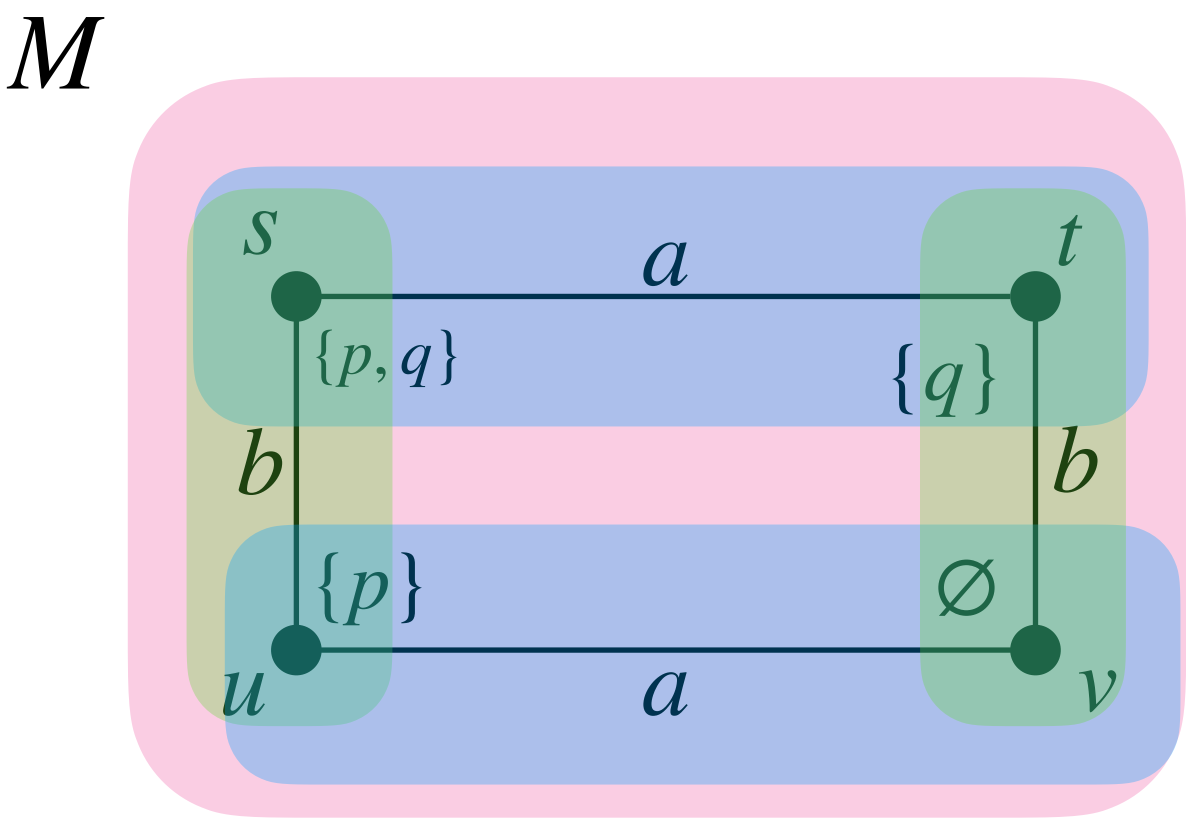
Truthful part

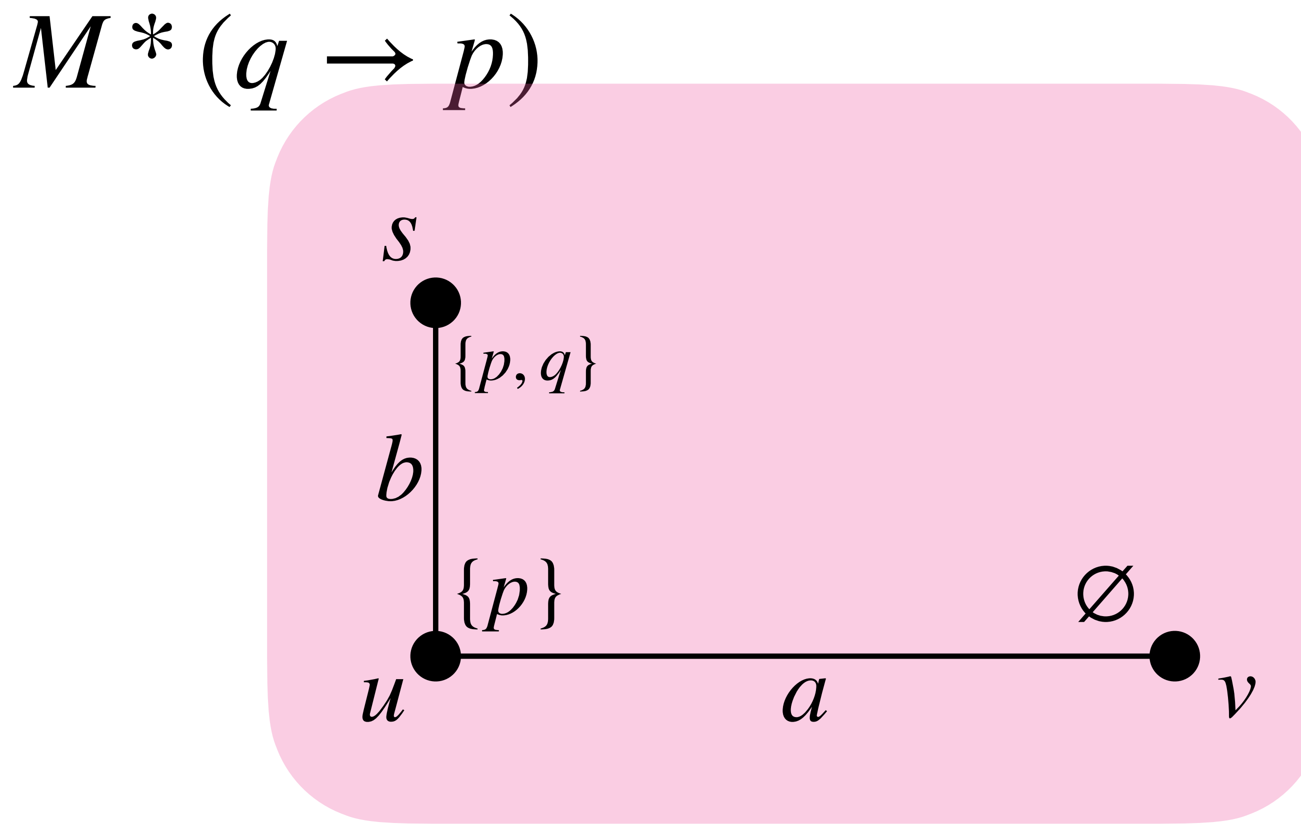
$$\varphi_a := \Box_a \varphi$$

Simultaneous part

$$\varphi_G := \bigwedge_{a \in G} \varphi_a$$

Letting agents do the work



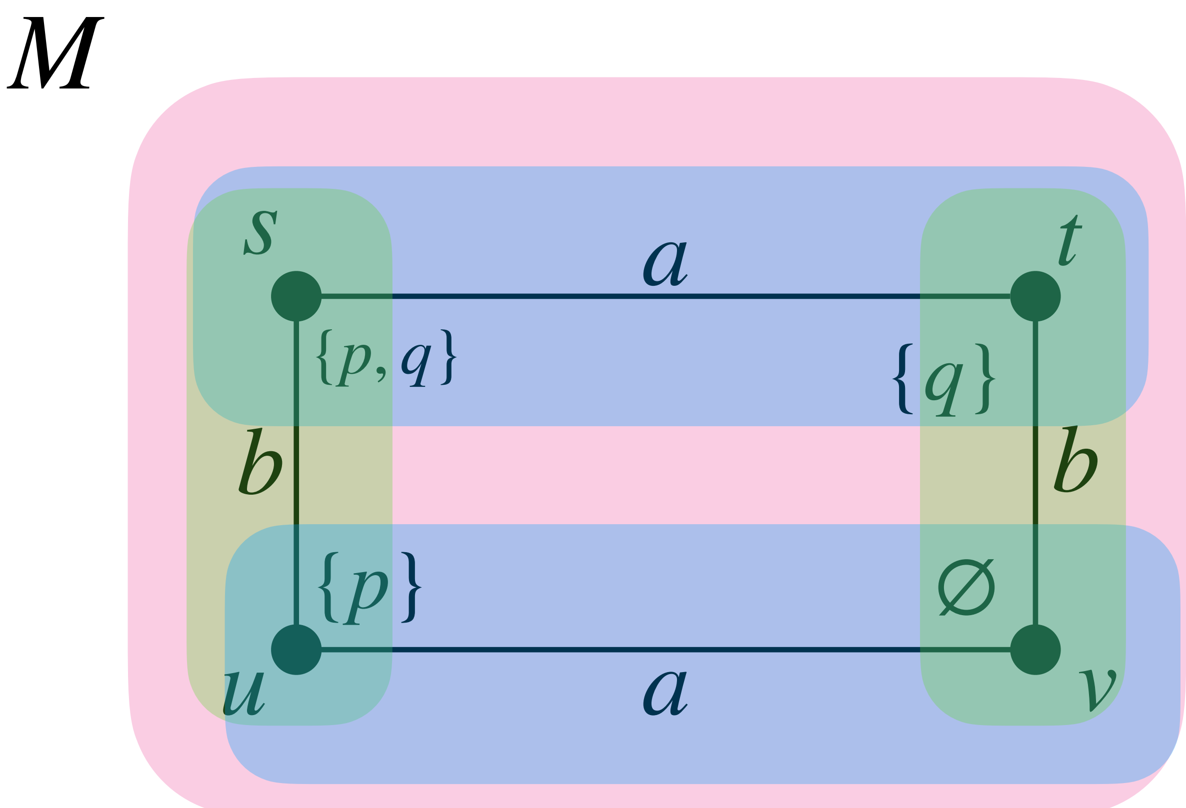


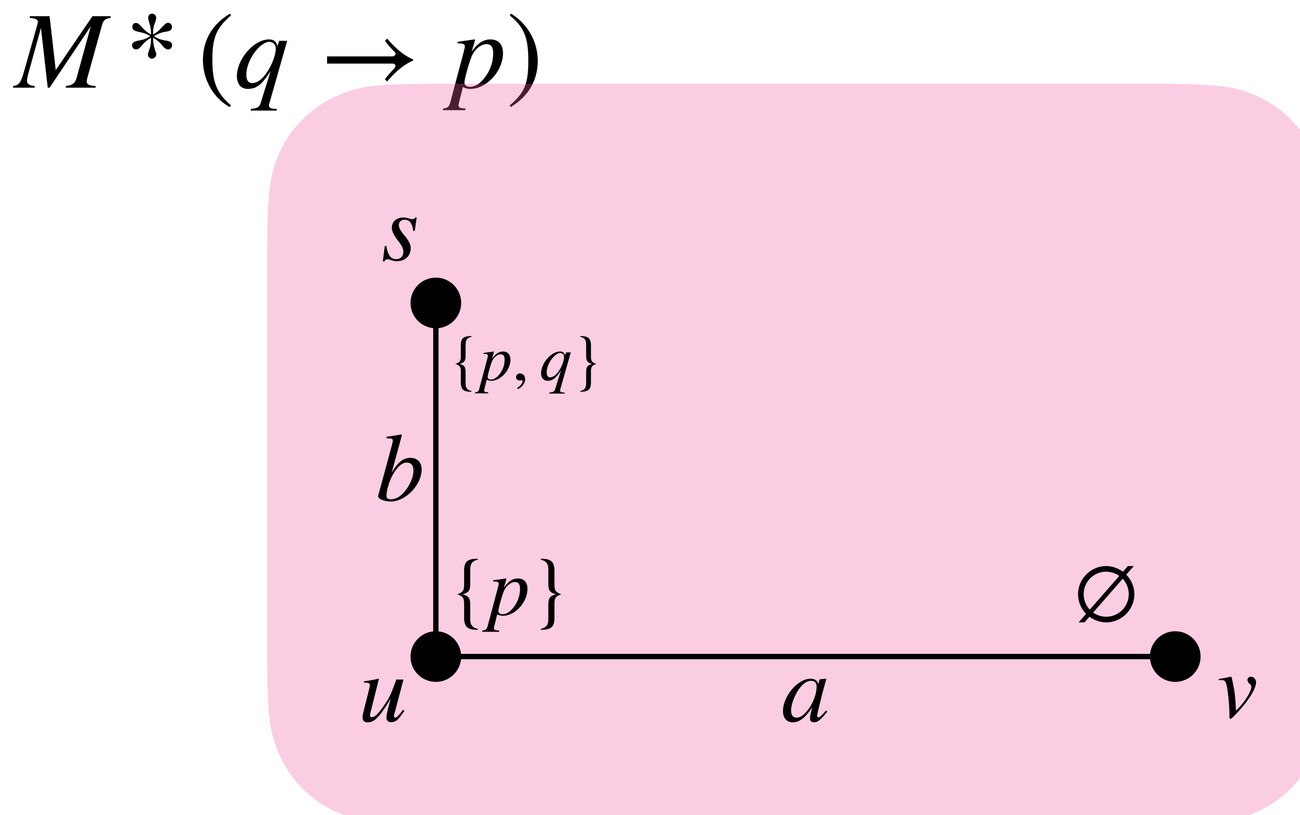
Announcement of $q \rightarrow p$ comes from an external source

Can agents a and b truthfully transform M the same way?

We can think of agents' announcement as them choosing which (union of) equivalence classes they want to preserve

Letting agents do the work





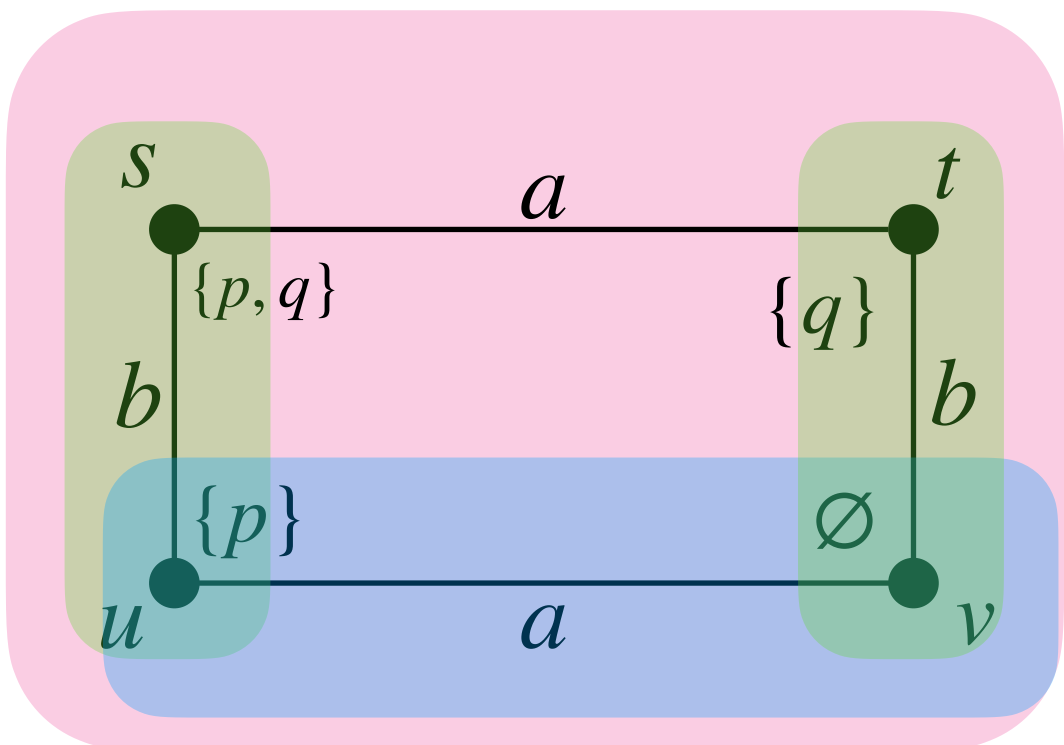
We can think of agents' announcement as them choosing which (union of) equivalence classes they want to preserve

States in the intersection of agents' choices will be preserved

We need to remove t from the intersection…

Letting agents do the work

M



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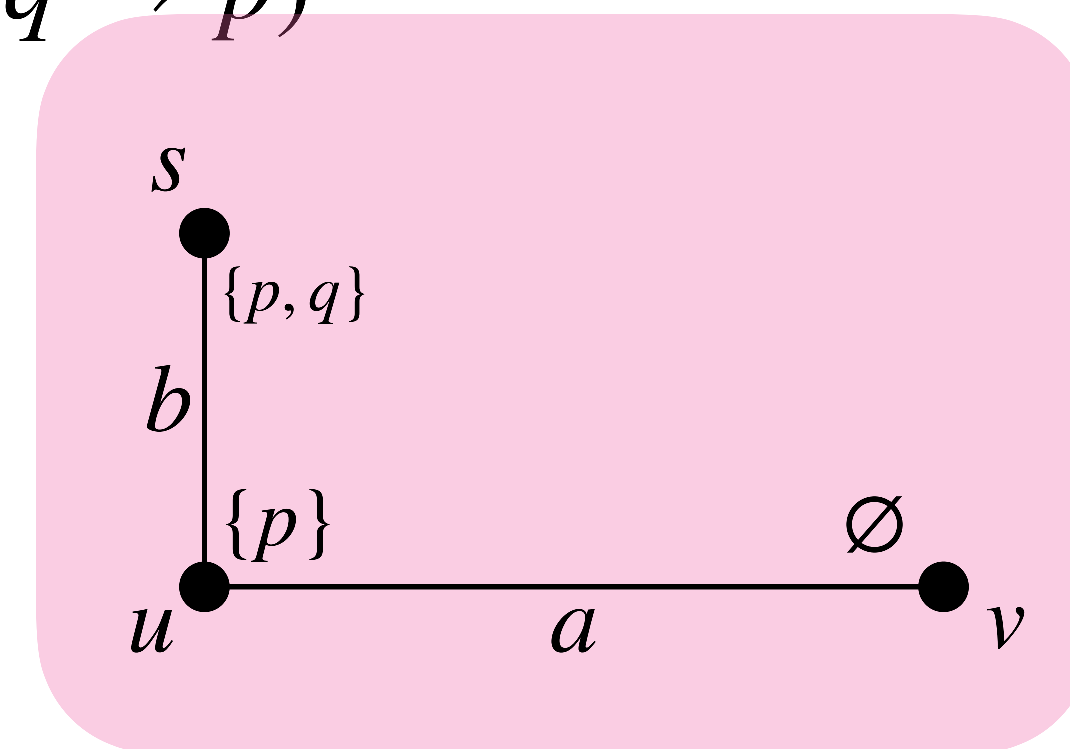

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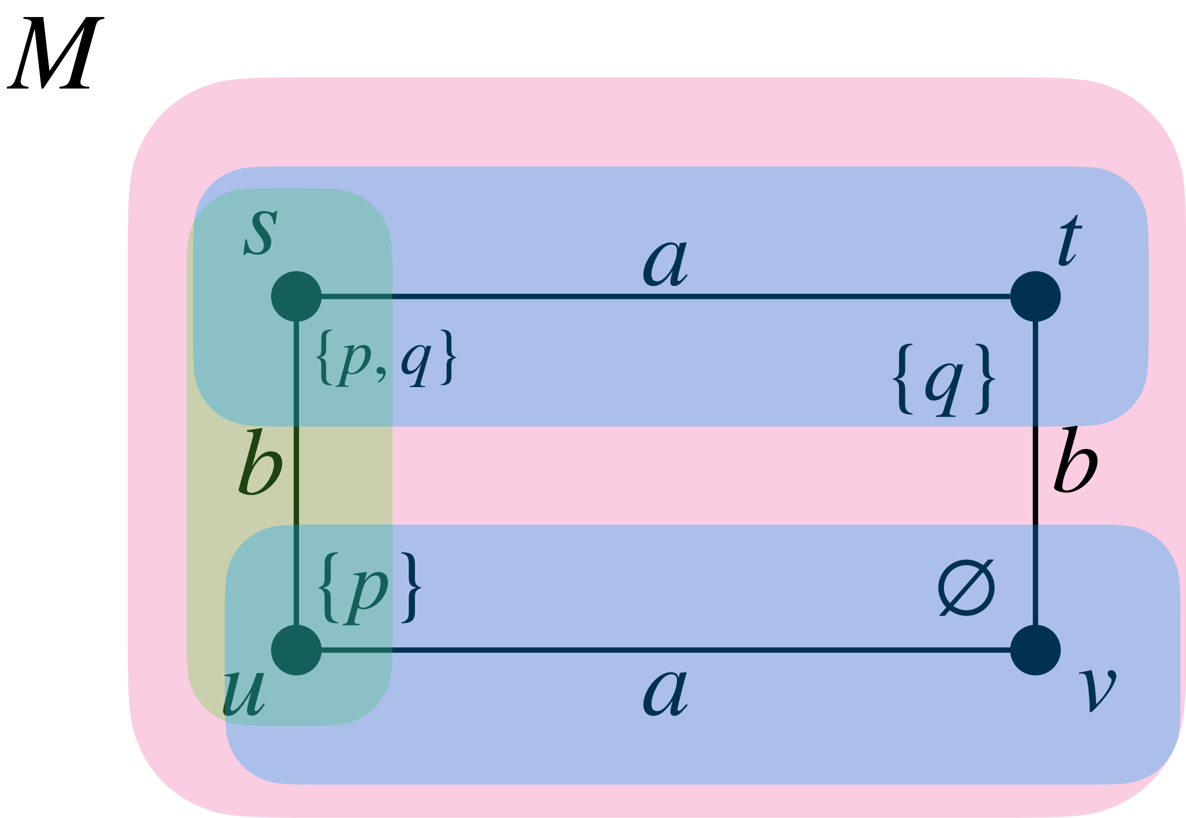

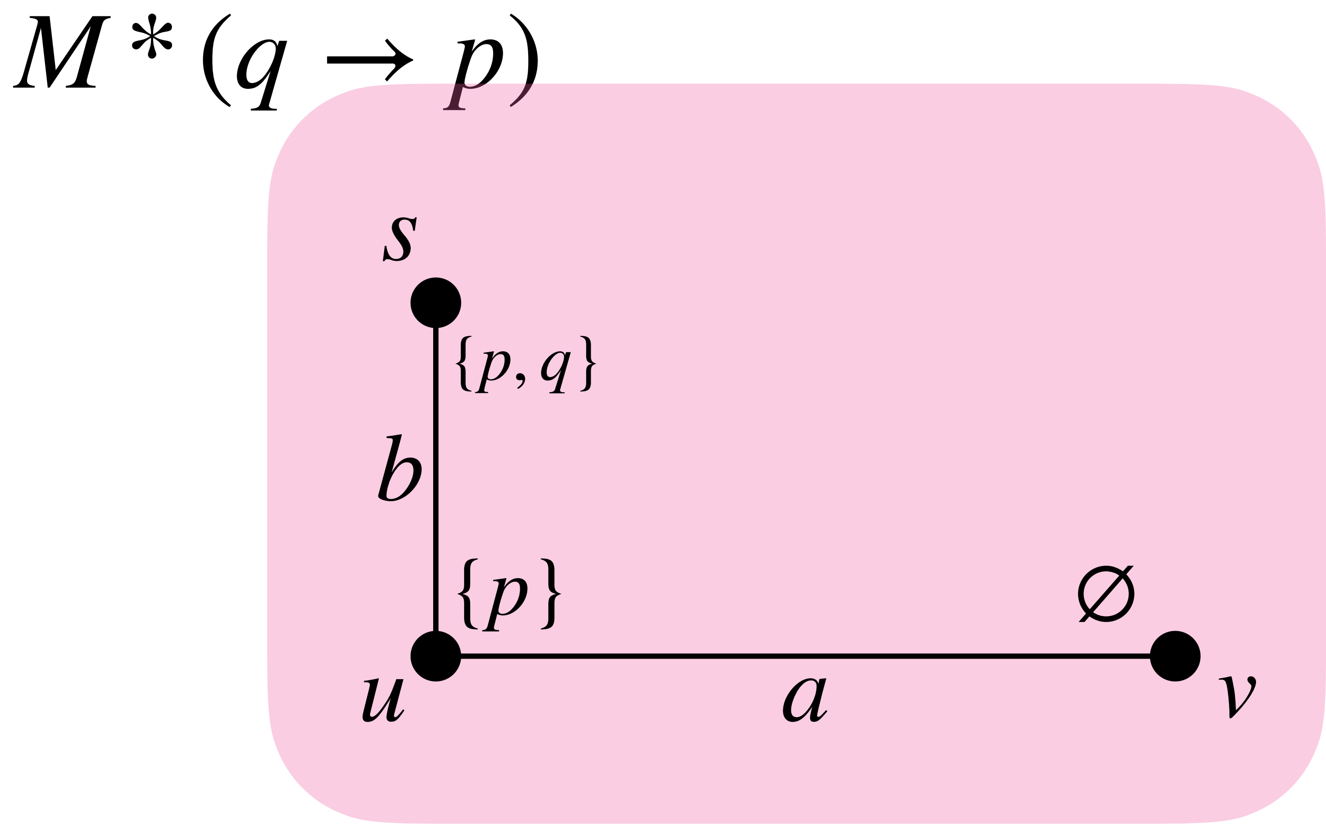

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We need to remove t from the intersection…

So, a and b cannot force the model on the right

Letting agents do the work

APAL allows quantification over all announcements

However, it does not specify whether such announcements can be made by any group of agents modelled in a system

$\langle G \rangle \varphi$: There is a truthful simultaneous announcement by agents from group G, such that φ is true after it

$[G]\varphi$: Whatever agents from group G truthfully and simultaneously announce, φ is true after it

Truthful part

$$\varphi_a := \Box_a \varphi$$

Simultaneous part

$$\varphi_G := \bigwedge_{a \in G} \varphi_a$$

Group Announcement Logic

Language of GAL

$$\mathscr{GAL} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\varphi]\varphi \mid [G]\varphi$$

Semantics

$$M, s \vDash [G]\varphi \text{ iff } \forall \psi_G \in \mathscr{PAL} : M, s \vDash [\psi_G]\varphi$$

$$M, s \vDash \langle G \rangle\varphi \text{ iff } \exists \psi_G \in \mathscr{PAL} : M, s \vDash \langle \psi_G \rangle\varphi$$

Some validities

$$\langle \psi_G \rangle \varphi \to \langle G \rangle \varphi \qquad [G]\varphi \to \varphi$$

$$\langle G \rangle \langle H \rangle \varphi \to \langle G \cup H \rangle \varphi \qquad \langle G \cup H \rangle \varphi \nrightarrow \langle G \rangle \langle H \rangle \varphi$$

Note that GAL quantifies over a subset of $\mathscr{PAL}$

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Ågotnes et al. *Group announcement logic*, 2010.

Virtues of Cooperation

$\langle G\rangle\langle H\rangle\varphi \to \langle G\cup H\rangle\varphi$ $\quad$ $\langle G\cup H\rangle\varphi \nrightarrow \langle G\rangle\langle H\rangle\varphi$

$\langle G\rangle\langle H\rangle\varphi \to \langle G\cup H\rangle\varphi$: If groups G and H can achieve φ by consecutive announcements, they can achieve φ by a simultaneous announcement

$\langle G\cup H\rangle\varphi \nrightarrow \langle G\rangle\langle H\rangle\varphi$: Splitting a group may decrease the discerning power of its subgroups

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Quantifying over Group Announcements

Security. Groups G and H can communicate a secret such that the outsiders do not learn it

Analysis of ability. Being able to achieve φ through communication as $\langle G \rangle \varphi$, or $\langle G \rangle \bigwedge_{a \in G} \Box_a \varphi$, or $\bigwedge_{a \in G} \Box_a \langle G \rangle \varphi$, and so on

Ågotnes et al. *Group announcement logic*, 2010.

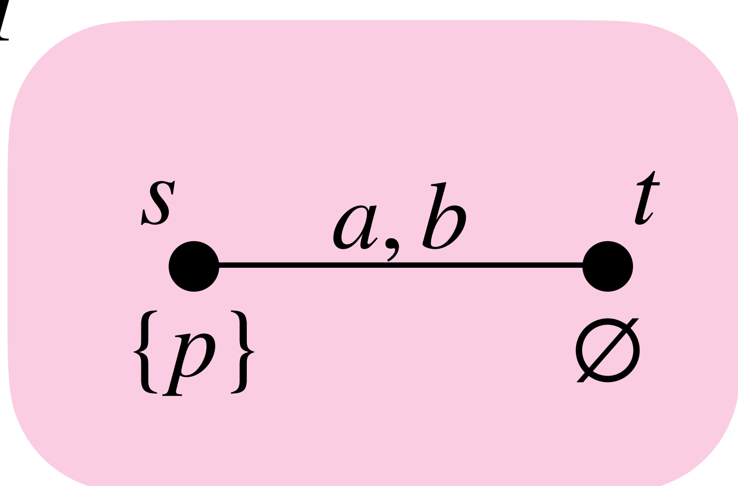
GAL versus PAL

Consider $\langle b \rangle \Box_a p$

Assume that there is a $\psi \in \mathcal{EL}$ which is equivalent to the given GAL formula

Since ψ is finite, there must be a $q \in P$ that does not appear in ψ

M



$M, s \models \langle b \rangle \Box_a p$?

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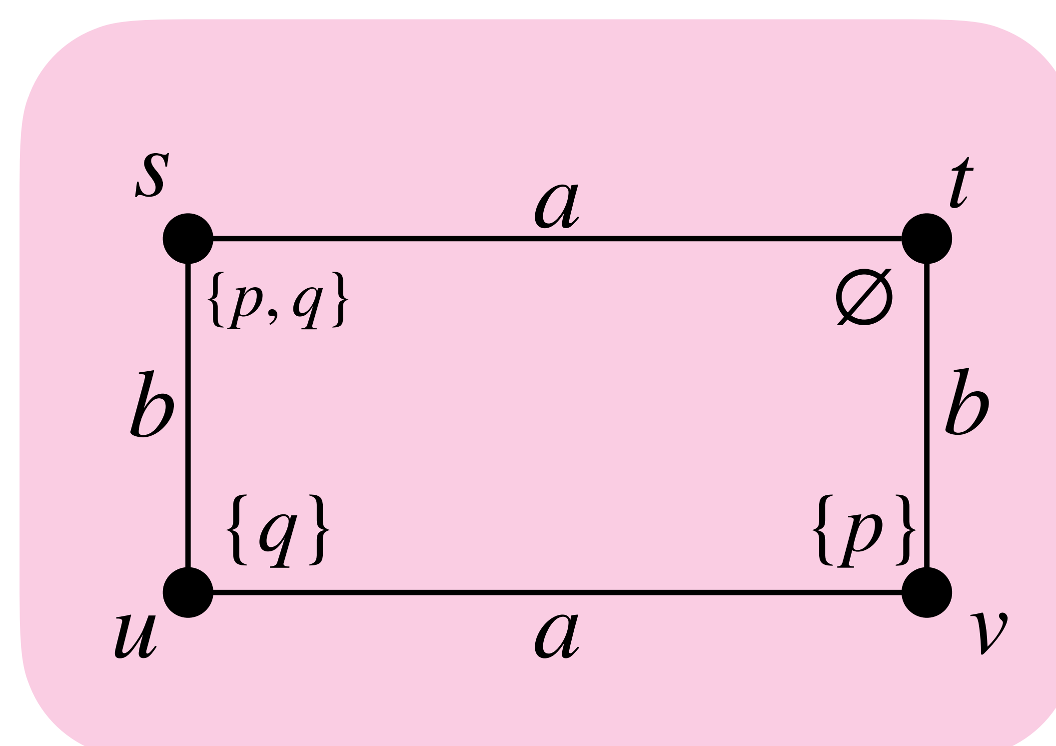

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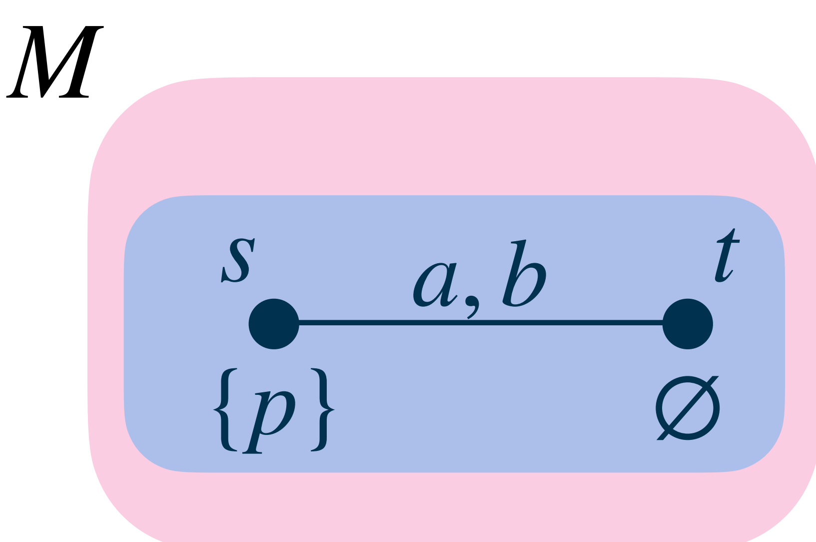
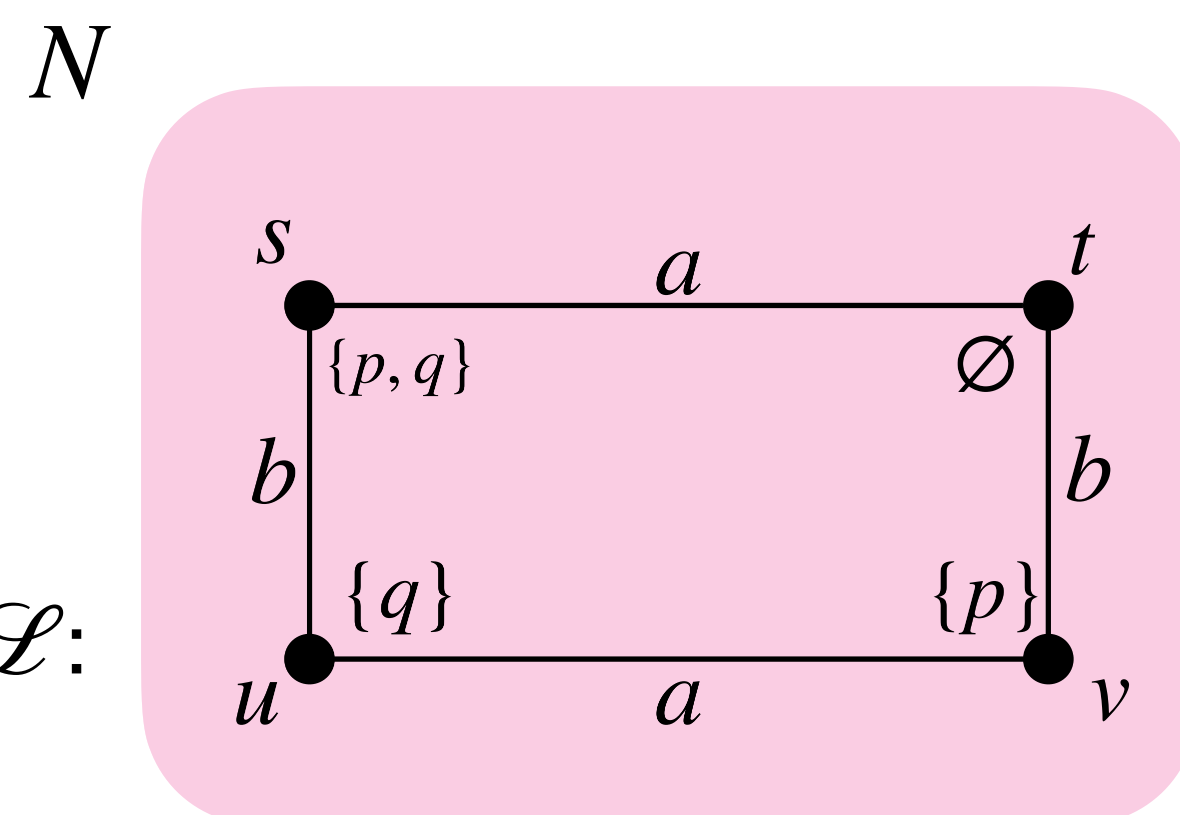


$M, s \models \langle b \rangle \Box_a p$ iff $\exists \psi \in \mathscr{PAL}$:
$M, s \models \langle \Box_b \psi \rangle \Box_a p$?



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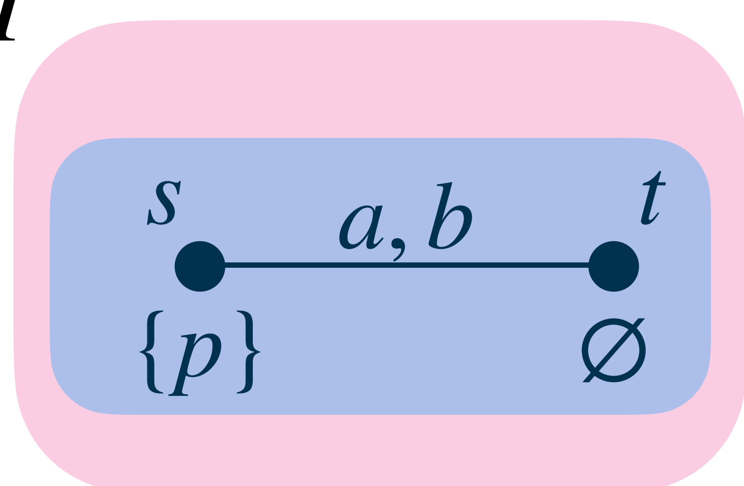

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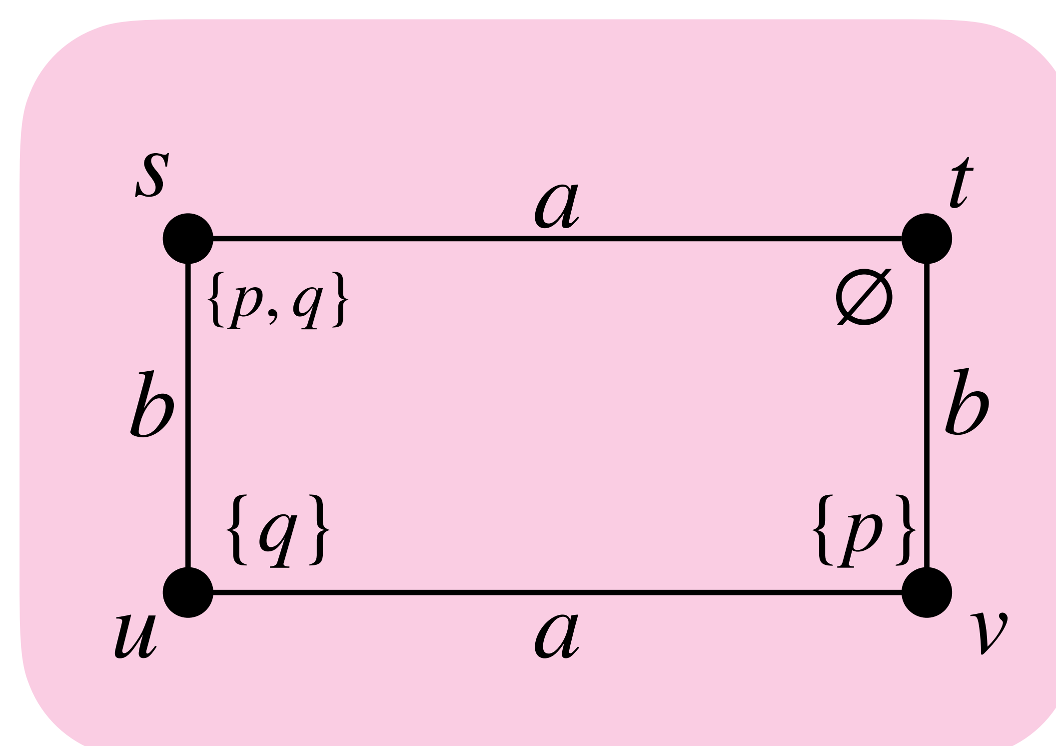

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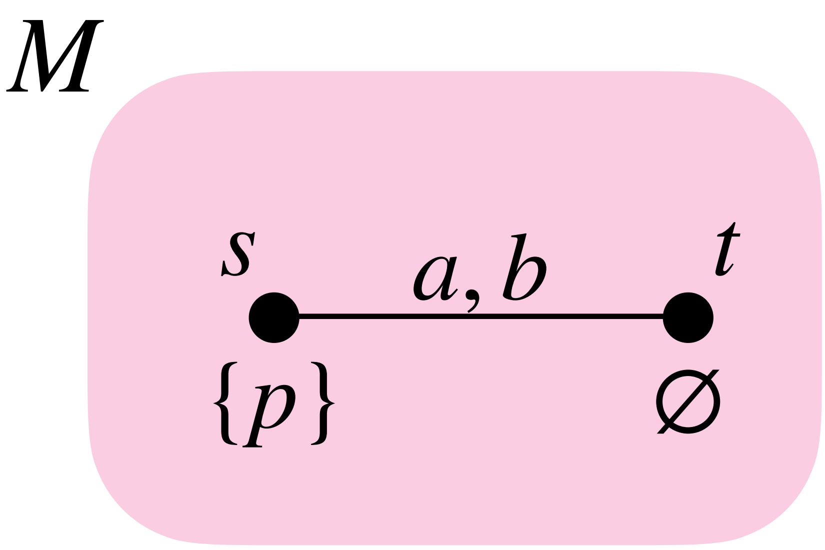

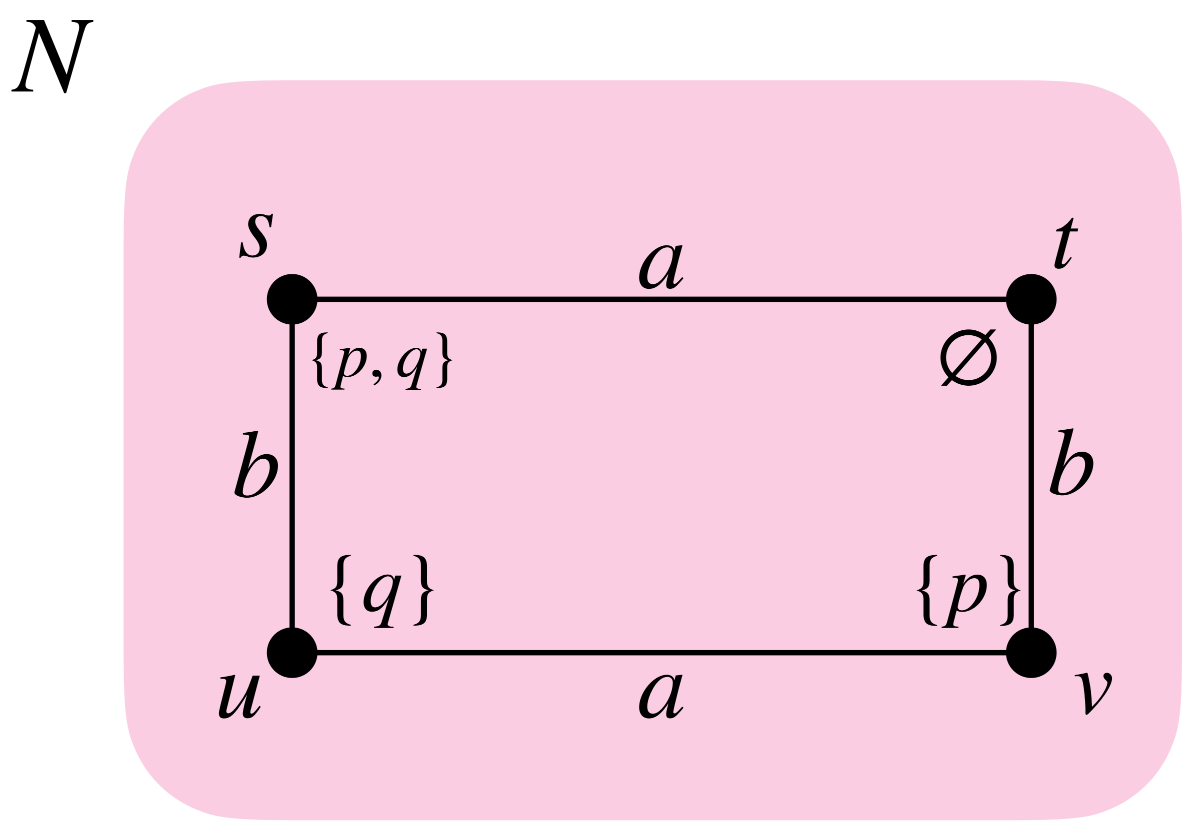

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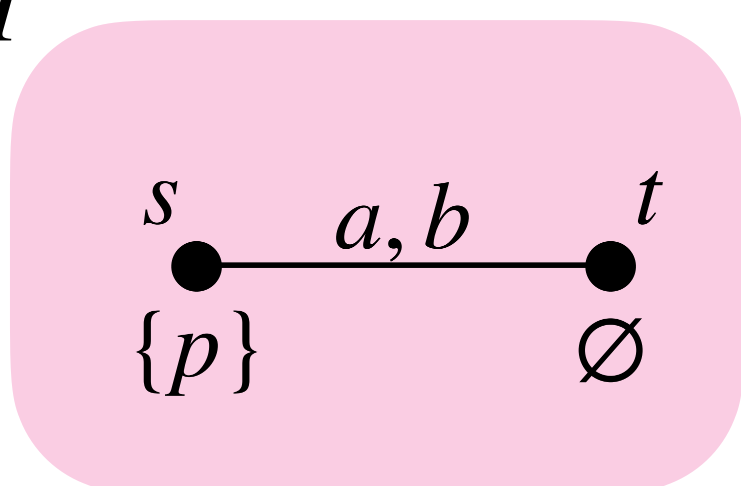

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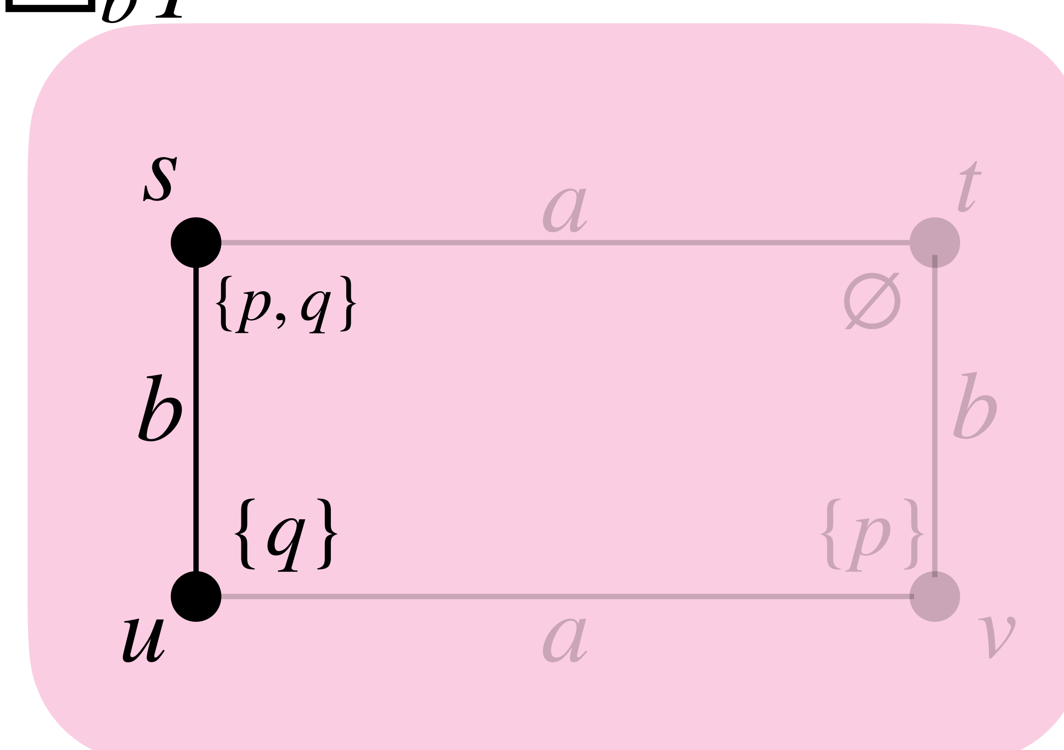

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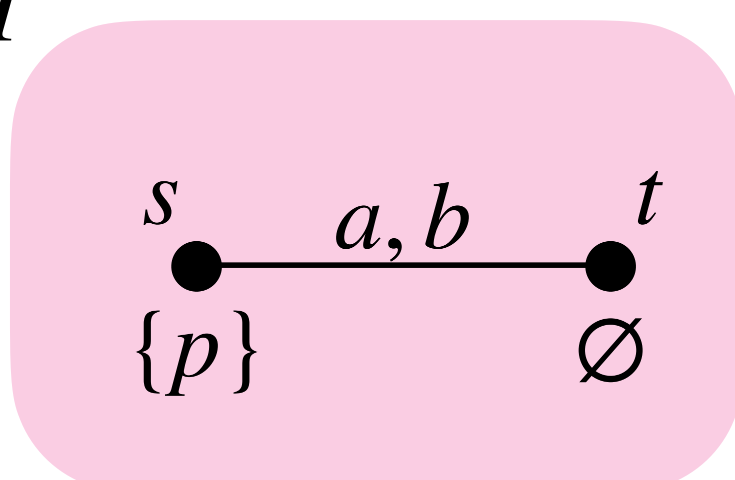

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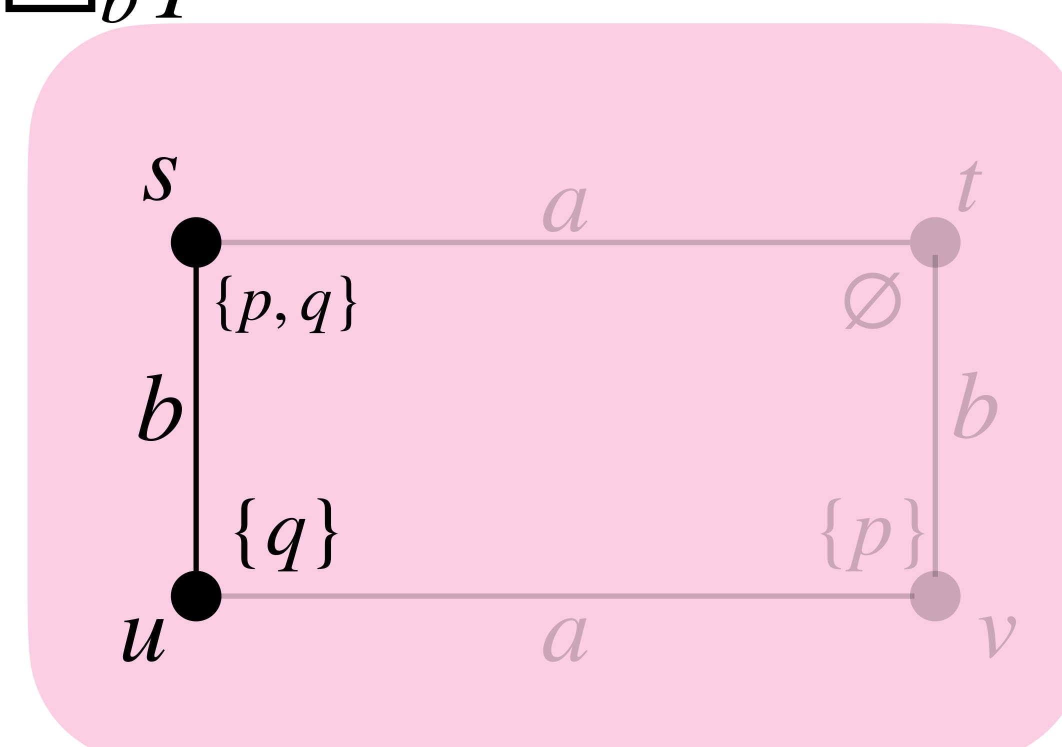

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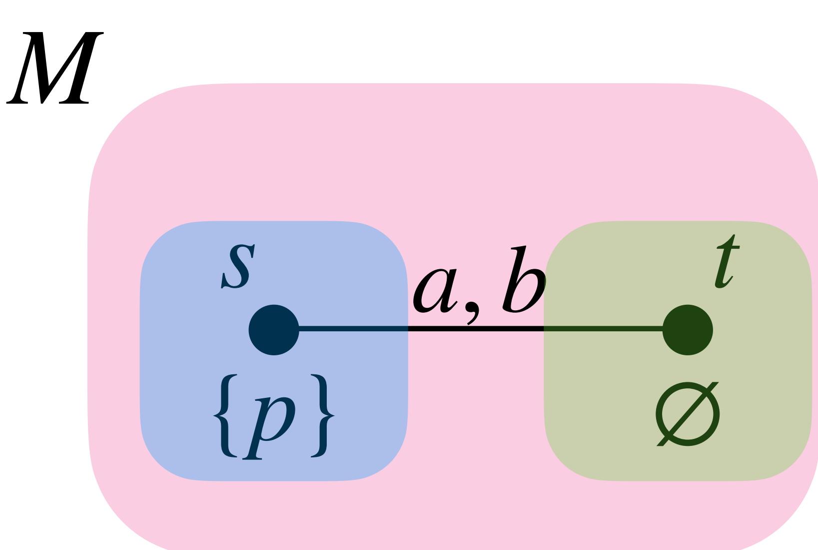
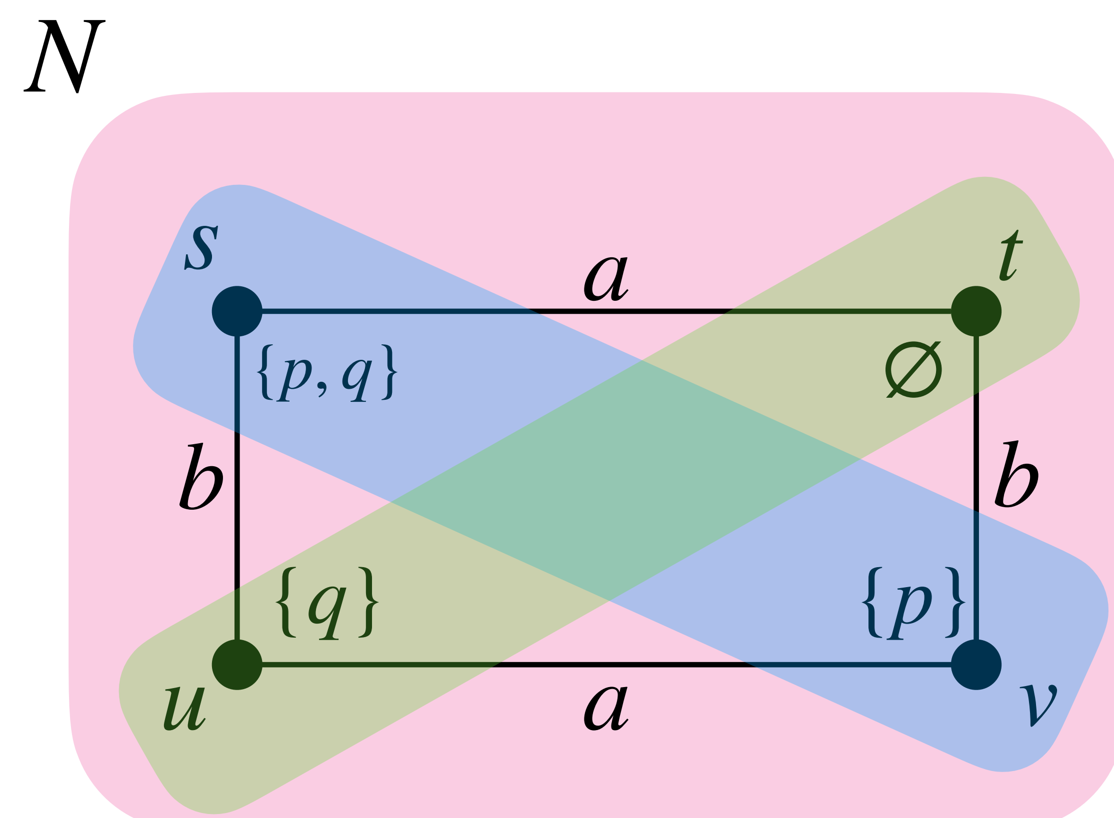


What about ψ?



GAL versus PAL

Consider $\langle b \rangle \Box_a p$

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Contradiction!

Since ψ is finite, there must be a $q \in P$ that does not appear in ψ

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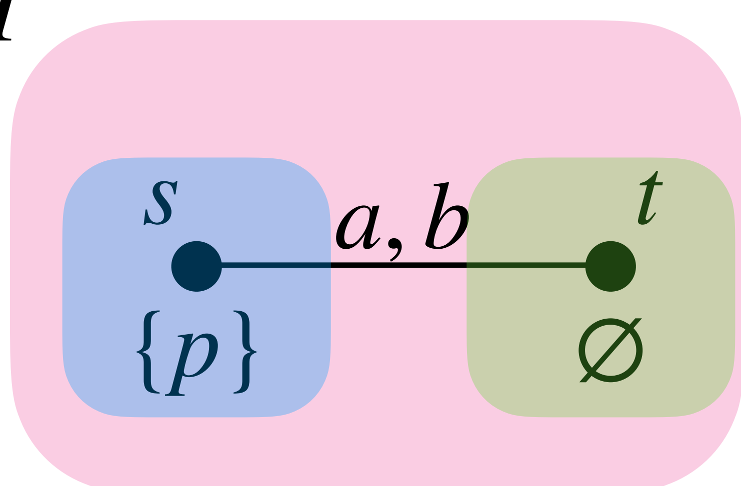
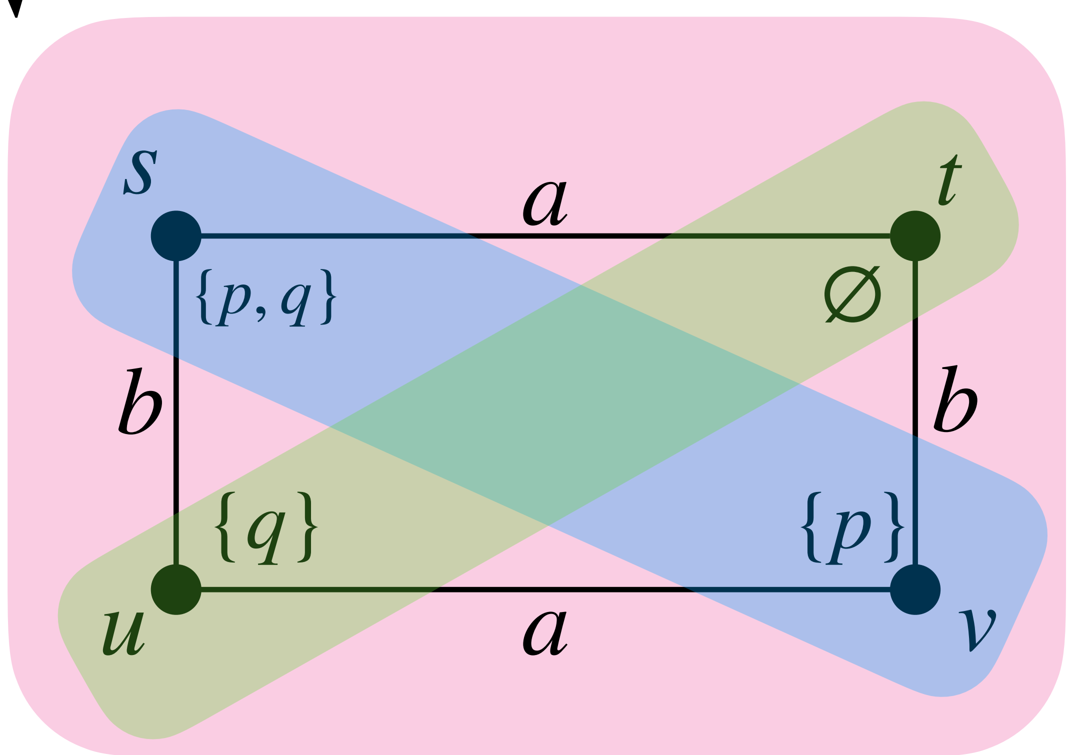


ψ can not tell the difference between M and N

N



GAL versus PAL

Theorem. PAL and EL are equally expressive

Theorem. APAL is more expressive than PAL and EL

$[G]\varphi$ quantifies over formulas $\bigwedge_{a \in G} \Box_a \psi_a$ that can contain all propositional variables (even those not explicitly present in φ) and have arbitrary finite modal depth

Theorem. GAL is more expressive than PAL and EL

There are no reduction axioms for GAL

Axiomatisation of GAL

Axioms of EL and PAL

$[G]\varphi \to [\psi_G]\varphi$ with $\psi_G \in \mathscr{PAL}$

From $\{\eta([\psi_G]\varphi) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([G]\varphi)$

Theorem. GAL is more expressive than PAL

Theorem. GAL is sound and complete

Theorem. SAT-GAL is undecidable

Theorem. Complexity of MC-GAL is PSPACE-complete

Open Problem. Is there a finitary axiomatisation of GAL?

Ågotnes et al. *Group announcement logic*, 2010.

Ågotnes, French, Van Ditmarsch. *The Undecidability of Quantified Announcements*, 2016.

GAL versus APAL

Consider $\langle ! \rangle(\Box_a p \wedge \neg \Box_b \Box_a p)$

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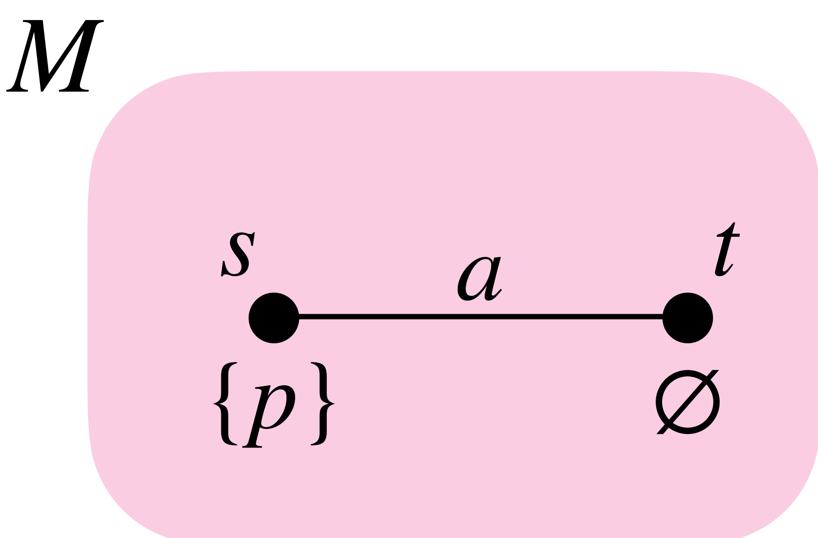

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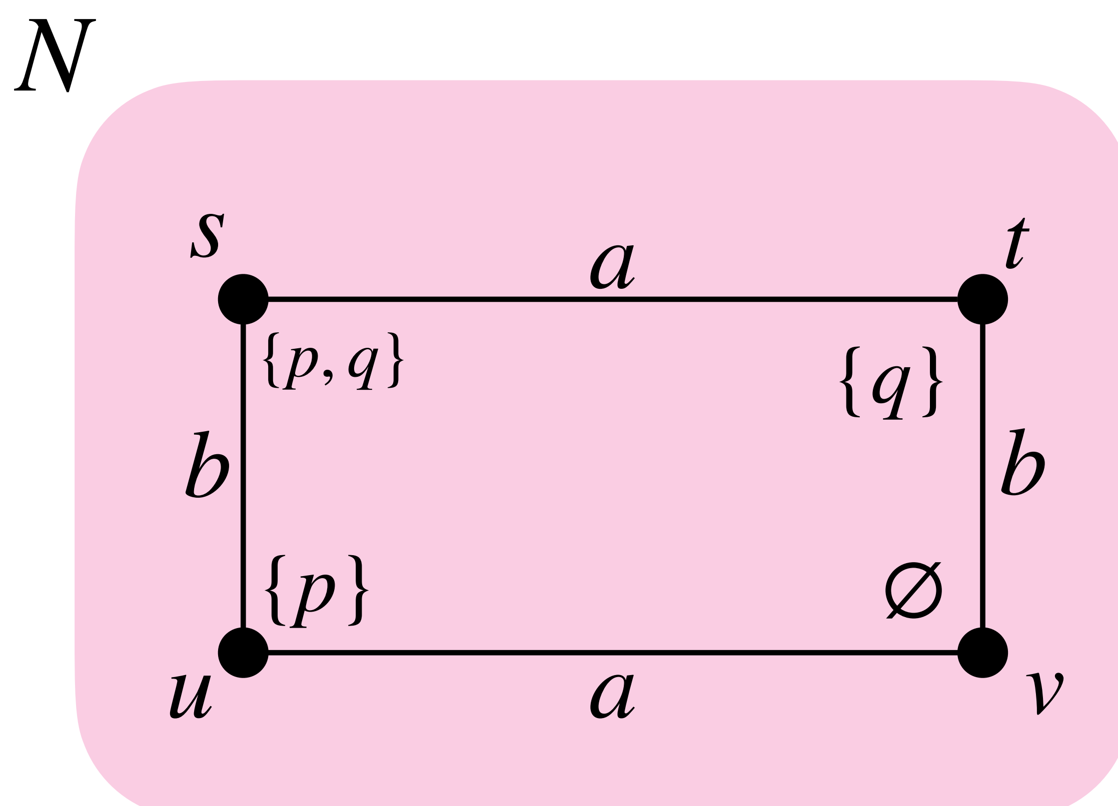

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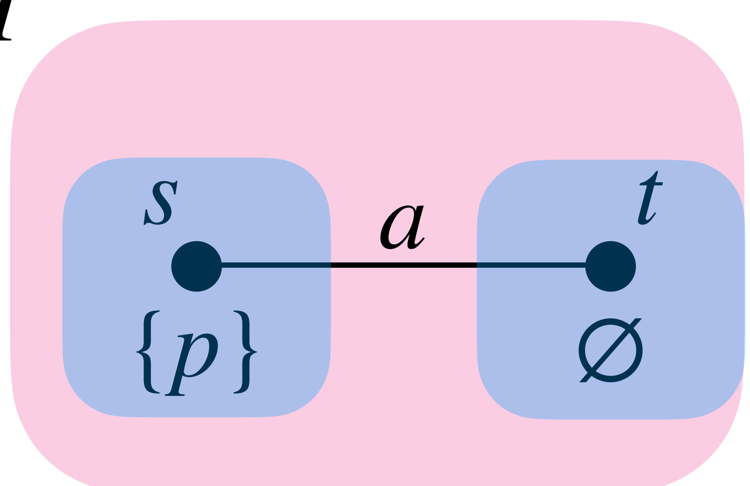

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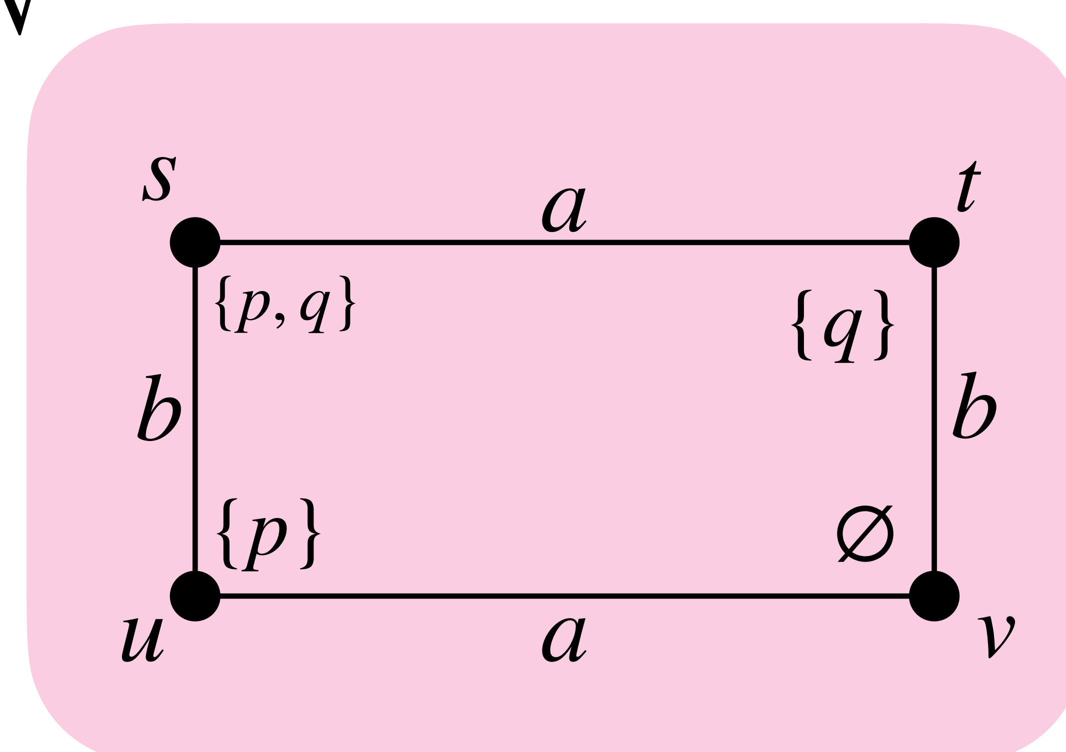

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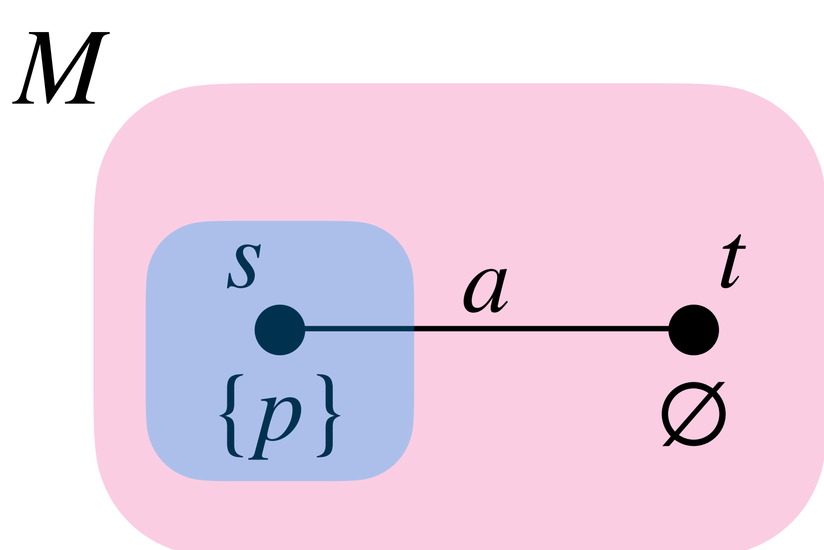

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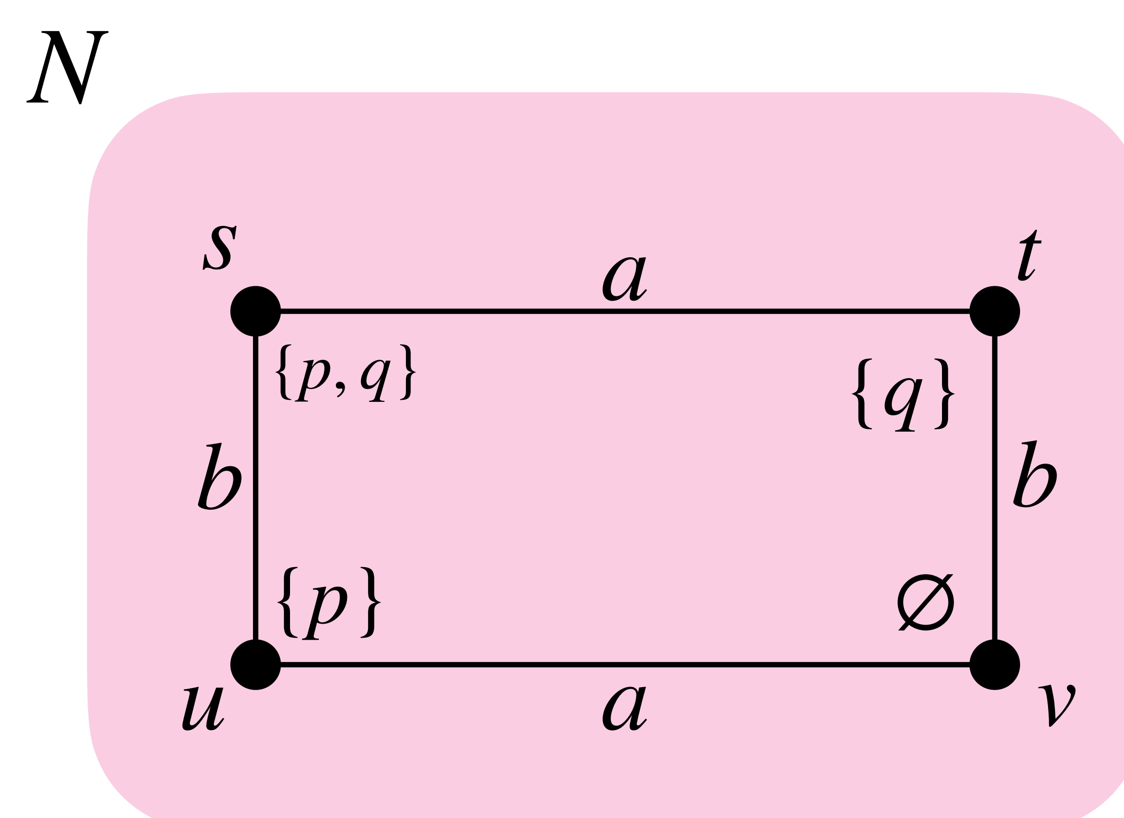

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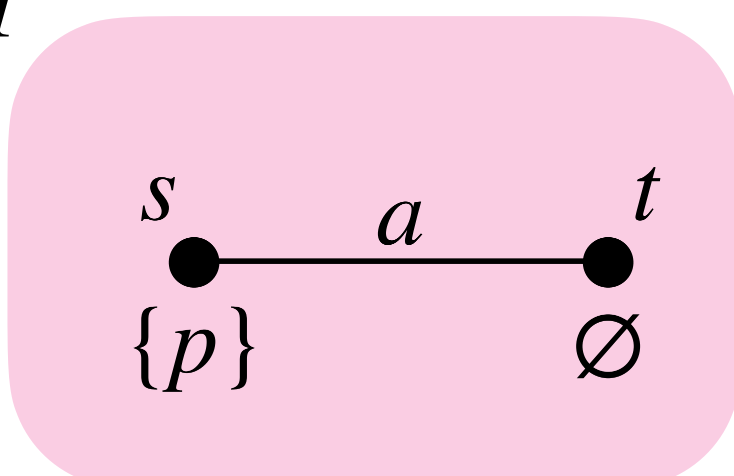

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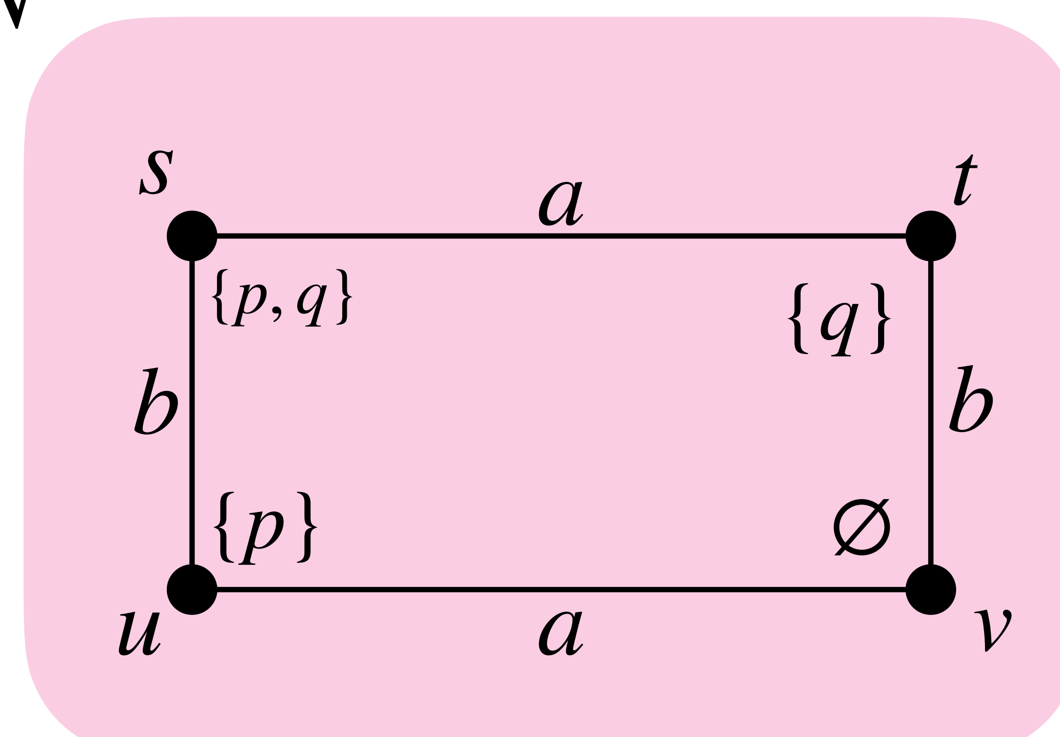

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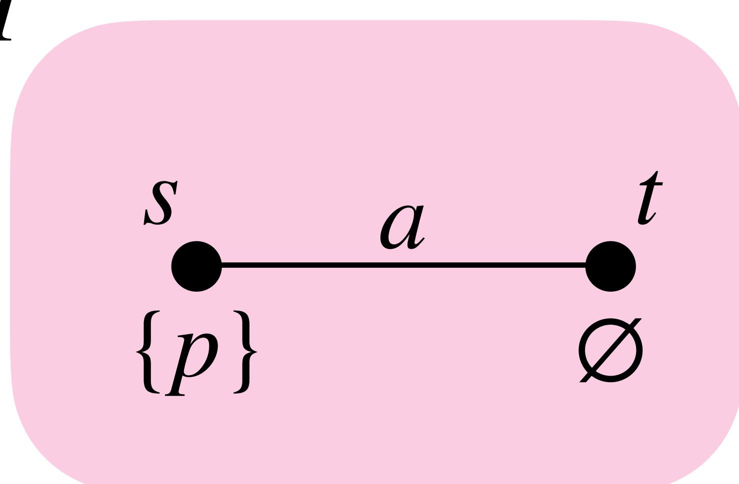

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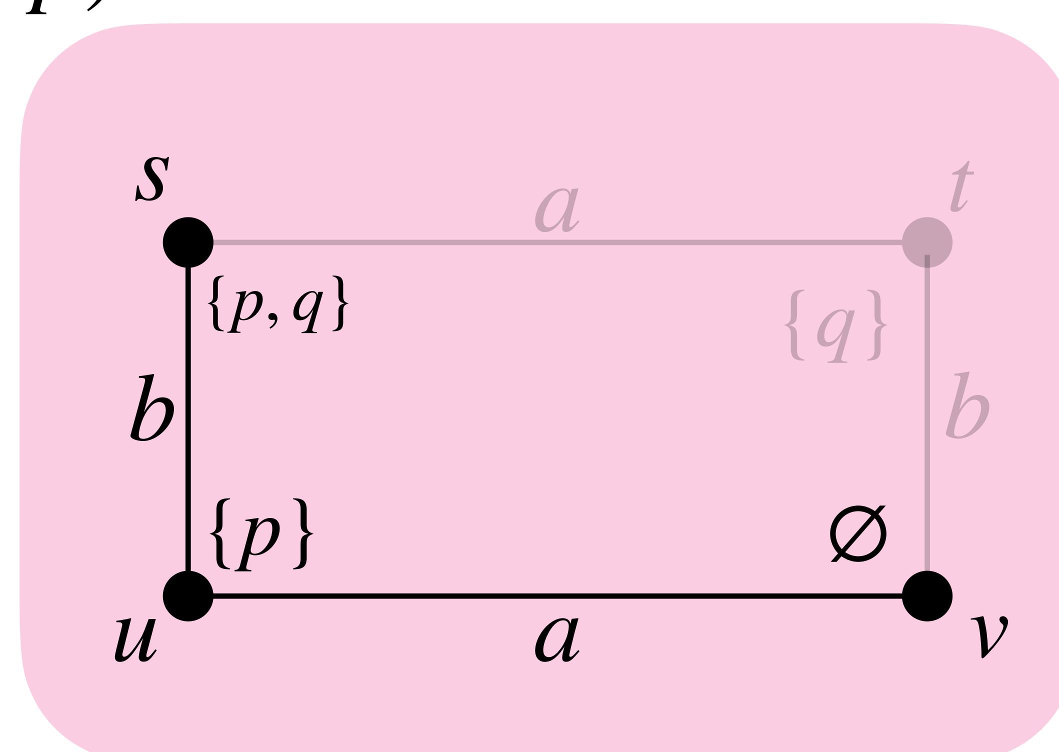

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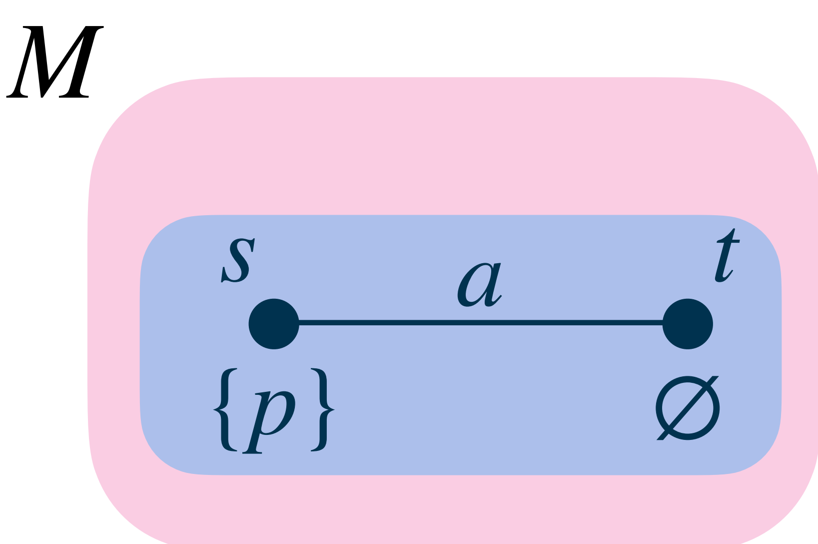

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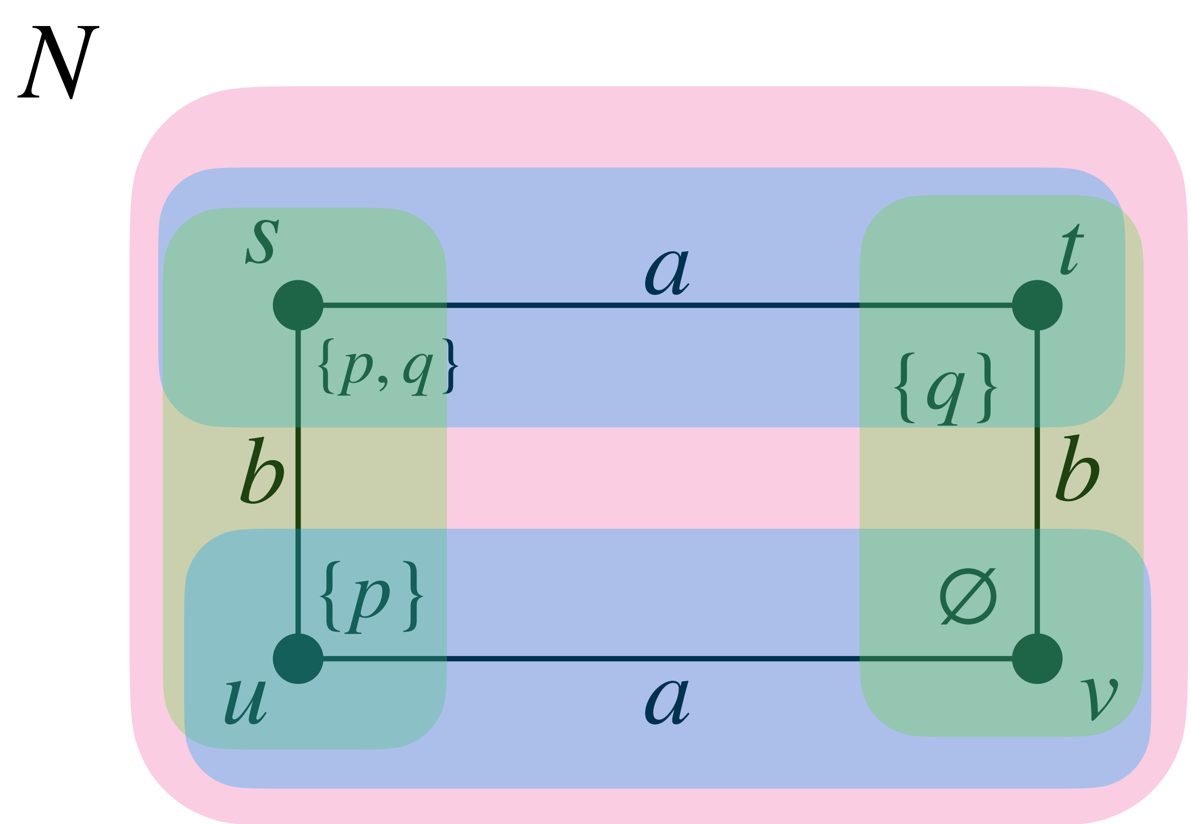

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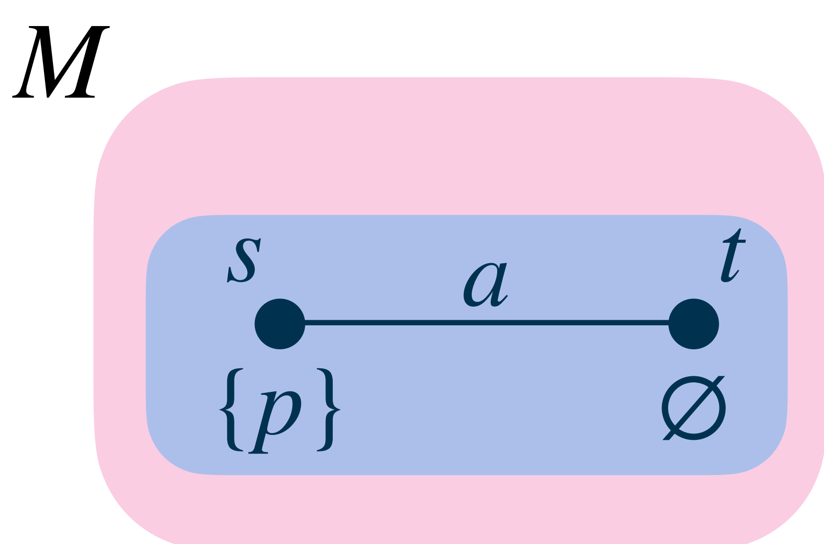
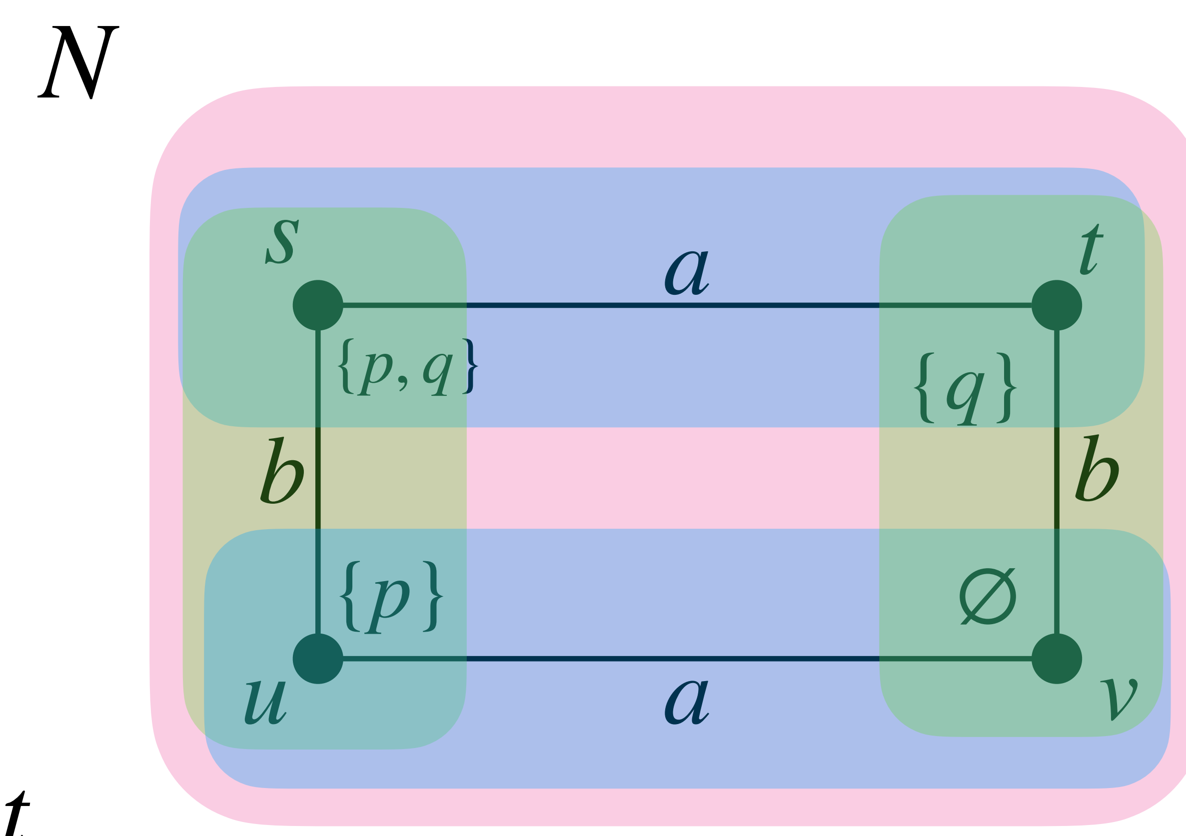




What about ψ?

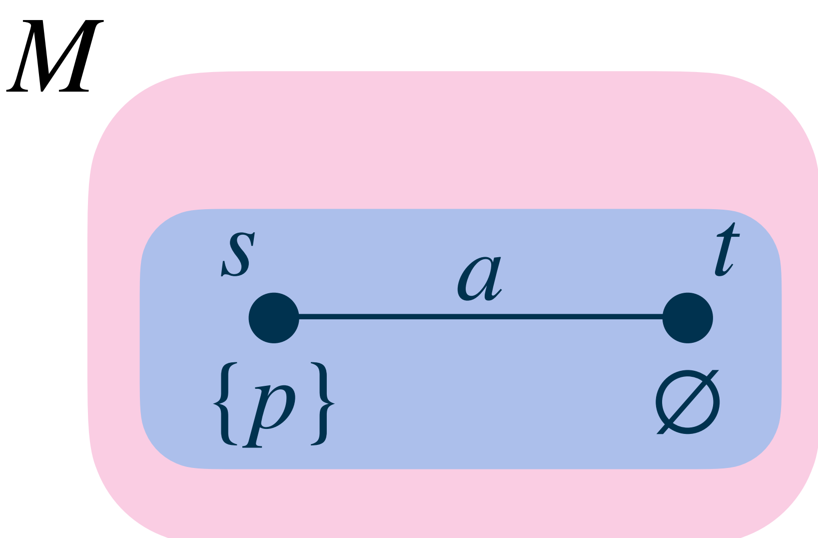
We need to cut out only state t

GAL versus APAL

Consider $\langle ! \rangle(\Box_a p \wedge \neg \Box_b \Box_a p)$

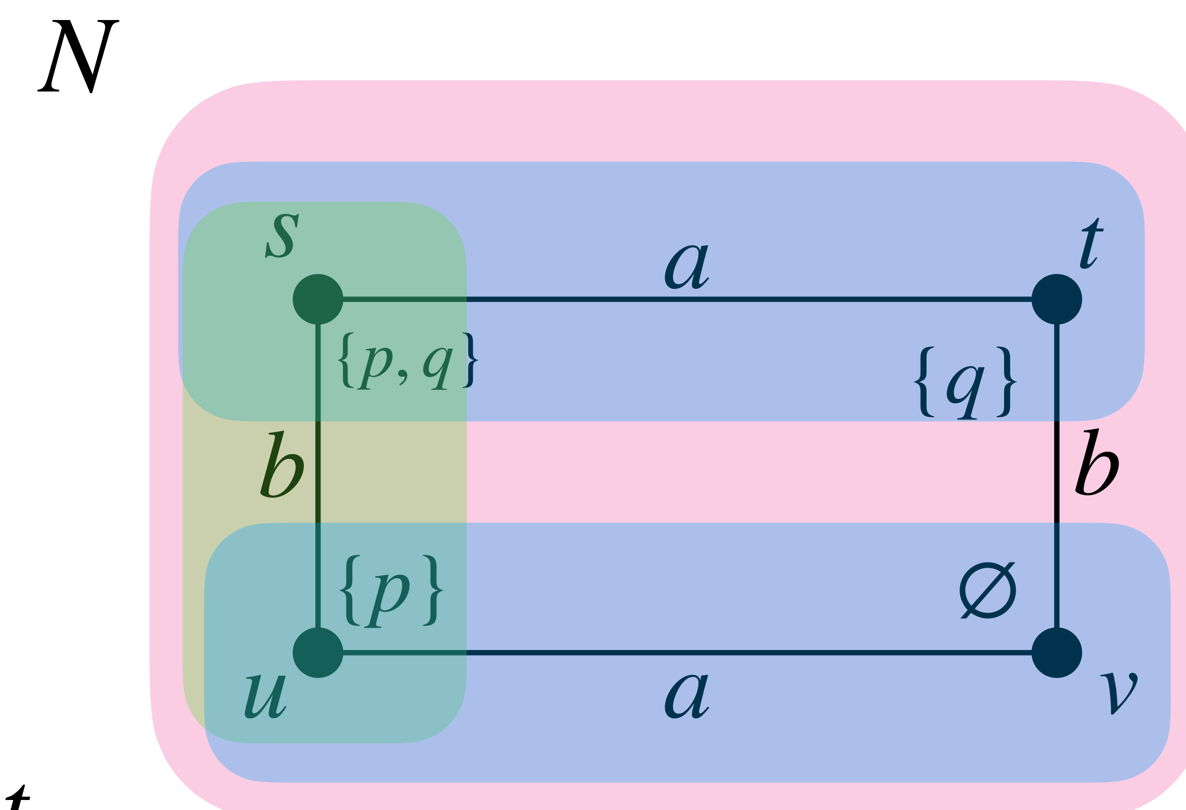
Assume that there is a $\psi \in \mathscr{GAL}$ which is equivalent to the given APAL formula

Since ψ is finite, there must be a $q \in P$ that does not appear in ψ



What about ψ?

We need to cut out only state t



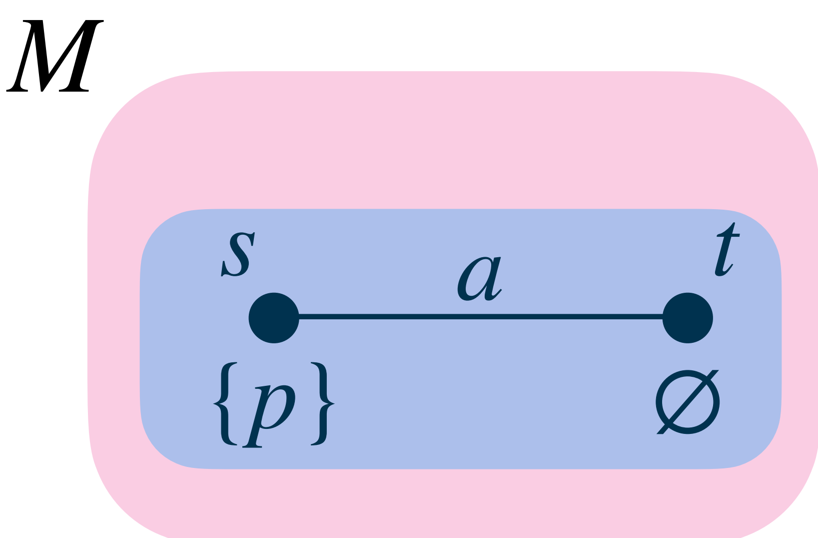
GAL versus APAL

Consider $\langle ! \rangle(\Box_a p \land \neg \Box_b \Box_a p)$

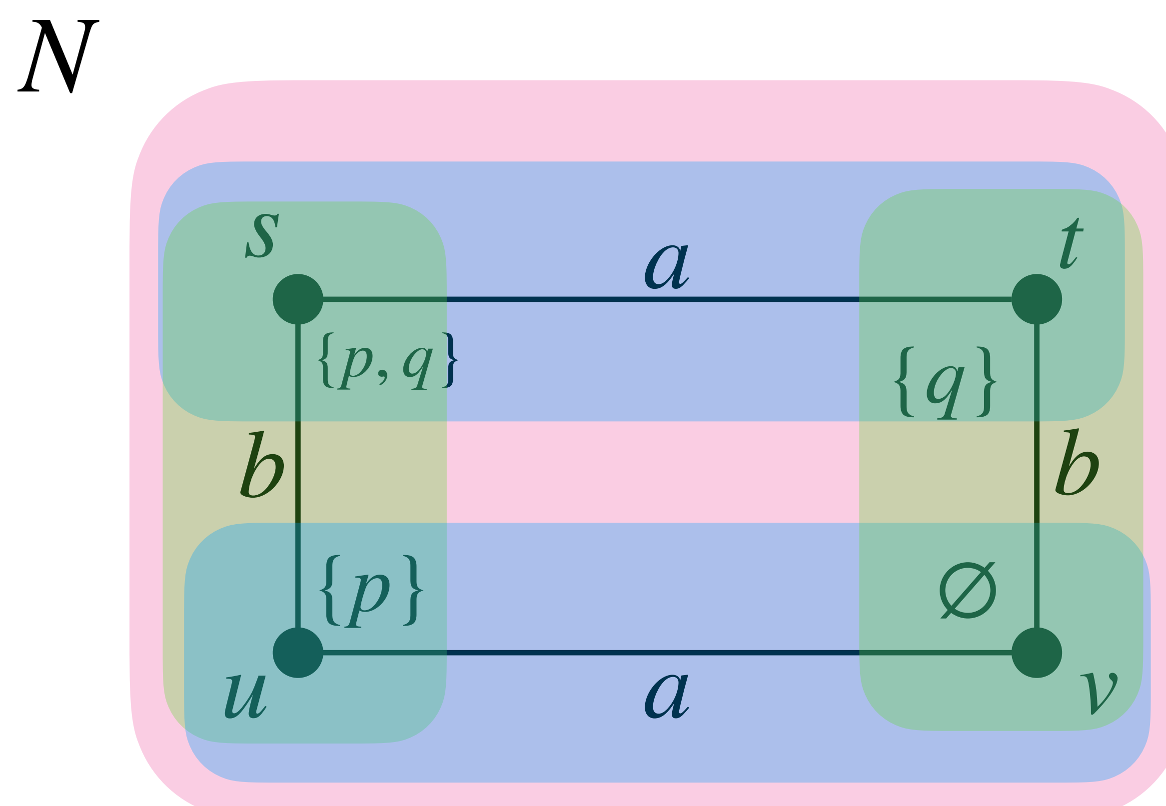
Assume that there is a $\psi \in \mathcal{GAL}$ which is equivalent to the given APAL formula

Contradiction!

Since ψ is finite, there must be a $q \in P$ that does not appear in ψ



What about ψ?



GAL versus APAL

Theorem. There are some properties expressible in APAL that are not expressible in GAL

What about the converse?

Theorem. There are some properties expressible in GAL that are not expressible in APAL

Corollary. APAL and GAL are incomparable

Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

Take-home message

- Group announcement logic (GAL) allows quantification over truthful and simultaneous announcements by groups of agents

- GAL is quite similar to APAL: axiomatisation

- GAL is quite different from APAL: incomparable expressivity

Open Problem. Is there a finitary axiomatisation of GAL?