Coalition Announcement Logic

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Overview of GAL

Axioms of EL and PAL

$[G]\varphi \to [\psi_G]\varphi$ with $\psi_G \in \mathscr{PAL}$

From $\{\eta([\psi_G]\varphi) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([G]\varphi)$

Open Problem. Is there a finitary axiomatisation of GAL?

Theorem. GAL and APAL are incomparable

Theorem. GAL is more expressive than PAL

Theorem. GAL is sound and complete

Theorem. SAT-GAL is undecidable

Theorem. Complexity of MC-GAL is PSPACE-complete

Ågotnes et al. *Group announcement logic*, 2010.

Ågotnes, French, Van Ditmarsch. *The Undecidability of Quantified Announcements*, 2016.

Strategic setting

In GAL only a specified group of agents makes an announcement

Following the lead of ATL, we can think of group announcements as one-step strategies to achieve an epistemic goal no matter what opponents do at the same time

$\langle [G] \rangle \varphi$: There is a truthful simultaneous announcement by agents from coalition G, such that no matter what agents in the anti-coalition announce at the same time, φ is true

$[\langle G \rangle] \varphi$: Whatever agents from coalition G announce, there is a counter-announcement by the anti-coalition, such that φ is true

Ågotnes, Van Ditmarsch. *Coalitions and Announcements*, 2008.
Alur, Henzinger, Kupferman. *Alternating-time Temporal Logic*, 2002.

Coalition Announcement Logic

Language of CAL

$$\mathscr{CAL} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\varphi]\varphi \mid [\langle G \rangle]\varphi$$

Semantics

$$M_s \models [\langle G \rangle]\varphi \text{ iff}$$
$$\forall \psi_G \exists \chi_{A \setminus G} : M_s \models \psi_G \to \langle \psi_G \wedge \chi_{A \setminus G} \rangle \varphi$$
$$M_s \models \langle [G] \rangle\varphi \text{ iff}$$
$$\exists \psi_G \forall \chi_{A \setminus G} : M_s \models \psi_G \wedge [\psi_G \wedge \chi_{A \setminus G}] \varphi$$

Truthful part

$$\varphi_a := \Box_a \varphi$$

Simultaneous part

$$\varphi_G := \bigwedge_{a \in G} \varphi_a$$

Ågotnes, Van Ditmarsch. *Coalitions and Announcements*, 2008.

Coalition Announcement Logic

Language of CAL

$$\mathscr{CAL} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\varphi]\varphi \mid [\langle G \rangle]\varphi$$

Semantics

$$M_s \models [\langle G \rangle]\varphi \ \text{ iff}$$
$$\forall \psi_G \exists \chi_{A \backslash G} : M_s \models \psi_G \rightarrow \langle \psi_G \wedge \chi_{A \backslash G} \rangle \varphi$$

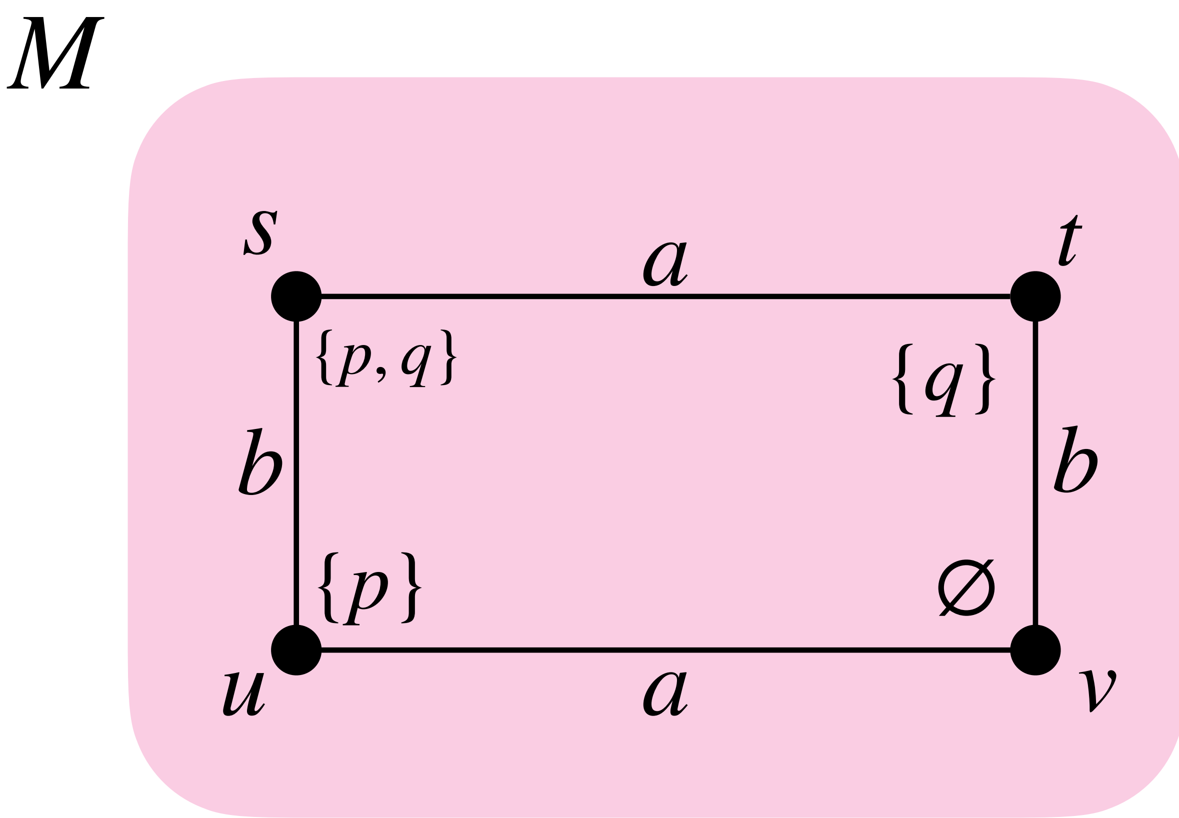
$$M_s \models \langle [G] \rangle\varphi \ \text{ iff}$$
$$\exists \psi_G \forall \chi_{A \backslash G} : M_s \models \psi_G \wedge [\psi_G \wedge \chi_{A \backslash G}]\varphi$$

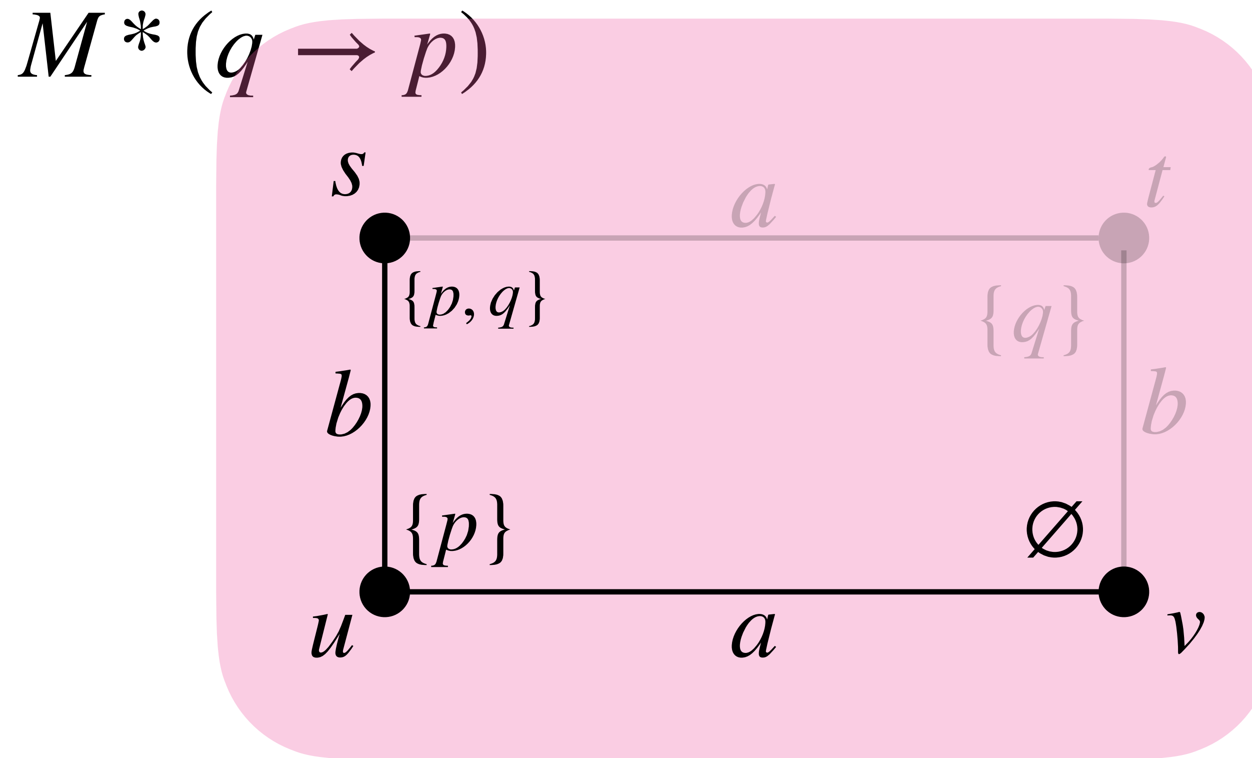
In $[\langle G \rangle]$ we will call G the coalition, $A \backslash G$ the anti-coalition, ψ_G the coalition announcement, and $\psi_{A \backslash G}$ the counter-announcement (or response)

Ågotnes, Van Ditmarsch. *Coalitions and Announcements*, 2008.

Example

APAL





$$M, s \models \langle ! \rangle (\Box_a p \land \neg \Box_b \Box_a p)$$

$$\exists \psi \in \mathscr{PAL} : M, s \models \langle \psi \rangle (\Box_a p \land \neg \Box_b \Box_a p)$$

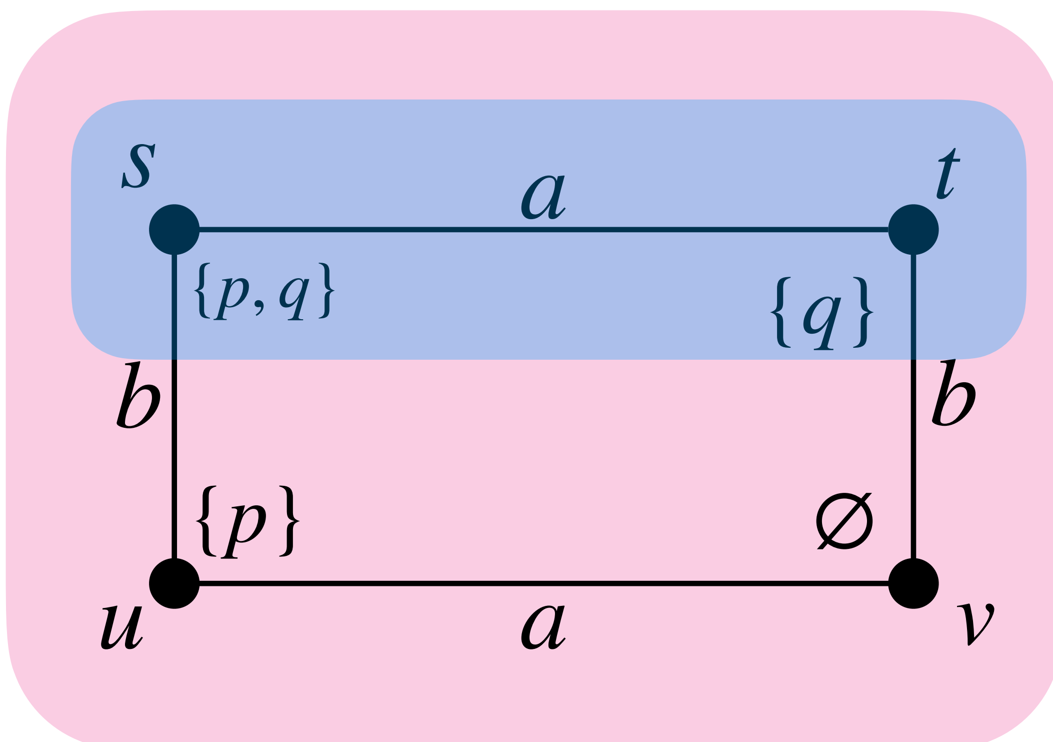
$$M, s \models \langle q \to p \rangle (\Box_a p \land \neg \Box_b \Box_a p)$$

$M, s \models q \to p$ and $M * (q \to p), s \models \Box_a p \land \neg \Box_b \Box_a p$

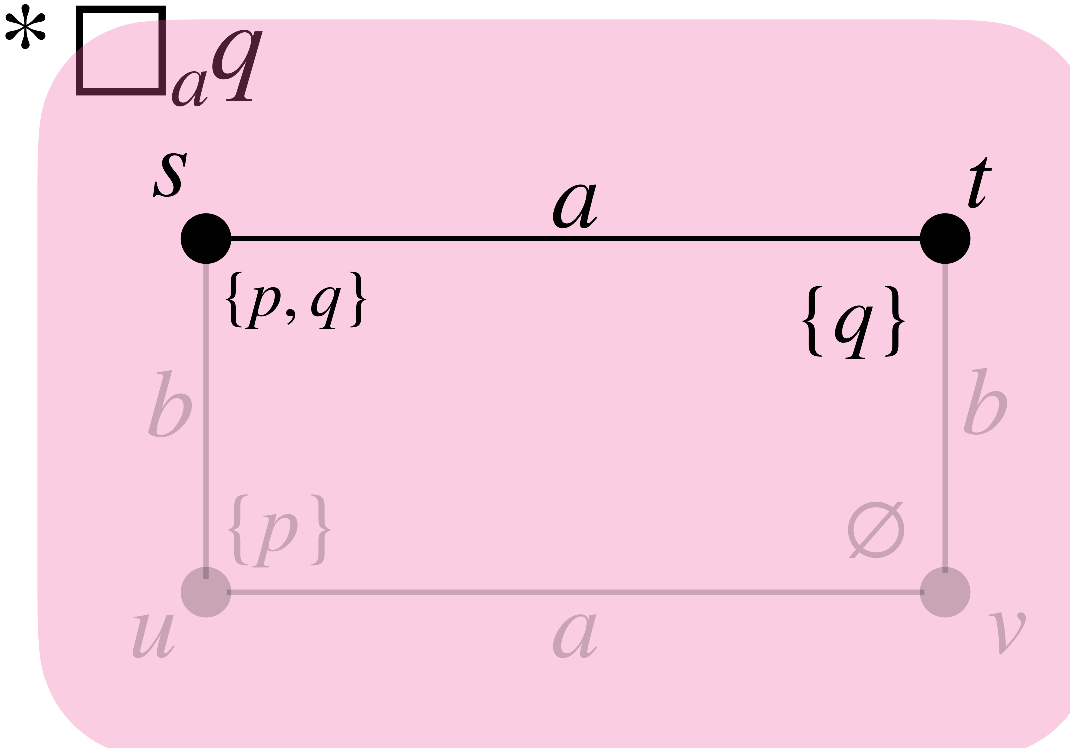
Example

GAL

M



$M * \Box_a q$



$$M, s \models \langle a \rangle (\Box_b q \wedge \neg \Box_a p)$$

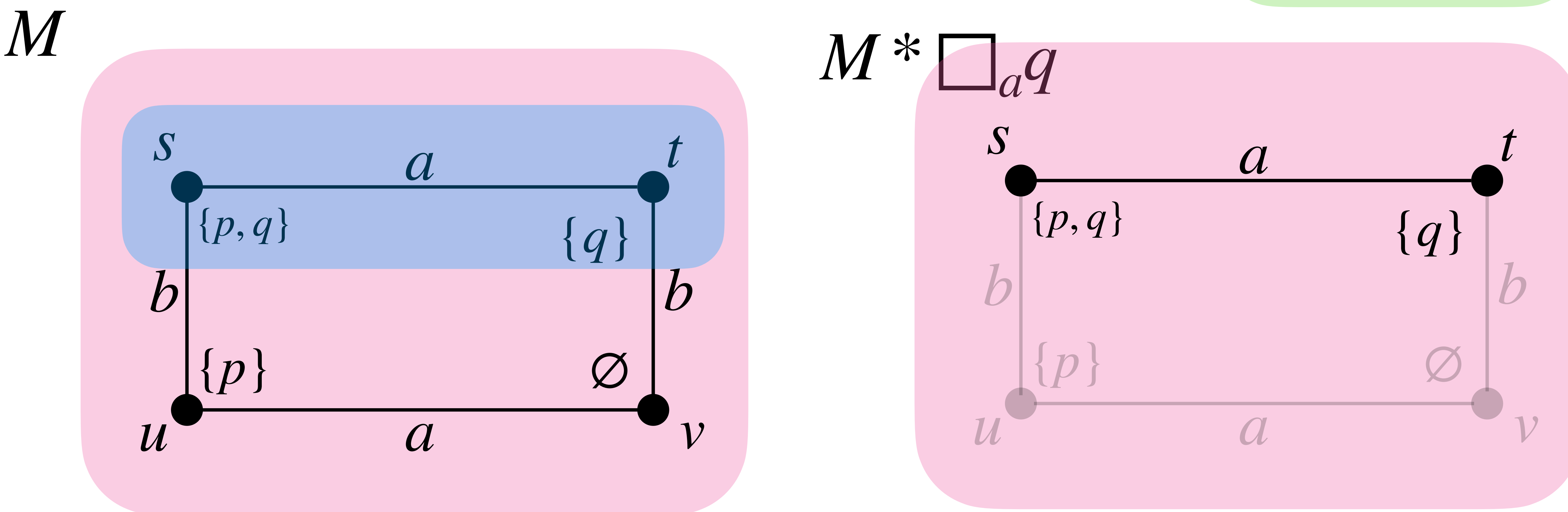
$$\exists \psi \in \mathscr{PAL} : M, s \models \langle \Box_a \psi \rangle (\Box_b q \wedge \neg \Box_a p)$$

$$M, s \models \langle \Box_a q \rangle (\Box_b q \wedge \neg \Box_a p)$$

$M, s \models \Box_a q$ and $M * \Box_a q, s \models \Box_b q \wedge \neg \Box_a p$

Example

CAL



$$M, s \vDash \langle\!\![a]\!\!\rangle(\Box_b q \wedge \neg \Box_a p)$$

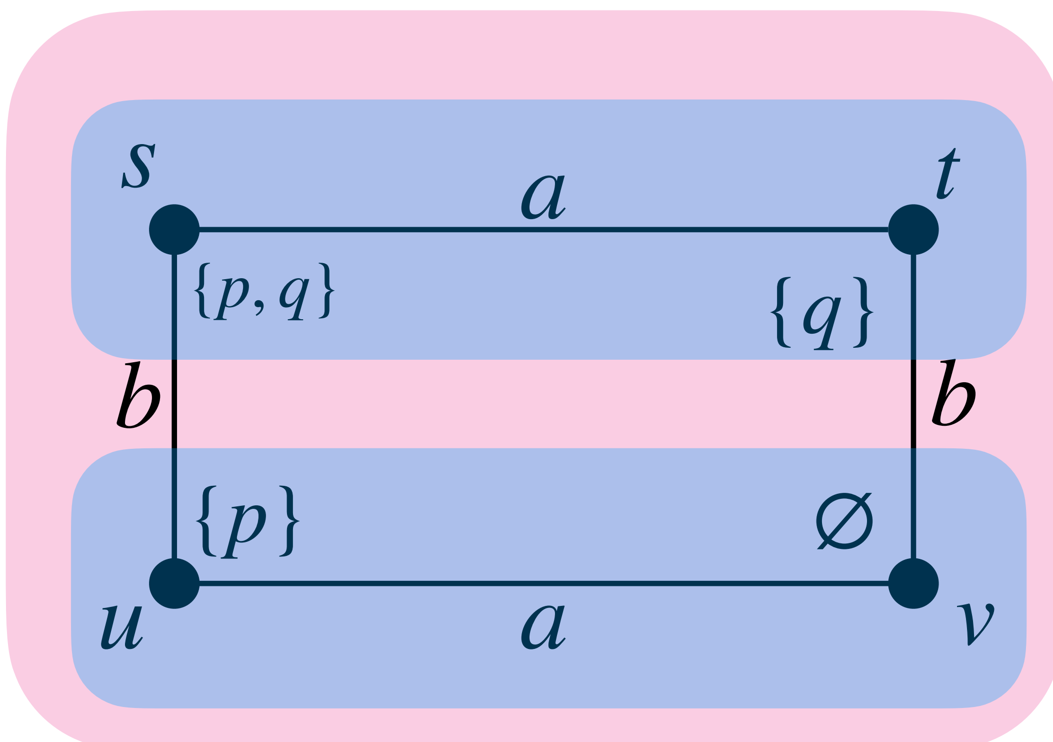
$$\exists \psi \in \mathscr{PAL}, \forall \chi \in \mathscr{PAL} : M, s \vDash \Box_a \psi \wedge [\Box_a \psi \wedge \Box_b \chi](\Box_b q \wedge \neg \Box_a p)$$

There are only two ways agent a can influence the model

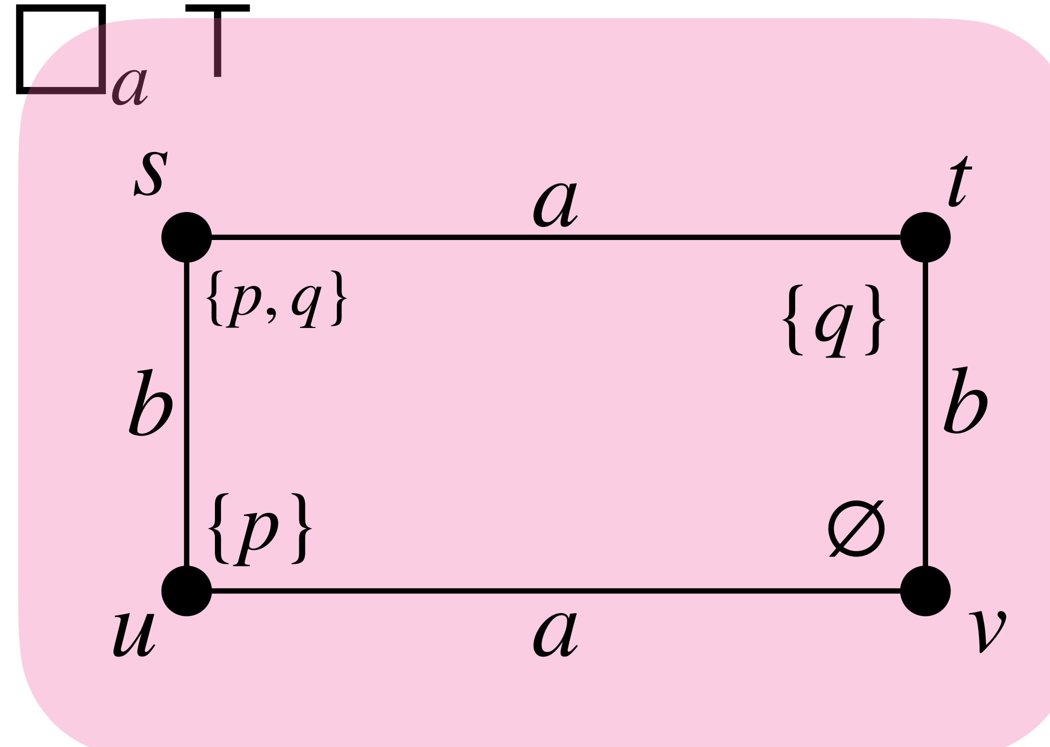
Example

CAL

M



$M * \Box_a \top$



$$M, s \models \langle [a] \rangle (\Box_b q \land \neg \Box_a p)$$

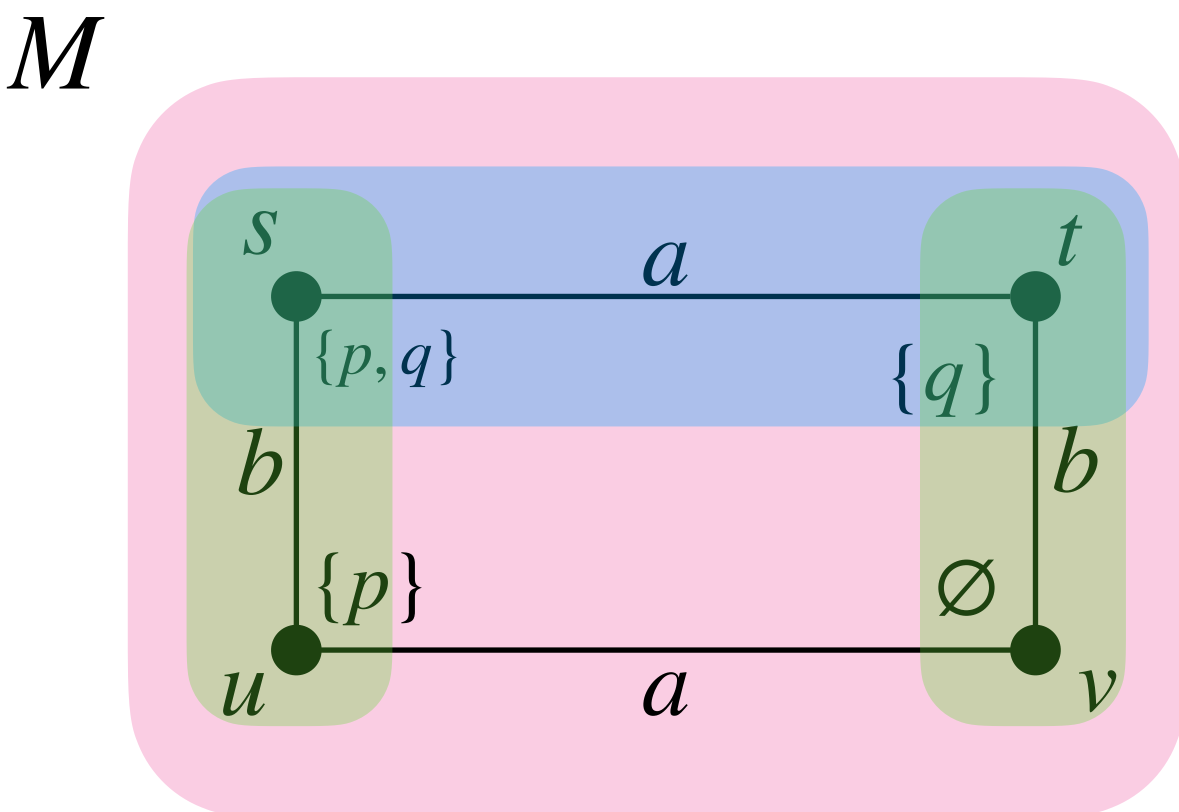
$$\exists \psi \in \mathscr{PAL}, \forall \chi \in \mathscr{PAL} : M, s \models \Box_a \psi \land [\Box_a \psi \land \Box_b \chi](\Box_b q \land \neg \Box_a p)$$

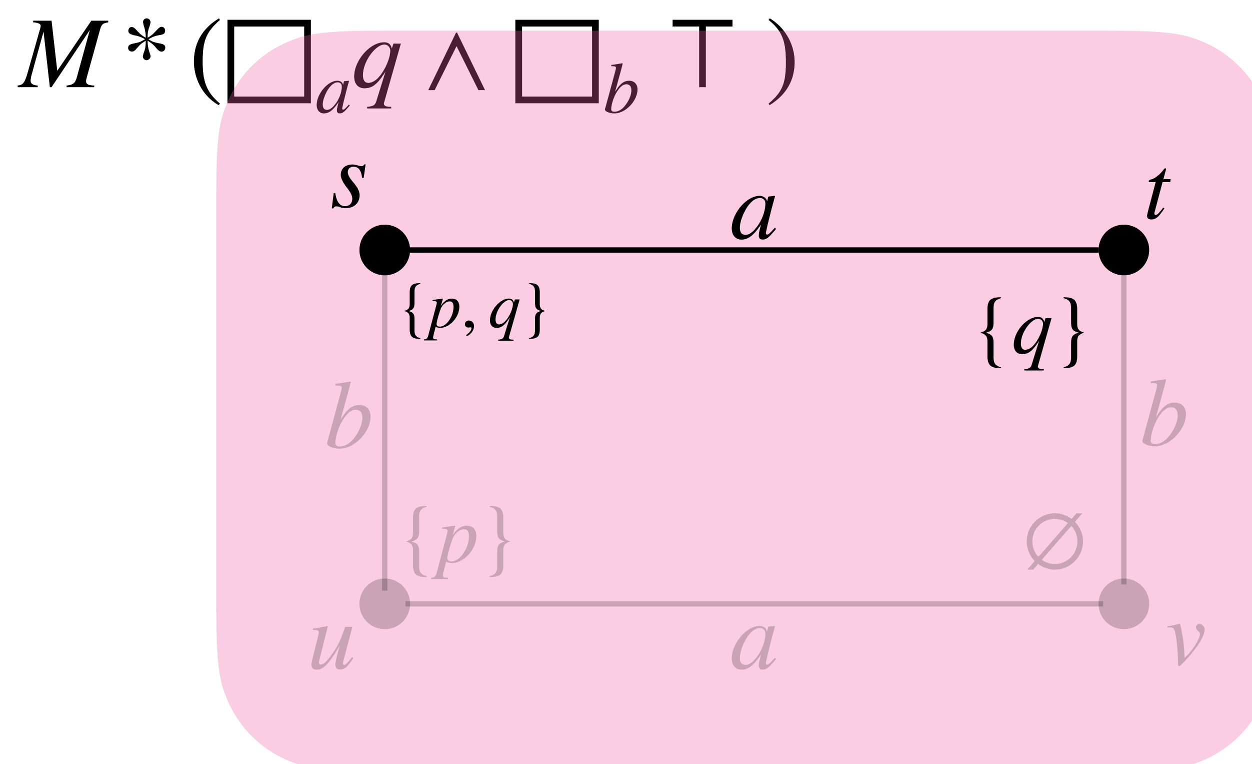
There are only two ways agent a can influence the model

This second option does not achieve $\Box_b q$

Example

CAL





$$M, s \models \langle\!\![a]\!\!\rangle(\Box_b q \wedge \neg \Box_a p)$$

$$\exists \psi \in \mathscr{PAL}, \forall \chi \in \mathscr{PAL} : M, s \models \Box_a \psi \wedge [\Box_a \psi \wedge \Box_b \chi](\Box_b q \wedge \neg \Box_a p)$$

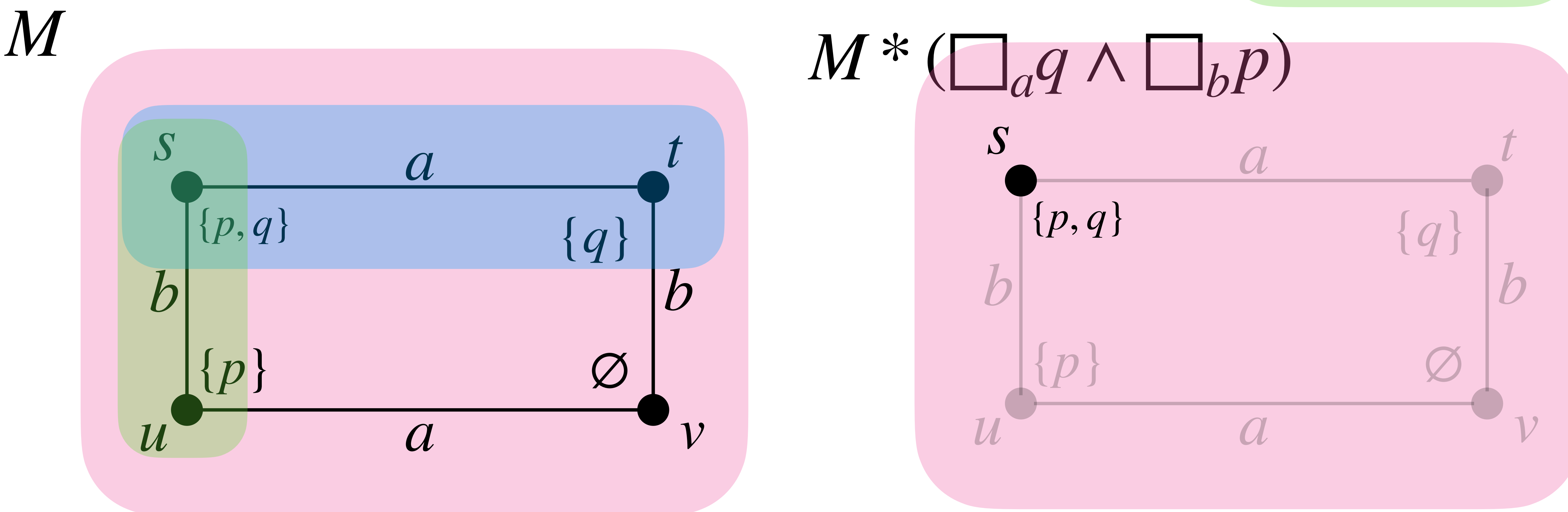
There are only two ways agent b can influence the model

In this case, the target formula $\Box_b q \wedge \neg \Box_a p$ is true

We need however, to quantify over all b's announcements

Example

CAL



$$M, s \models \langle\!\![a]\!\!\rangle(\Box_b q \wedge \neg \Box_a p)$$

$$\exists \psi \in \mathscr{PAL}, \forall \chi \in \mathscr{PAL} : M, s \models \Box_a \psi \wedge [\Box_a \psi \wedge \Box_b \chi](\Box_b q \wedge \neg \Box_a p)$$

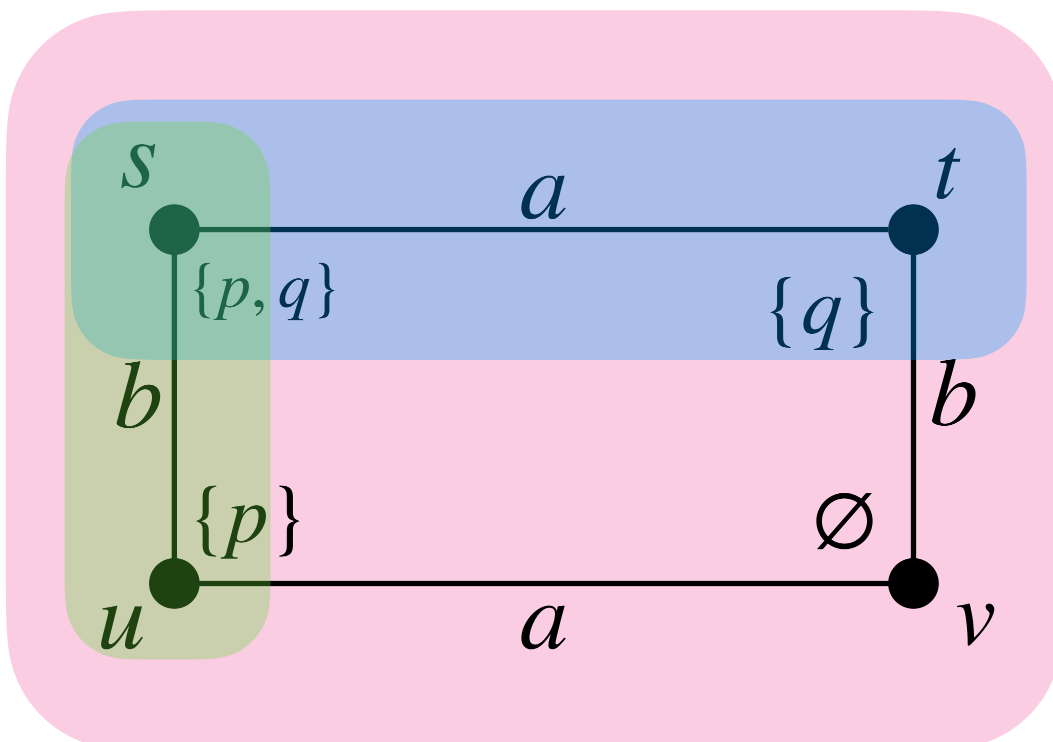
There are only two ways agent b can influence the model

In this case, $\neg \Box_a p$ is not true, i.e. b can make agent a learn p no matter what

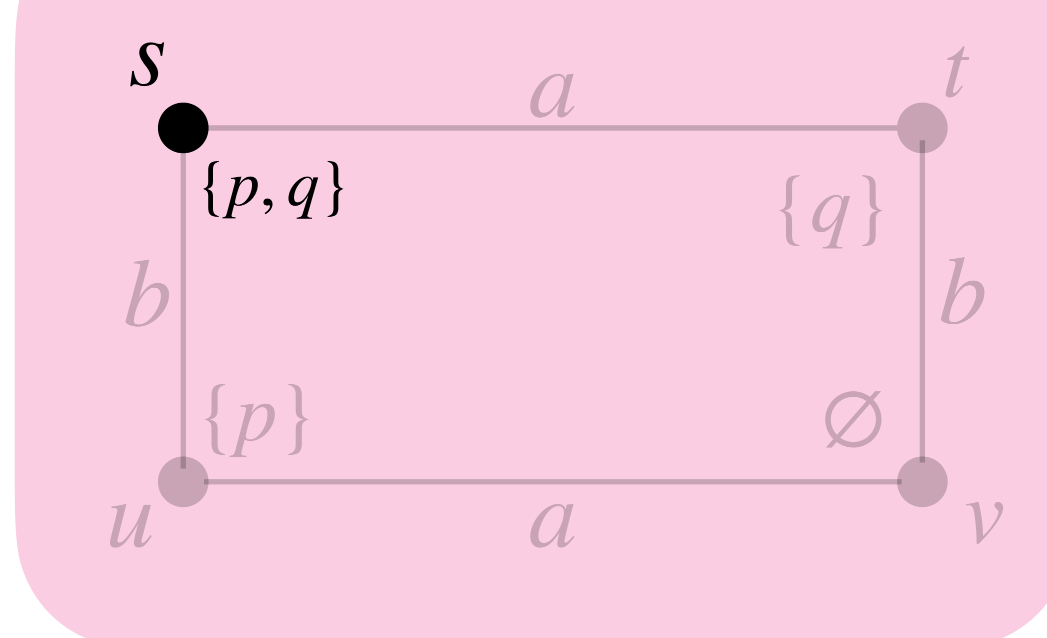
Example

CAL

M



$M * (\Box_a q \wedge \Box_b p)$



$$M, s \nvDash \langle\![a]\!\rangle(\Box_b q \wedge \neg \Box_a p)$$ ✗

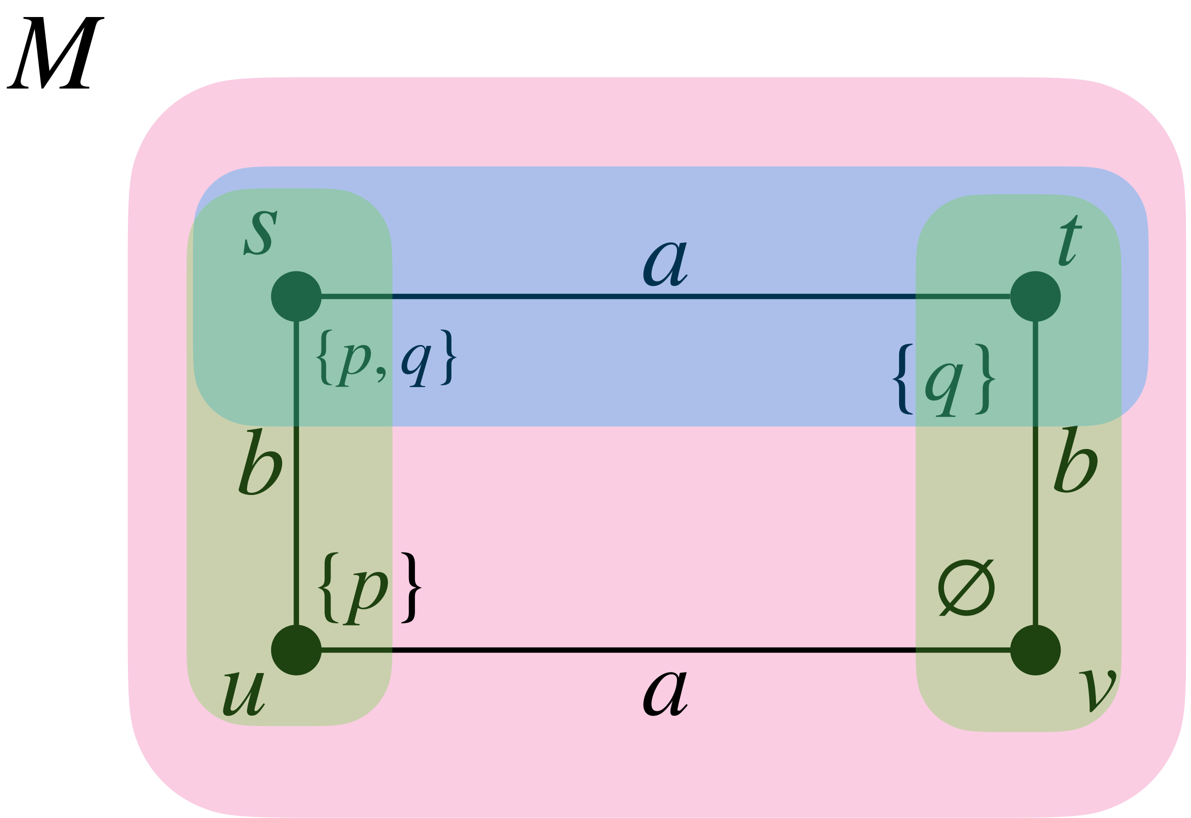
$$\exists \psi \in \mathscr{PAL}, \forall \chi \in \mathscr{PAL} : M, s \vDash \Box_a \psi \wedge [\Box_a \psi \wedge \Box_b \chi](\Box_b q \wedge \neg \Box_a p)$$

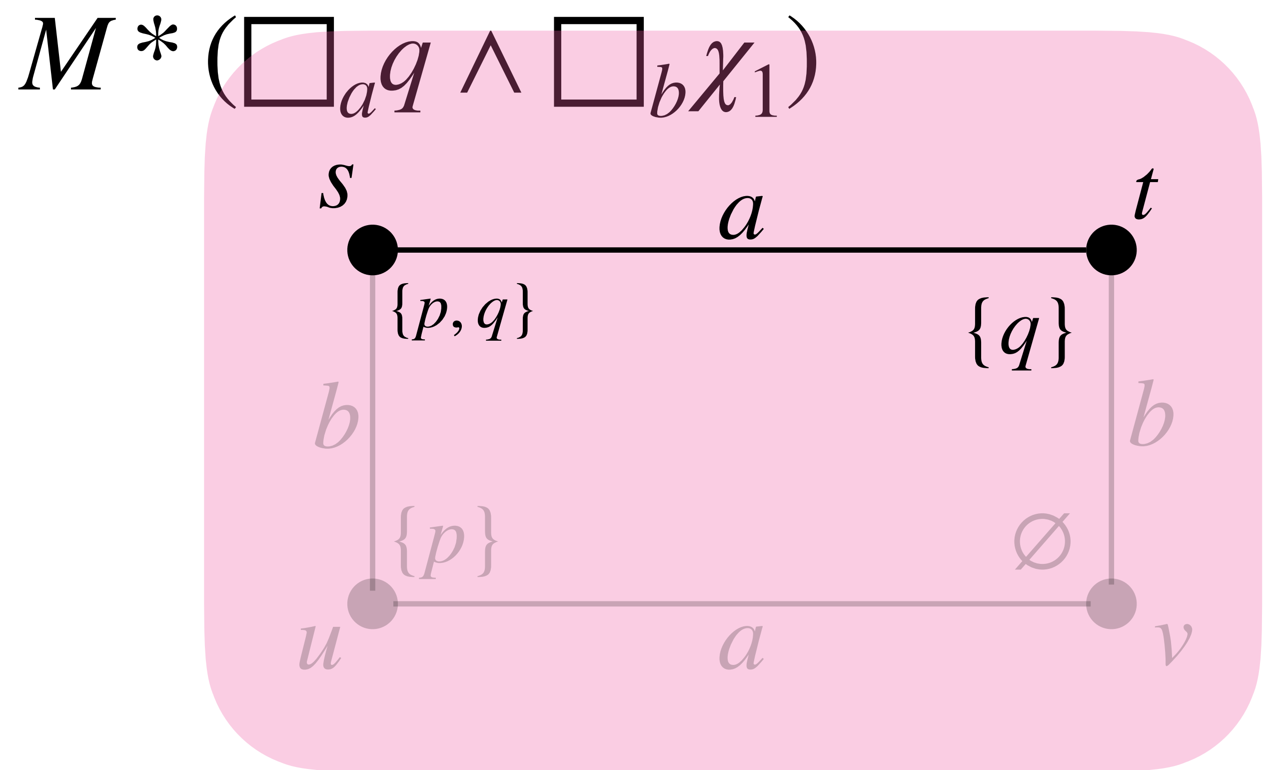
There are only two ways agent b can influence the model

In this case, $\neg \Box_a p$ is not true, i.e. b can make agent a learn p no matter what

Example

CAL





$$M, s \models \langle\![a]\!\rangle \Box_b q$$

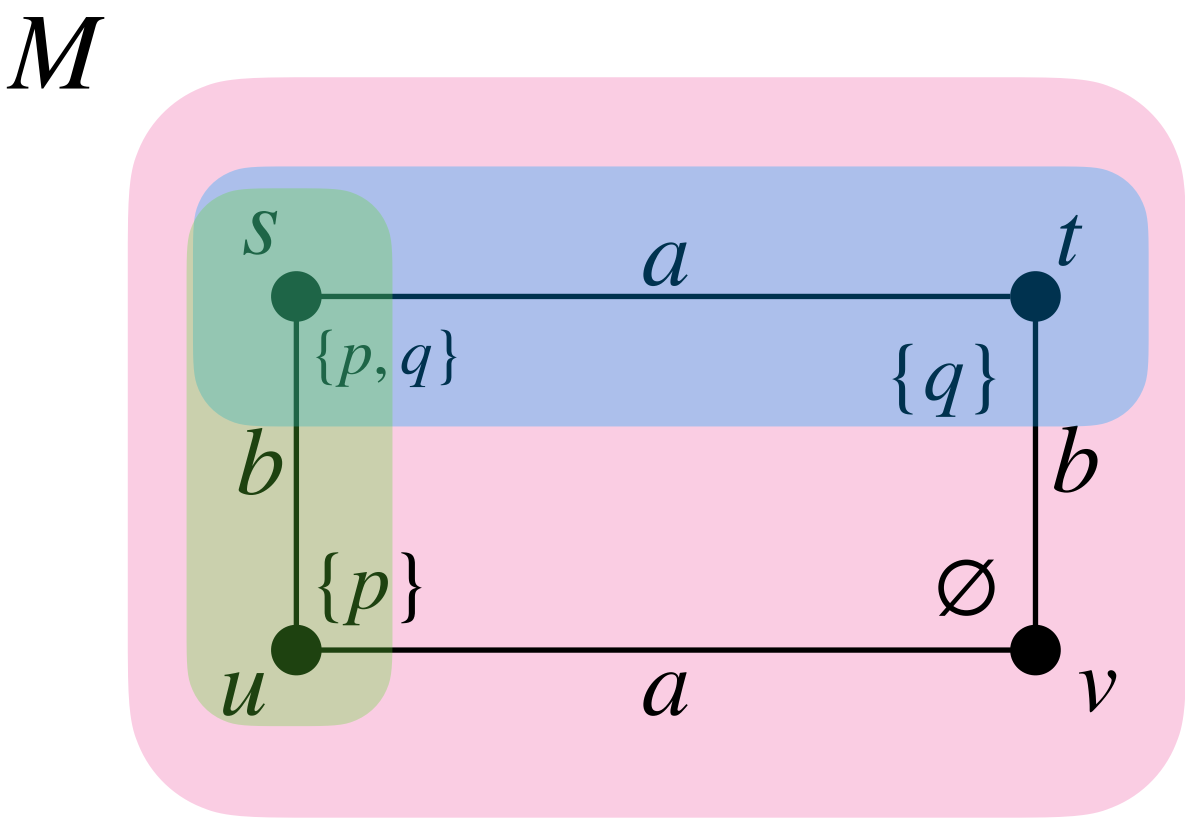
$$\exists \psi \in \mathscr{PAL}, \forall \chi \in \mathscr{PAL} : M, s \models \Box_a \psi \wedge [\Box_a \psi \wedge \Box_b \chi] \Box_b q$$

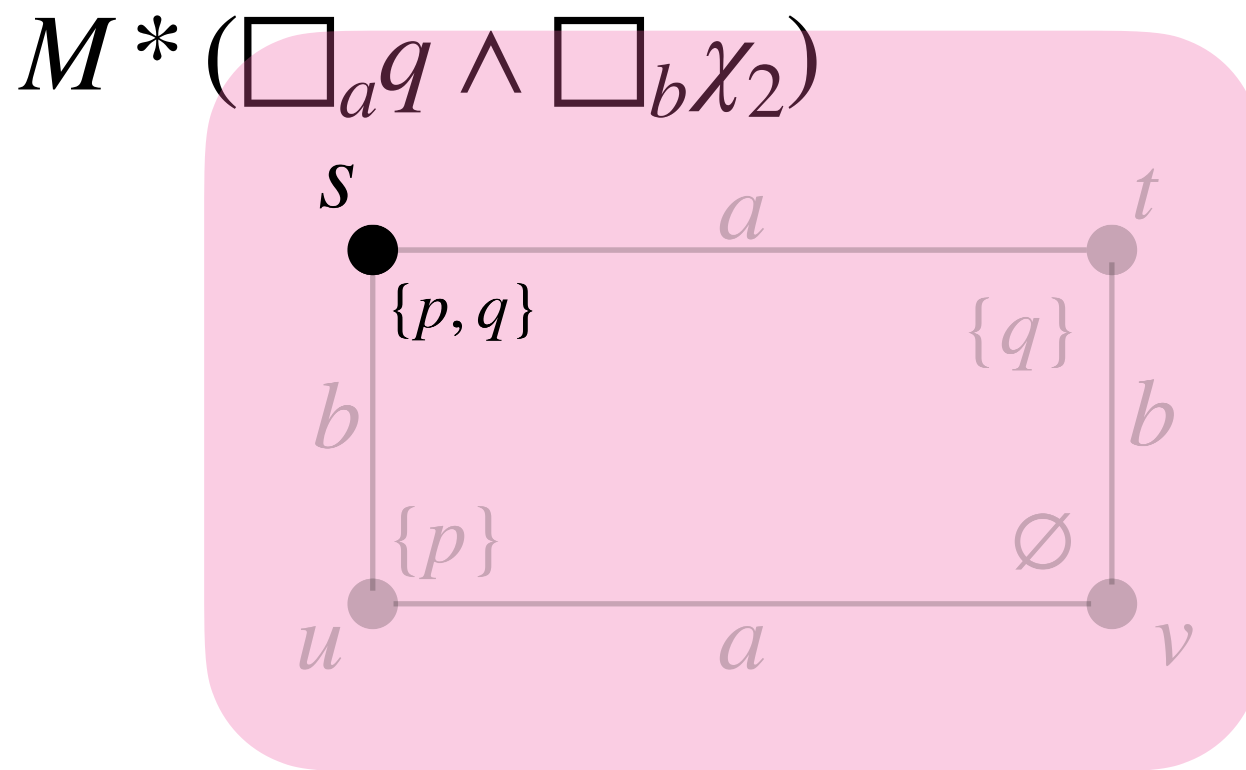
$$\forall \chi \in \mathscr{PAL} : M, s \models \Box_a q \wedge [\Box_a q \wedge \Box_b \chi] \Box_b q$$

Now, whatever b announces at the same time (only two options), she cannot avoid $\Box_b q$

Example

CAL





$$M, s \models \langle\!\![a]\!\!\rangle \Box_b q$$

$$\exists \psi \in \mathscr{PAL}, \forall \chi \in \mathscr{PAL} : M, s \models \Box_a \psi \wedge [\Box_a \psi \wedge \Box_b \chi] \Box_b q$$

$$\forall \chi \in \mathscr{PAL} : M, s \models \Box_a q \wedge [\Box_a q \wedge \Box_b \chi] \Box_b q$$

Now, whatever b announces at the same time (only two options), she cannot avoid $\Box_b q$

CAL as a Strategy Logic

Modalities of CAL we inspired by the modalities of Coalition Logic (CL) $[\![G]\!]\varphi$ and $\langle\!\langle G\rangle\!\rangle\varphi$

$\langle\!\langle G\rangle\!\rangle\varphi$: There is an action by agents from coalition G, such that no matter what agents in the anti-coalition do at the same time, φ is true

$\langle\!\langle G\rangle\!\rangle\varphi$: coalition G can force φ

Modalities of CL capture strategies in normal form games (one shot games)

Pauly. *A modal logic for coalitional power in games*, 2002.

CAL as a Strategy Logic

Axioms of CL

$\neg \langle\!\langle G \rangle\!\rangle \bot \qquad \langle\!\langle G \rangle\!\rangle \top$

$\neg \langle\!\langle \varnothing \rangle\!\rangle \neg \varphi \rightarrow \langle\!\langle A \rangle\!\rangle \varphi$

$\langle\!\langle G \rangle\!\rangle (\varphi \land \psi) \rightarrow \langle\!\langle G \rangle\!\rangle \varphi$

$\langle\!\langle G \rangle\!\rangle \varphi \land \langle\!\langle H \rangle\!\rangle \psi \rightarrow$
$\langle\!\langle G \cup H \rangle\!\rangle (\varphi \land \psi)$, if
$G \cap H = \varnothing$

Validities of CAL

$\neg \langle\![G]\!\rangle \bot \qquad \langle\![G]\!\rangle \top$

$\neg \langle\![\varnothing]\!\rangle \neg \varphi \rightarrow \langle\![A]\!\rangle \varphi$

$\langle\![G]\!\rangle (\varphi \land \psi) \rightarrow \langle\![G]\!\rangle \varphi$

$\langle\![G]\!\rangle \varphi \land \langle\![H]\!\rangle \psi \rightarrow$
$\langle\![G \cup H]\!\rangle (\varphi \land \psi)$, if
$G \cap H = \varnothing$

Theorem. CAL subsumes CL

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Pauly. *A modal logic for coalitional power in games*, 2002.

Virtues of Cooperation

$\langle\![G]\!\rangle\langle\![H]\!\rangle\varphi \to \langle\![G \cup H]\!\rangle\varphi$ $\quad$ $\langle\![G \cup H]\!\rangle\varphi \nrightarrow \langle\![G]\!\rangle\langle\![H]\!\rangle\varphi$

$\langle\![G]\!\rangle\langle\![H]\!\rangle\varphi \to \langle\![G \cup H]\!\rangle\varphi$: If coalitions G and H can achieve φ by consecutively, they can achieve φ simultaneously

$\langle\![G \cup H]\!\rangle\varphi \nrightarrow \langle\![G]\!\rangle\langle\![H]\!\rangle\varphi$: Competing coalitions can spoil each others' strategies

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Axiomatisation of CAL

Theorem. CAL is more expressive than PAL; there are some properties that can be expressed in APAL but not in CAL

Theorem. Complexity of MC-CAL is PSPACE-complete

Theorem. SAT-CAL is undecidable

Open Problem. Is there an axiomatisation, finitary or infinitary, of CAL?

Alechina et al. *The Expressivity of Quantified Group Announcements*, 2022.

Alechina et al. *Verification and Strategy Synthesis for Coalition Announcement Logic*, 2021.

Ågotnes, French, Van Ditmarsch. *The Undecidability of Quantified Announcements*, 2016.

Why the OP is hard

While proving completeness using the canonical model construction, one usually has to prove a Lindenbaum type lemma

Recall APAL

$$M, s \vDash [!]\varphi \text{ iff } \forall \psi \in \mathscr{PAL} : M, s \vDash [\psi]\varphi$$

$$M, s \vDash \langle ! \rangle\varphi \text{ iff } \exists \psi \in \mathscr{PAL} : M, s \vDash \langle \psi \rangle\varphi$$

$$[!]\varphi$$

MCS

$$[!]\varphi \to [\psi_1]\varphi$$
$$[!]\varphi \to [\psi_2]\varphi$$
$$[!]\varphi \to [\psi_3]\varphi$$
$$\dots$$

Instances of an axiom schema

Why the OP is hard

While proving completeness using the canonical model construction, one usually has to prove a Lindenbaum type lemma

Recall APAL

$$M, s \vDash [!]\varphi \text{ iff } \forall \psi \in \mathscr{PAL} : M, s \vDash [\psi]\varphi$$

$$M, s \vDash \langle ! \rangle\varphi \text{ iff } \exists \psi \in \mathscr{PAL} : M, s \vDash \langle \psi \rangle\varphi$$

MCS $[!]\varphi$

$[\psi_1]\varphi$

$[\psi_2]\varphi$

$[\psi_3]\varphi$

$\ldots$

By closure under MP

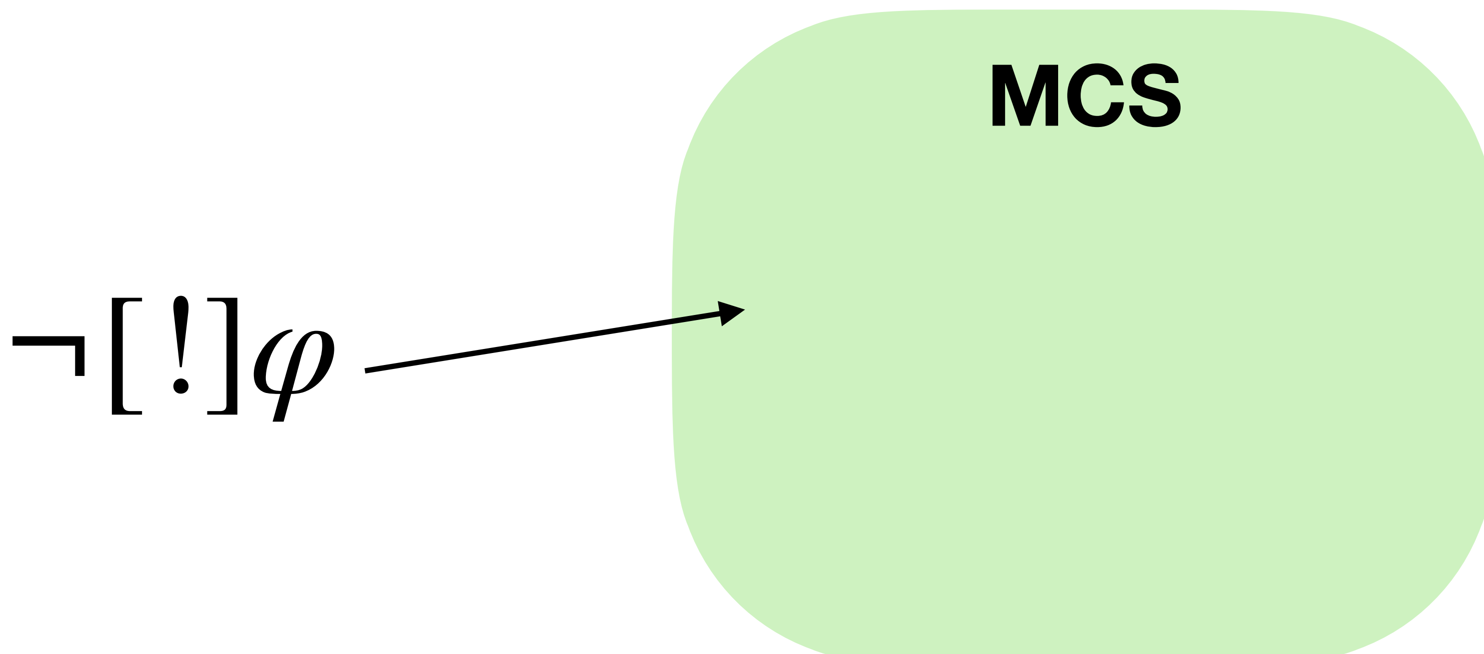
Why the OP is hard

While proving completeness using the canonical model construction, one usually has to prove a Lindenbaum type lemma

Recall APAL

$$M, s \models [\,!\,]\varphi \text{ iff } \forall \psi \in \mathscr{PAL} : M, s \models [\psi]\varphi$$

$$M, s \models \langle\,!\,\rangle\varphi \text{ iff } \exists \psi \in \mathscr{PAL} : M, s \models \langle\psi\rangle\varphi$$



Add a witness

Why the OP is hard

While proving completeness using the canonical model construction, one usually has to prove a Lindenbaum type lemma

Recall APAL

$M, s \models [!]\varphi$ iff $\forall \psi \in \mathscr{PAL} : M, s \models [\psi]\varphi$

$M, s \models \langle ! \rangle\varphi$ iff $\exists \psi \in \mathscr{PAL} : M, s \models \langle \psi \rangle\varphi$

MCS

$\neg[!]\varphi$

$\neg[\psi_n]\varphi$

Add a witness

Why the OP is hard

While proving completeness using the canonical model construction, one usually has to prove a Lindenbaum type lemma

Recall CAL

$$M, s \models [\langle G \rangle]\varphi \text{ iff}$$
$$\forall \psi_G \exists \chi_{A \setminus G} : M, s \models \psi_G \to \langle \psi_G \wedge \chi_{A \setminus G} \rangle \varphi$$

$$M, s \models \langle [G] \rangle \varphi \text{ iff}$$
$$\exists \psi_G \forall \chi_{A \setminus G} : M, s \models \psi_G \wedge [\psi_G \wedge \chi_{A \setminus G}]\varphi$$

Note double quantification in both box and diamond operators

It is not clear how to deal with the double quantification

Why the OP is hard

While proving completeness using the canonical model construction, one usually has to prove a Lindenbaum type lemma

Recall CAL

$$M, s \models [\langle G \rangle]\varphi \text{ iff}$$
$$\forall \psi_G \exists \chi_{A\setminus G} : M, s \models \psi_G \to \langle \psi_G \land \chi_{A\setminus G} \rangle \varphi$$
$$M, s \models \langle [G] \rangle\varphi \text{ iff}$$
$$\exists \psi_G \forall \chi_{A\setminus G} : M, s \models \psi_G \land [\psi_G \land \chi_{A\setminus G}] \varphi$$

MCS

$[\langle G \rangle]\varphi$ → ???

For each ψ_G there may be a unique corresponding $\chi_{A\setminus G}$

Why the OP is hard

While proving completeness using the canonical model construction, one usually has to prove a Lindenbaum type lemma

Recall CAL

$$M, s \models [\langle G \rangle]\varphi \text{ iff}$$

$$\forall \psi_G \exists \chi_{A \setminus G} : M, s \models \psi_G \rightarrow \langle \psi_G \wedge \chi_{A \setminus G} \rangle \varphi$$

$$M, s \models \langle [G] \rangle \varphi \text{ iff}$$

$$\exists \psi_G \forall \chi_{A \setminus G} : M, s \models \psi_G \wedge [\psi_G \wedge \chi_{A \setminus G}]\varphi$$

$\neg [\langle G \rangle]\varphi$ →

MCS

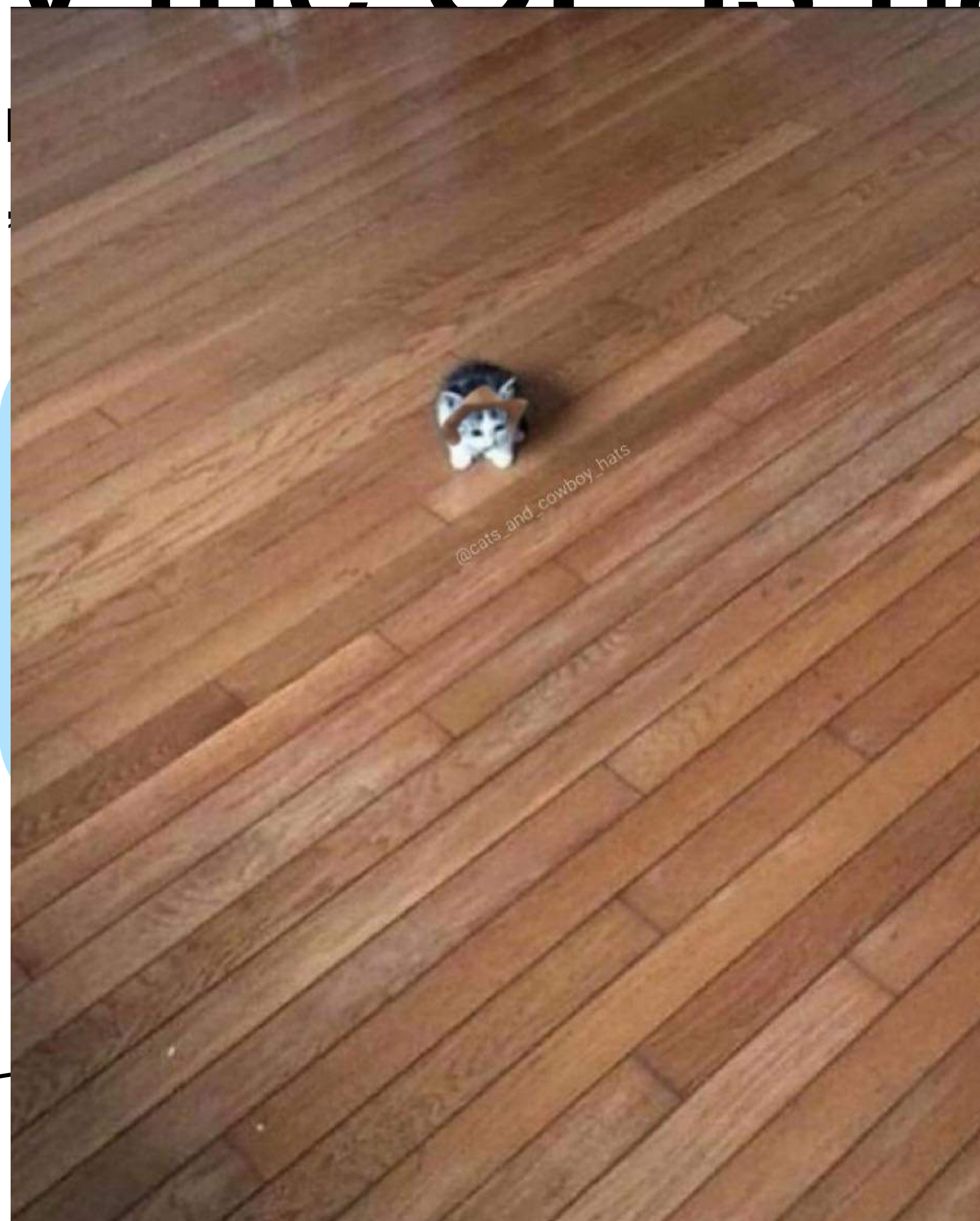
???

We need to add an infinite number of witnesses

Why the OP is hard

While provi … cal model construction, … nbaum type

Recall CAL



$\ldots \wedge \chi_{A \setminus G} \rangle \varphi$

$\ldots \wedge \chi_{A \setminus G}] \varphi$

$\neg [\langle G \rangle] \varphi$

… eed to add an … te number of … vitnesses

Partial Solution

We can use additional operators to split the quantification in CAL modalities

$[G, \chi]\varphi$: given a true announcement χ, whatever agents from coalition G announce in conjunction with χ, φ is true

$\langle G, \chi \rangle \varphi$: given any announcement χ, there is a simultaneous announcement by agents from coalition G, such that φ is true

Observe only single quantifiers

Formula χ is used as a placeholder (or memory) for announcements by a coalition

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Coalition and Relativised GAL

Language of CoRGAL

$$\mathscr{CORGAL} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\varphi]\varphi \mid [G,\varphi]\varphi \mid [\langle G \rangle]\varphi$$

Semantics

$$M, s \models [G, \chi]\varphi \text{ iff } \forall \psi_G : M, s \models \chi \wedge [\chi \wedge \psi_G]\varphi$$

$$M, s \models \langle G, \chi \rangle\varphi \text{ iff } \exists \psi_G : M, s \models \chi \rightarrow \langle \chi \wedge \psi_G \rangle\varphi$$

$$M_s \models [\langle G \rangle]\varphi \text{ iff } \forall \psi_G : M_s \models \langle A \setminus G, \psi_G \rangle\varphi$$

$$M_s \models \langle [G] \rangle\varphi \text{ iff } \exists \psi_G : M_s \models [A \setminus G, \psi_G]\varphi$$

Coalition operators now have only one quantifier

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Axiomatisation of CoRGAL

Axioms of EL and PAL

$[G, \chi]\varphi \to \chi \land [\psi_G \land \chi]\varphi$ with $\psi_G \in \mathcal{PAL}$

$[\langle G \rangle]\varphi \to \langle A \backslash G, \psi_G \rangle \varphi$ with $\psi_G \in \mathcal{PAL}$

From $\{\eta(\chi \land [\psi_G \land \chi]\varphi) \mid \psi_G \in \mathcal{PAL}\}$ infer $\eta([G, \chi]\varphi)$

From $\{\eta(\langle A \backslash G, \psi_G \rangle) \mid \psi_G \in \mathcal{PAL}\}$ infer $\eta([\langle G \rangle]\varphi)$

$[\langle G \rangle]\varphi$ →

MCS

$[\langle G \rangle]\varphi \to \langle A \backslash G, \psi_G^1 \rangle \varphi$

$[\langle G \rangle]\varphi \to \langle A \backslash G, \psi_G^2 \rangle \varphi$

$[\langle G \rangle]\varphi \to \langle A \backslash G, \psi_G^3 \rangle \varphi$

$\dots$

Instances of an axiom schema

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Axiomatisation of CoRGAL

Axioms of EL and PAL

$[G, \chi]\varphi \rightarrow \chi \wedge [\psi_G \wedge \chi]\varphi$ with $\psi_G \in \mathscr{PAL}$

$[\langle G \rangle]\varphi \rightarrow \langle A \backslash G, \psi_G \rangle \varphi$ with $\psi_G \in \mathscr{PAL}$

From $\{\eta(\chi \wedge [\psi_G \wedge \chi]\varphi) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([G, \chi]\varphi)$

From $\{\eta(\langle A \backslash G, \psi_G \rangle) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([\langle G \rangle]\varphi)$

MCS $[\langle G \rangle]\varphi$

$\langle A \backslash G, \psi_G^1 \rangle \varphi$

$\langle A \backslash G, \psi_G^2 \rangle \varphi$

$\langle A \backslash G, \psi_G^3 \rangle \varphi$

$\ldots$

Closure under MP

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Axiomatisation of CoRGAL

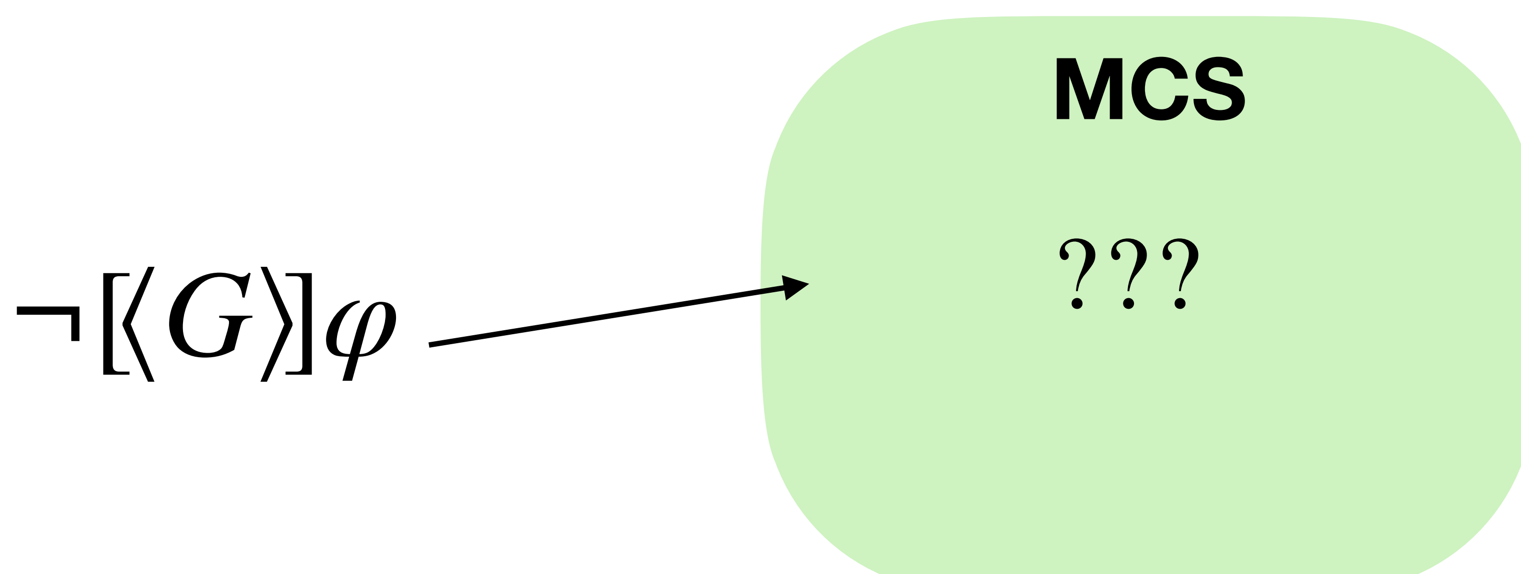
Axioms of EL and PAL

$[G,\chi]\varphi \to \chi \land [\psi_G \land \chi]\varphi$ with $\psi_G \in \mathscr{PAL}$

$[\langle G \rangle]\varphi \to \langle A \backslash G, \psi_G \rangle\varphi$ with $\psi_G \in \mathscr{PAL}$

From $\{\eta(\chi \land [\psi_G \land \chi]\varphi) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([G,\chi]\varphi)$

From $\{\eta(\langle A \backslash G, \psi_G \rangle) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([\langle G \rangle]\varphi)$



RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Axiomatisation of CoRGAL

Axioms of EL and PAL

$[G, \chi]\varphi \rightarrow \chi \wedge [\psi_G \wedge \chi]\varphi$ with $\psi_G \in \mathscr{PAL}$

$[\langle G \rangle]\varphi \rightarrow \langle A \backslash G, \psi_G \rangle \varphi$ with $\psi_G \in \mathscr{PAL}$

From $\{\eta(\chi \wedge [\psi_G \wedge \chi]\varphi) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([G, \chi]\varphi)$

From $\{\eta(\langle A \backslash G, \psi_G \rangle) \mid \psi_G \in \mathscr{PAL}\}$ infer $\eta([\langle G \rangle]\varphi)$

MCS

$\neg [\langle G \rangle]\varphi$

$\neg \langle A \backslash G, \psi_G^n \rangle \varphi$

Add a witness

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Back to the OP

Theorem. CoRGAL, a logic with coalition modalities, is sound and complete

Open Problem. Is there an axiomatisation, finitary or infinitary, of CAL (without additional modalities)?

RG. *Coalition and Relativised Group Announcement Logic*, 2021.

Take-home message

- Coalition announcement logic (CAL) allows quantification over truthful and simultaneous announcements by coaltioins of agents and simultaneous counter-announcements by the anti-coalition

- CAL is quite different from APAL and GAL: double quantification

- CAL with additional modalities is sound and complete

Open Problem. Is there an axiomatisation, finitary or infinitary, of CAL (without additional modalities)?