Arbitrary Public Announcement Logic

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We are dealing with S5 models (agents' relation is equivalence)

Public Announcement Logic

Language of PAL

$\mathscr{PAL} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\varphi]\varphi$

Semantics

$M, s \models [\psi]\varphi$ iff $M, s \models \psi$ implies $M * \psi, s \models \varphi$

$M, s \models \langle\psi\rangle\varphi$ iff $M, s \models \psi$ and $M * \psi, s \models \varphi$

Updated model

Let $M = (S, \sim, V)$ and $\varphi \in \mathscr{PAL}$. An updated model $M * \varphi$ is a tuple $(S^\varphi, \sim^\varphi, V^\varphi)$, where

- $S^\varphi = \{s \in S \mid M, s \models \varphi\}$;
- $\sim_a^\varphi = \sim_a \cap (S^\varphi \times S^\varphi)$;
- $V^\varphi(p) = V(p) \cap S^\varphi$.

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

Axiomatisation of PAL

Axioms of EL

$[\varphi]p \leftrightarrow (\varphi \rightarrow p)$

$[\varphi]\neg\psi \leftrightarrow (\varphi \rightarrow \neg[\varphi]\psi)$

$[\varphi](\psi \wedge \chi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\chi)$

$[\varphi]\Box_a\psi \leftrightarrow (\varphi \rightarrow \Box_a[\varphi]\psi)$

$[\varphi][\psi]\chi \leftrightarrow ([\varphi \wedge [\varphi]\psi]\chi)$

From φ infer $[\psi]\varphi$

Theorem. PAL and EL are equally expressive

Theorem. PAL is sound and complete

Theorem. Complexity of SAT-PAL is PSPACE-complete

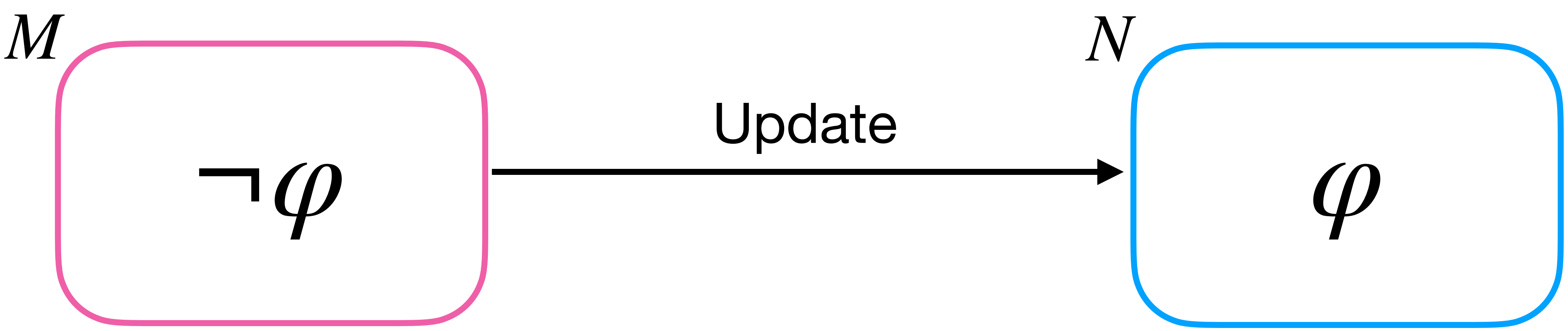
Theorem. Complexity of MC-PAL is P-complete

Van Benthem, Kooi. *Reduction axioms for epistemic actions*, 2004.

Lutz. *Complexity and Succinctness of Public Announcement Logic*, 2006.

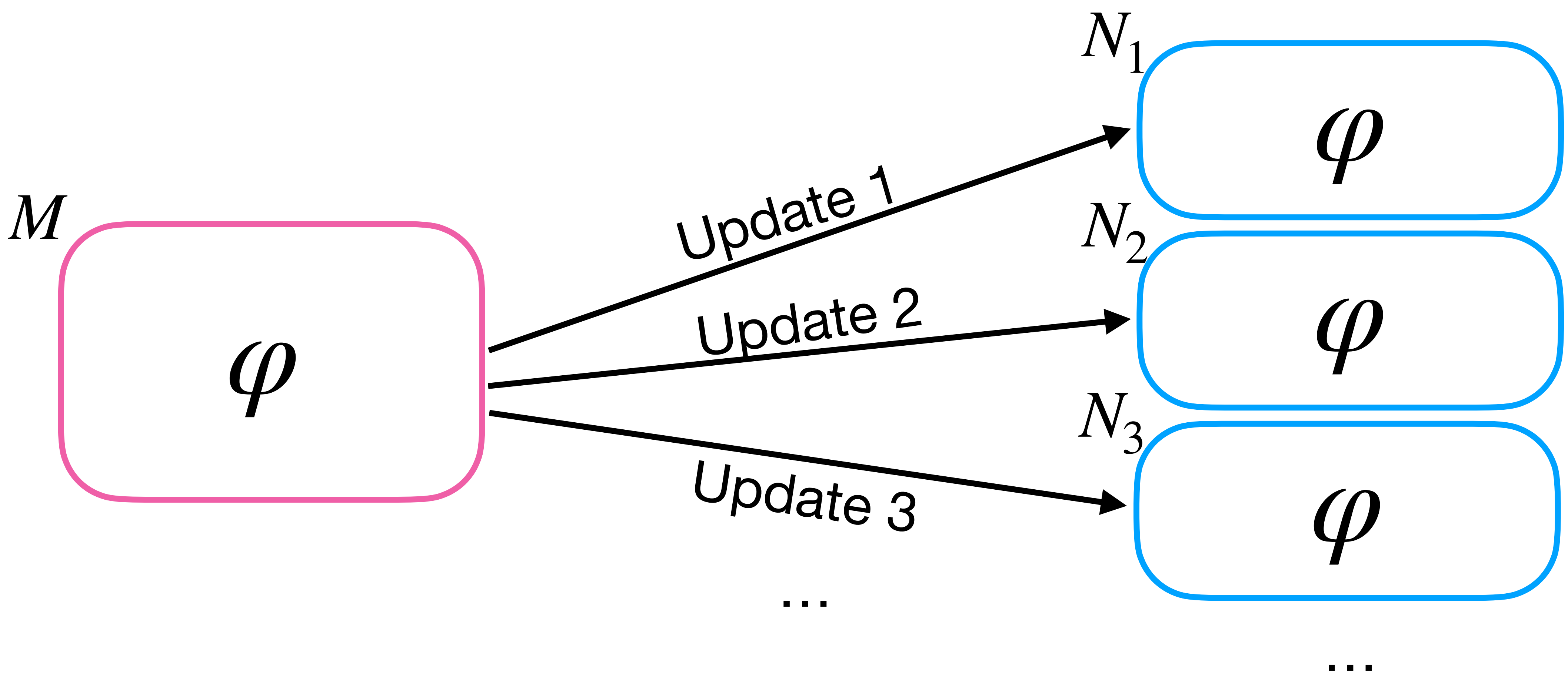
Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

Quantifying Over Updates



Existence: Having a starting configuration M and a property φ we would like to have, there is an epistemic action that results in configuration N satisfying φ

Quantifying Over Updates



Universality: Having a starting configuration M satisfying φ, we would like to ensure that all epistemic actions result in some configuration N satisfying φ

Why Quantification in DEL?

- Verification of functionality and security of a system

Functionality. There is a protocol that allows agents to achieve their goals

Why Quantification in DEL?

- Verification of functionality and security of a system

Security. No matter what agents do, they cannot reach some undesirable state

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- Use in other DEL-inspired logics, e.g. social networks and awareness

Why Quantification in DEL?

- Verification of functionality and security of a system
- Use in other DEL-inspired logics, e.g. social networks and awareness
- Protocol synthesis

Protocol synthesis. Given a goal state, provide an action (or their sequence), that takes any give state to the goal one

Why Quantification in DEL?

- Verification of functionality and security of a system
- Use in other DEL-inspired logics, e.g. social networks and awareness
- Protocol synthesis
- Capturing the notion of knowability in philosophy

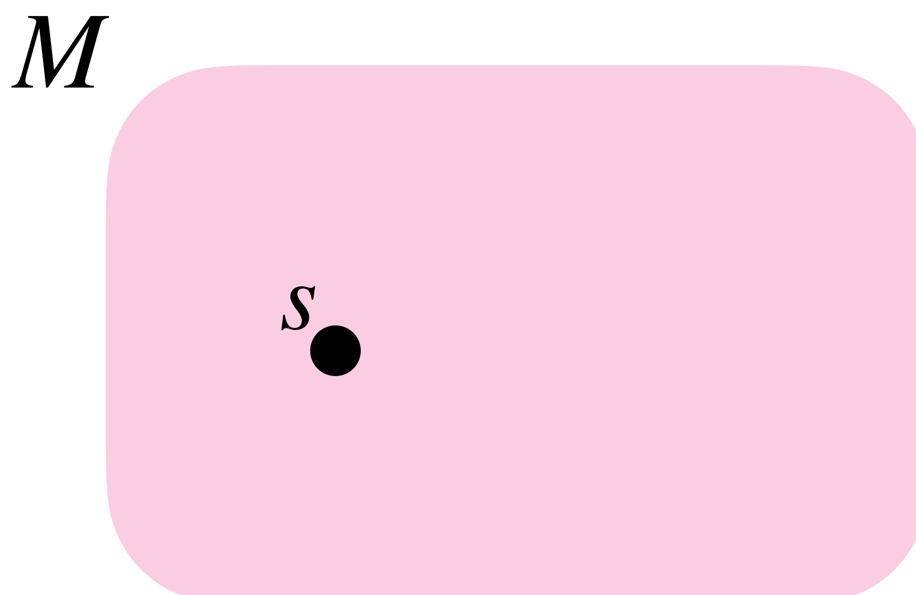
Knowability. Every true statement is knowable, in principle

Why Quantification in DEL?

- Verification of functionality and security of a system
- Use in other DEL-inspired logics, e.g. social networks and awareness
- Protocol synthesis
- Capturing the notion of knowability in philosophy
- And so on and so on and so on and so on…

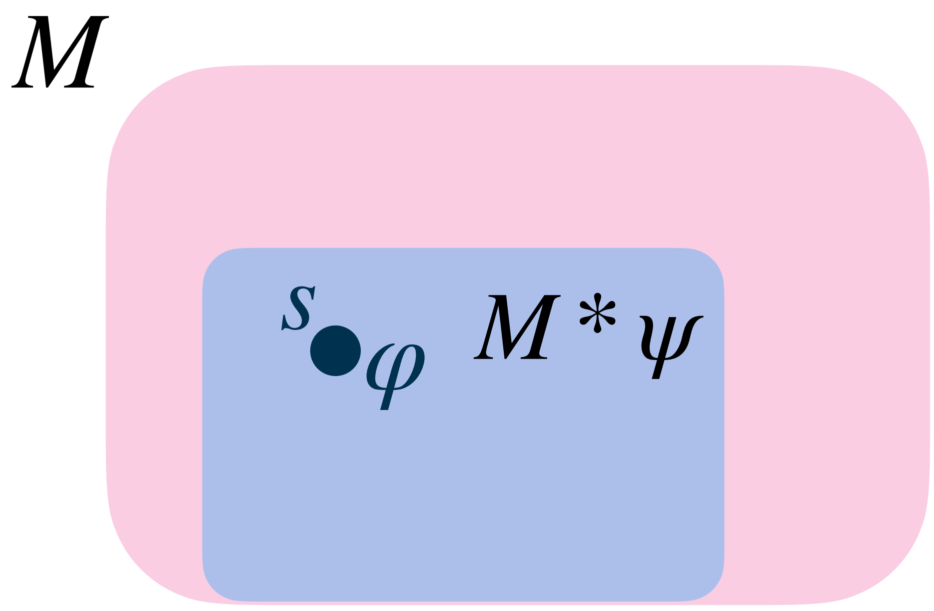
Knowability. Every true statement is knowable, in principle

Quantifying Over Public Announcements



$\langle ! \rangle \varphi$: There is a public announcement, after which φ is true

Quantifying Over Public Announcements



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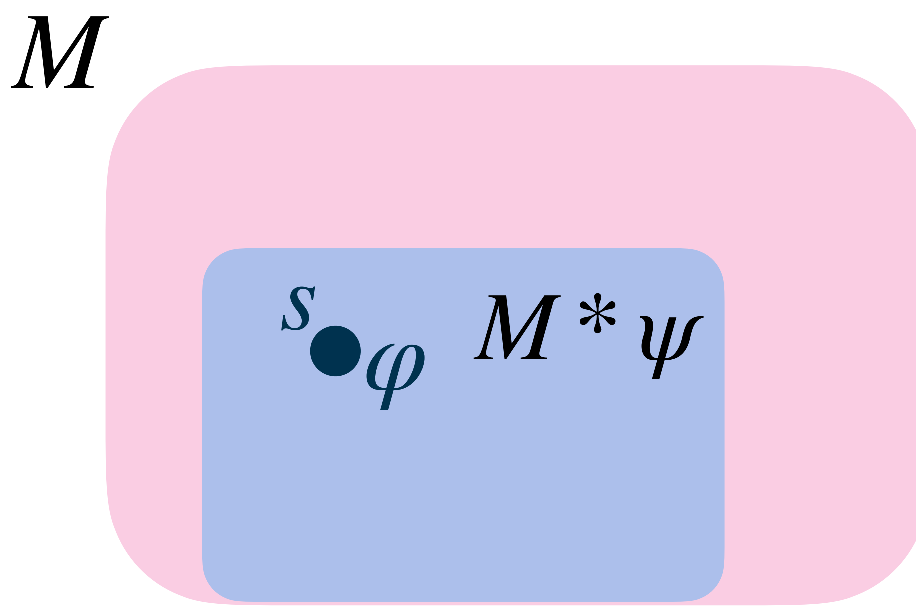
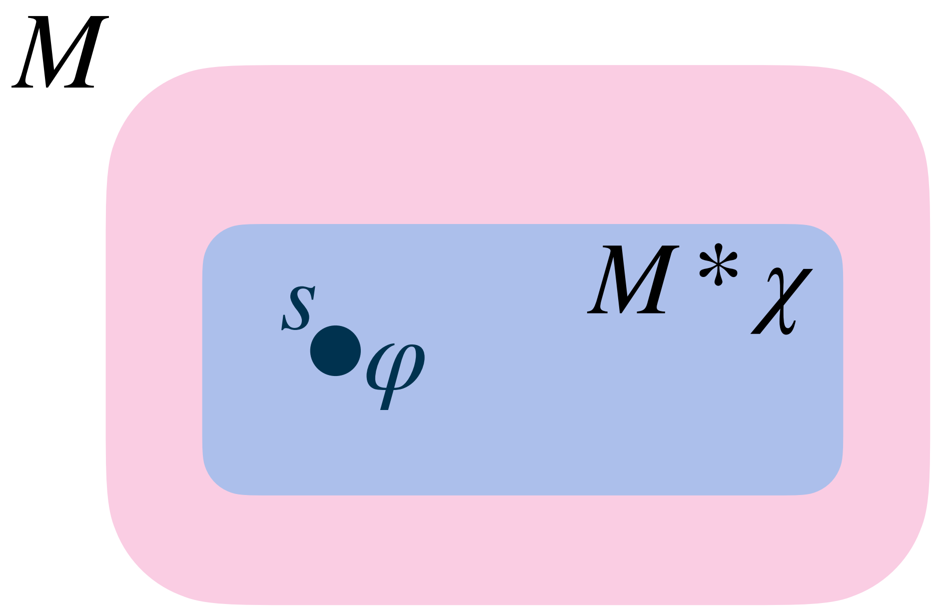


$[!]\varphi$: After all public announcements, φ is true

Quantifying Over Public Announcements



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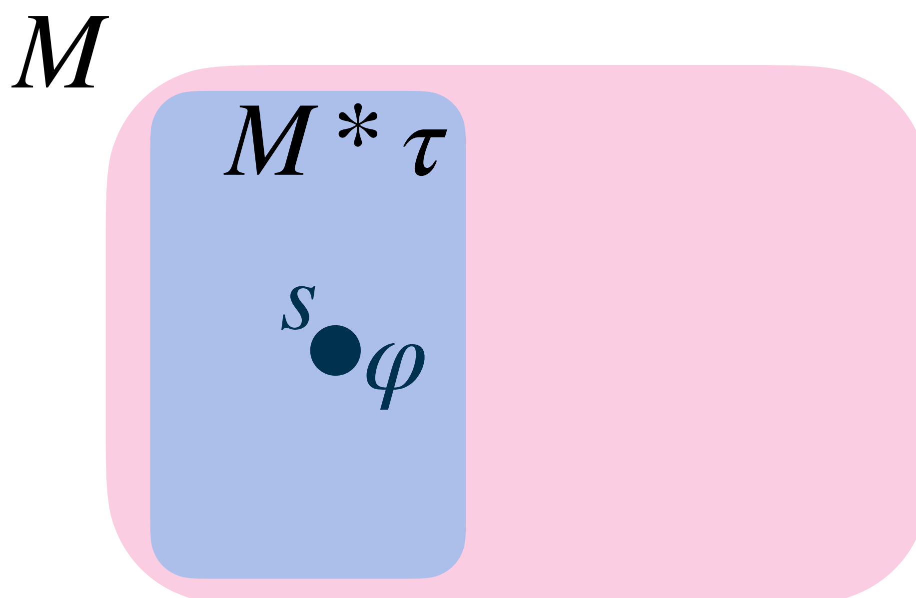
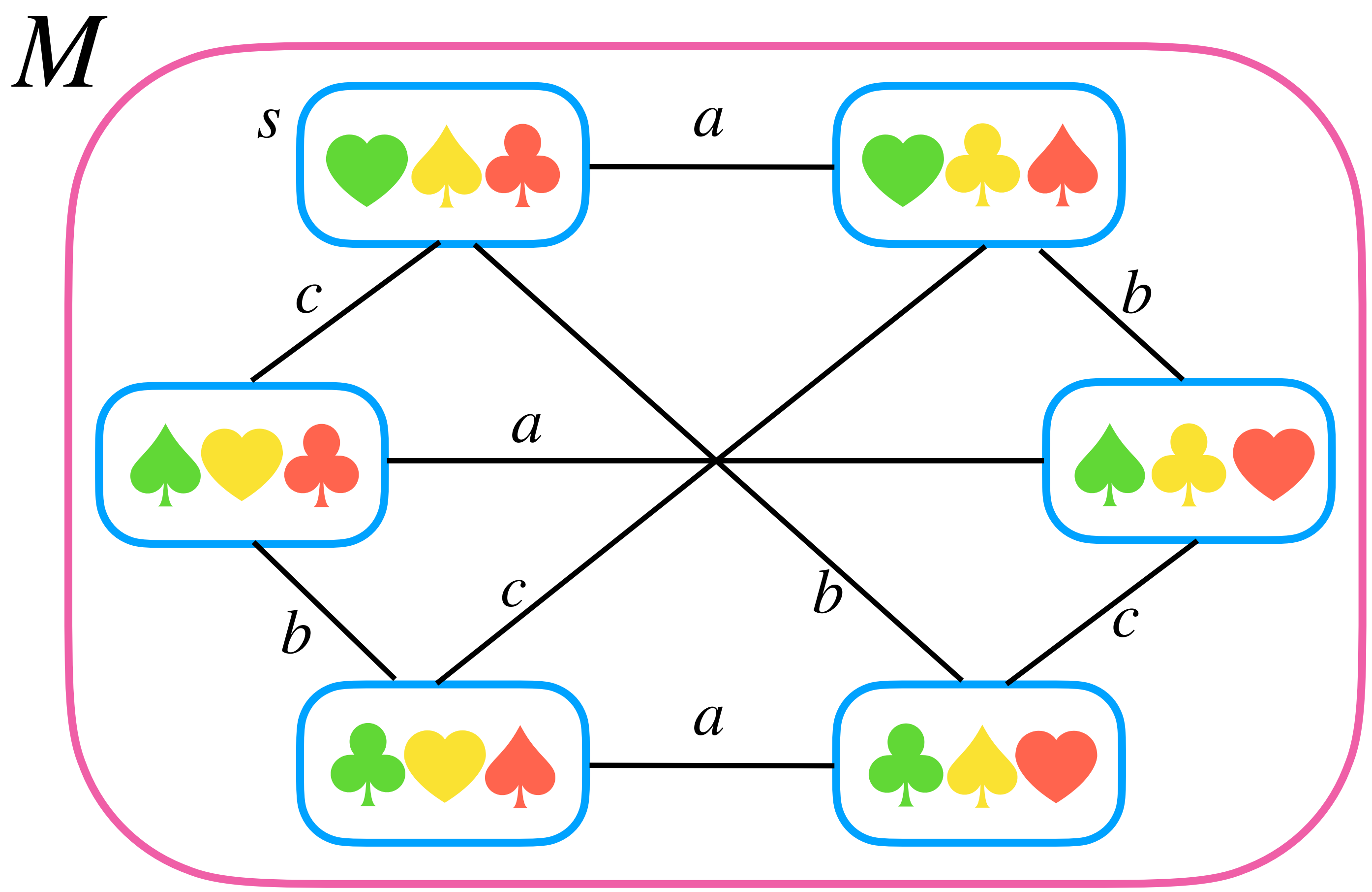


$[!]\varphi$: After all public announcements, φ is true

Card Example

There is an announcement such that Asgeir knows the deal, and Bendik and Caroline do not



$$M, s \vDash \langle ! \rangle (\Box_a \text{deal} \wedge \neg \Box_b \text{deal} \wedge \neg \Box_c \text{deal})$$

$$\varphi := (\spadesuit_b \vee \heartsuit_b) \wedge (\clubsuit_c \vee \heartsuit_c)$$

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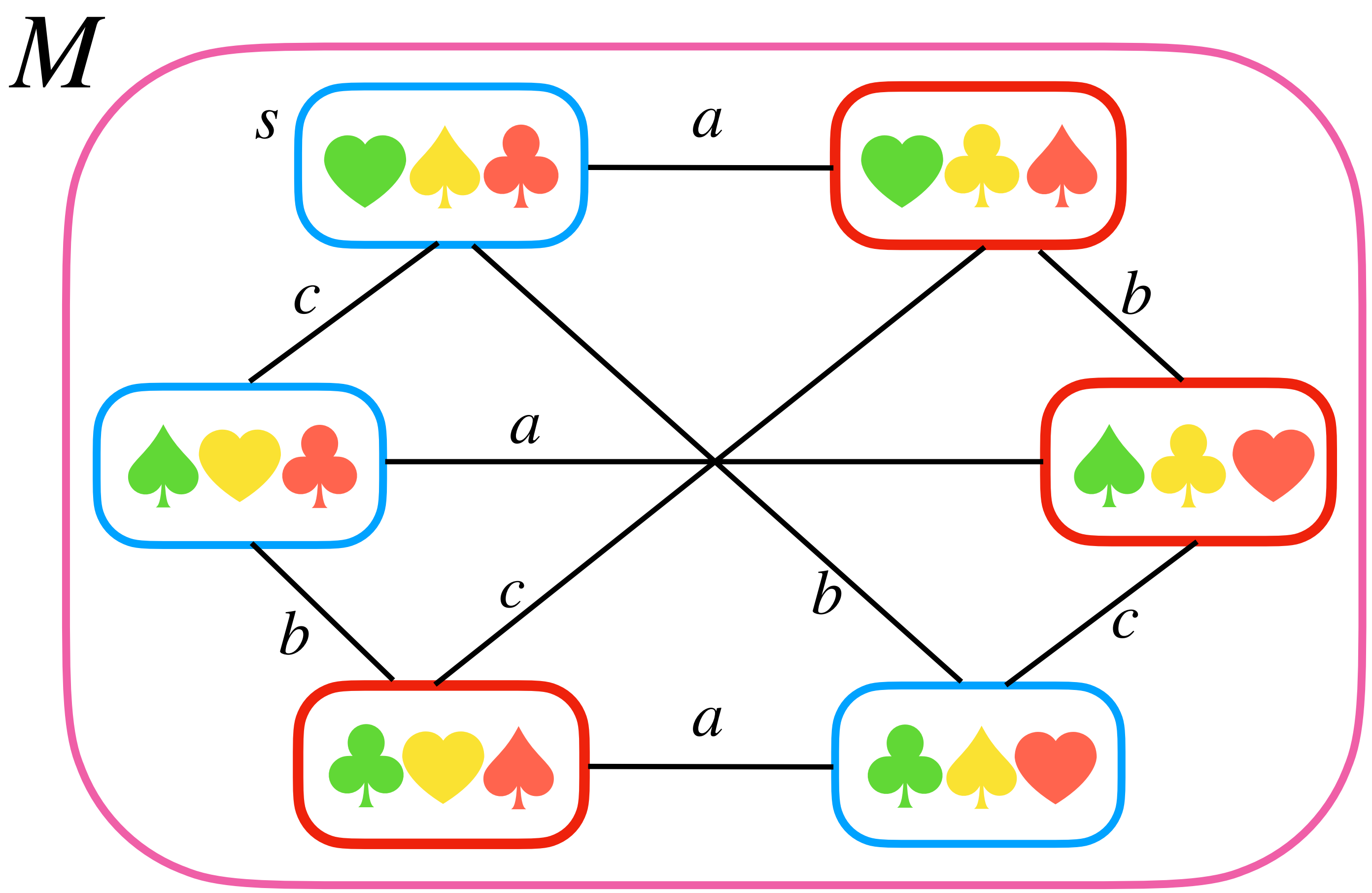

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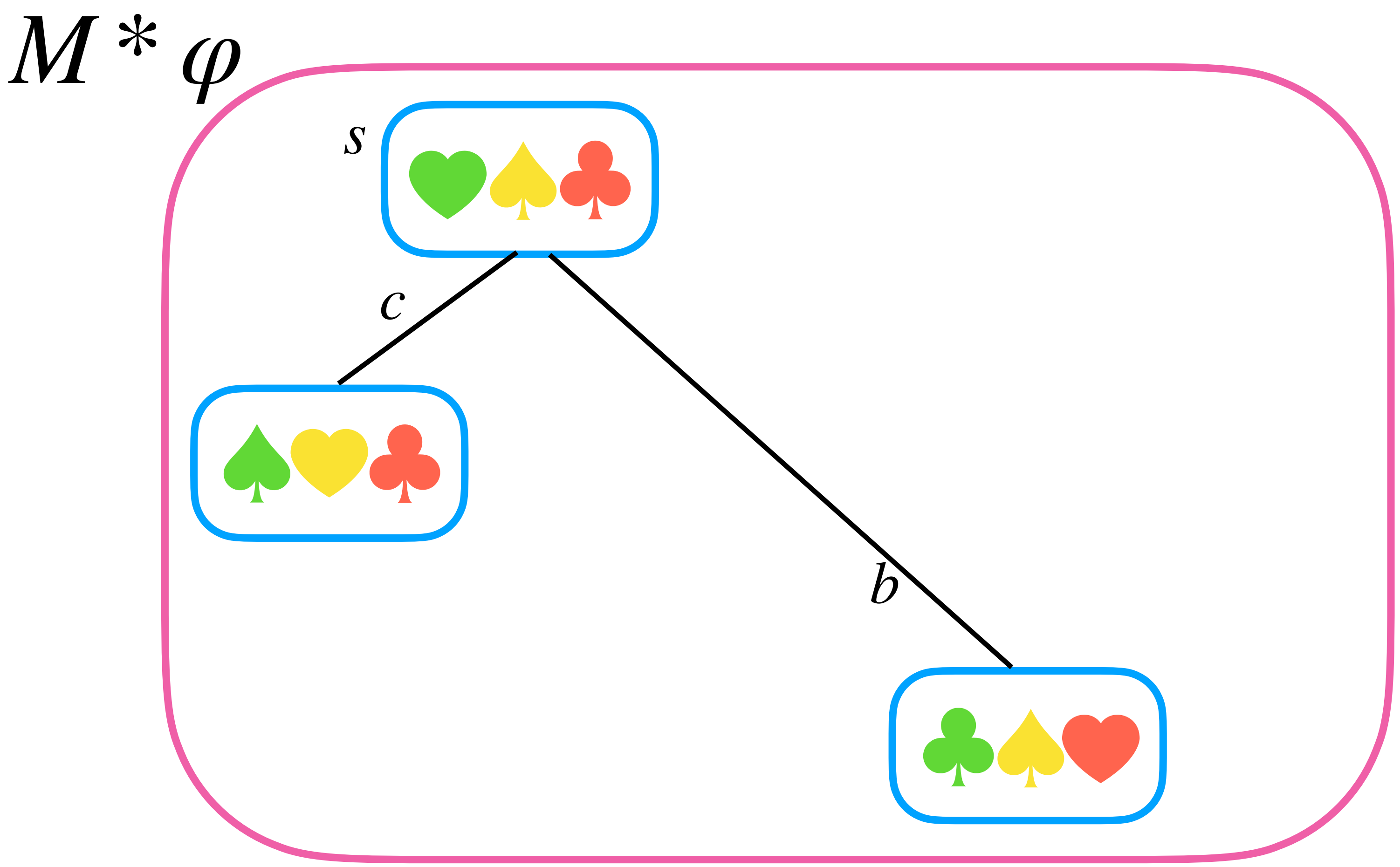

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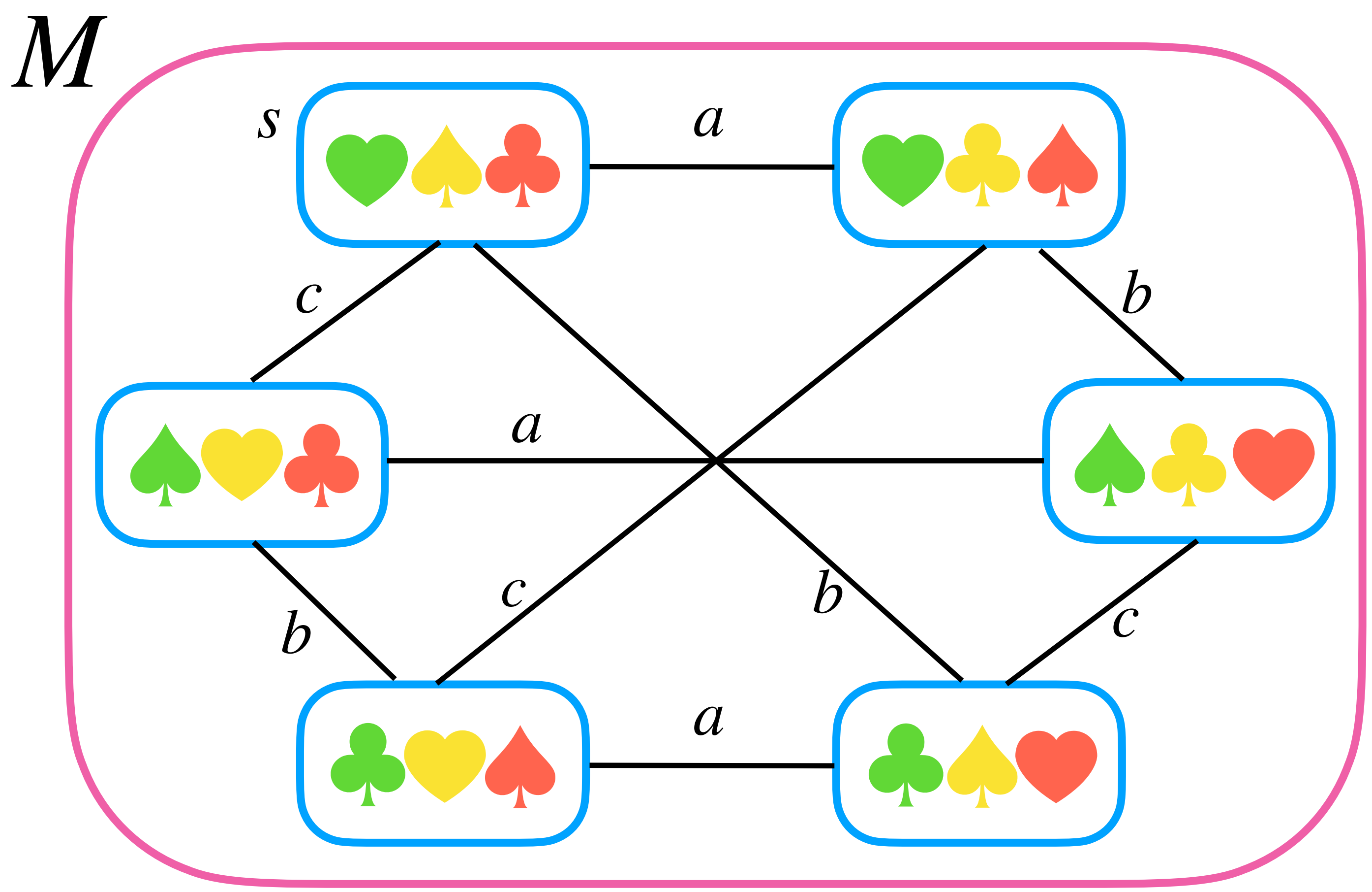
Card Example

After any announcement, Asgeir has one of the cards



$$M, s \models [!](\heartsuit_a \vee \clubsuit_a \vee \spadesuit_a)$$

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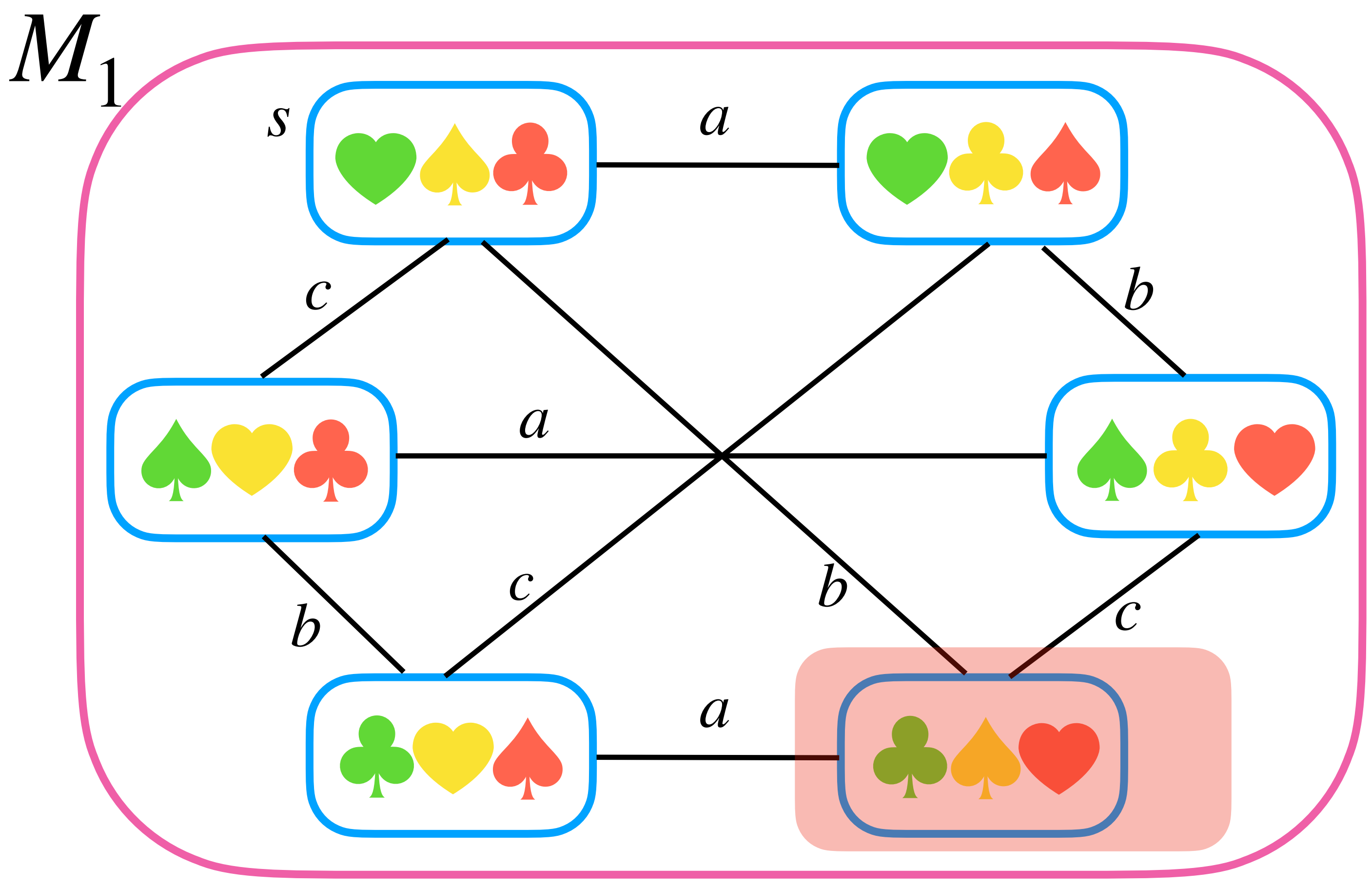

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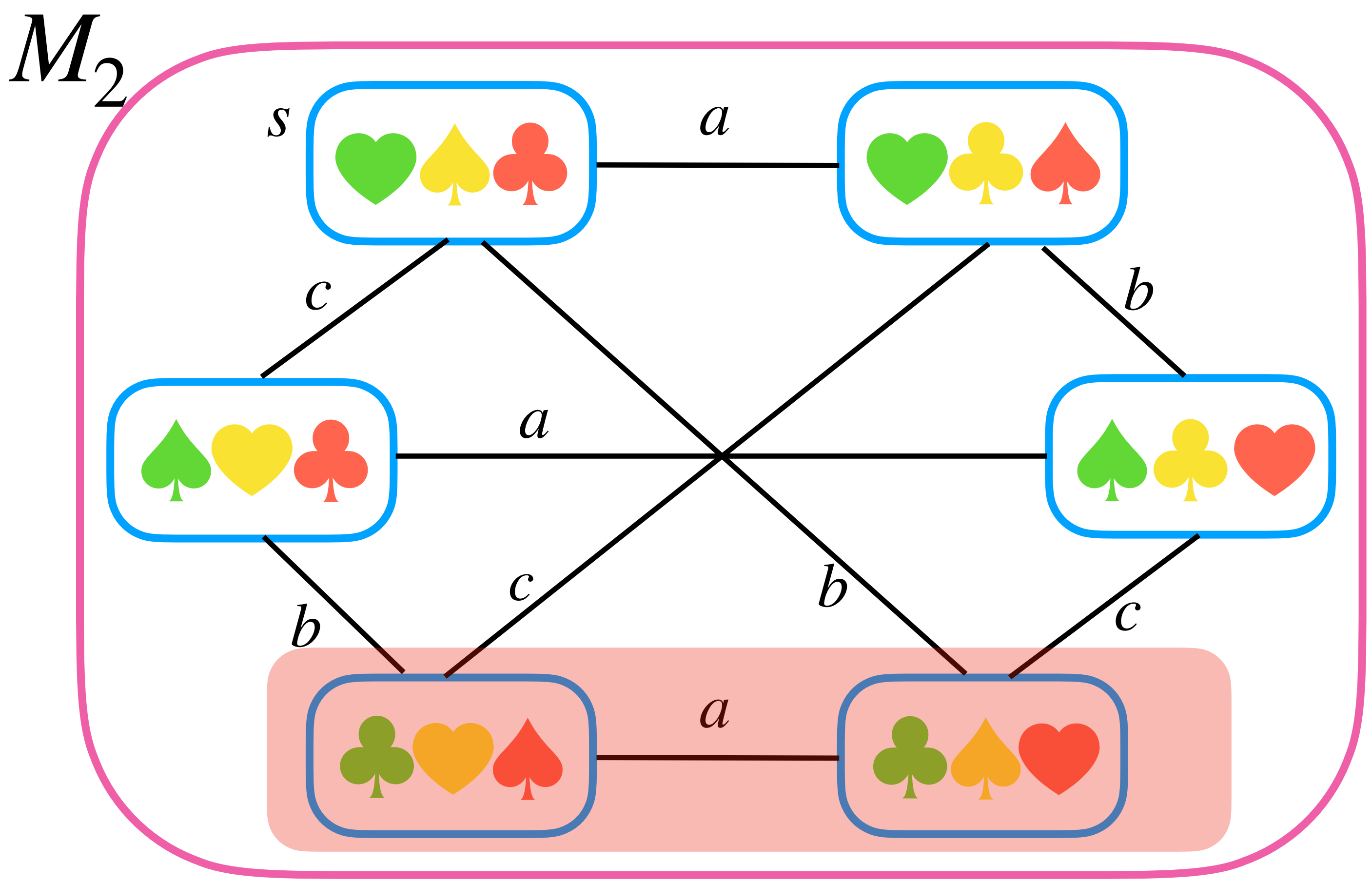

$$M, s \vDash [\,!\,](\heartsuit_a \vee \clubsuit_a \vee \spadesuit_a)$$

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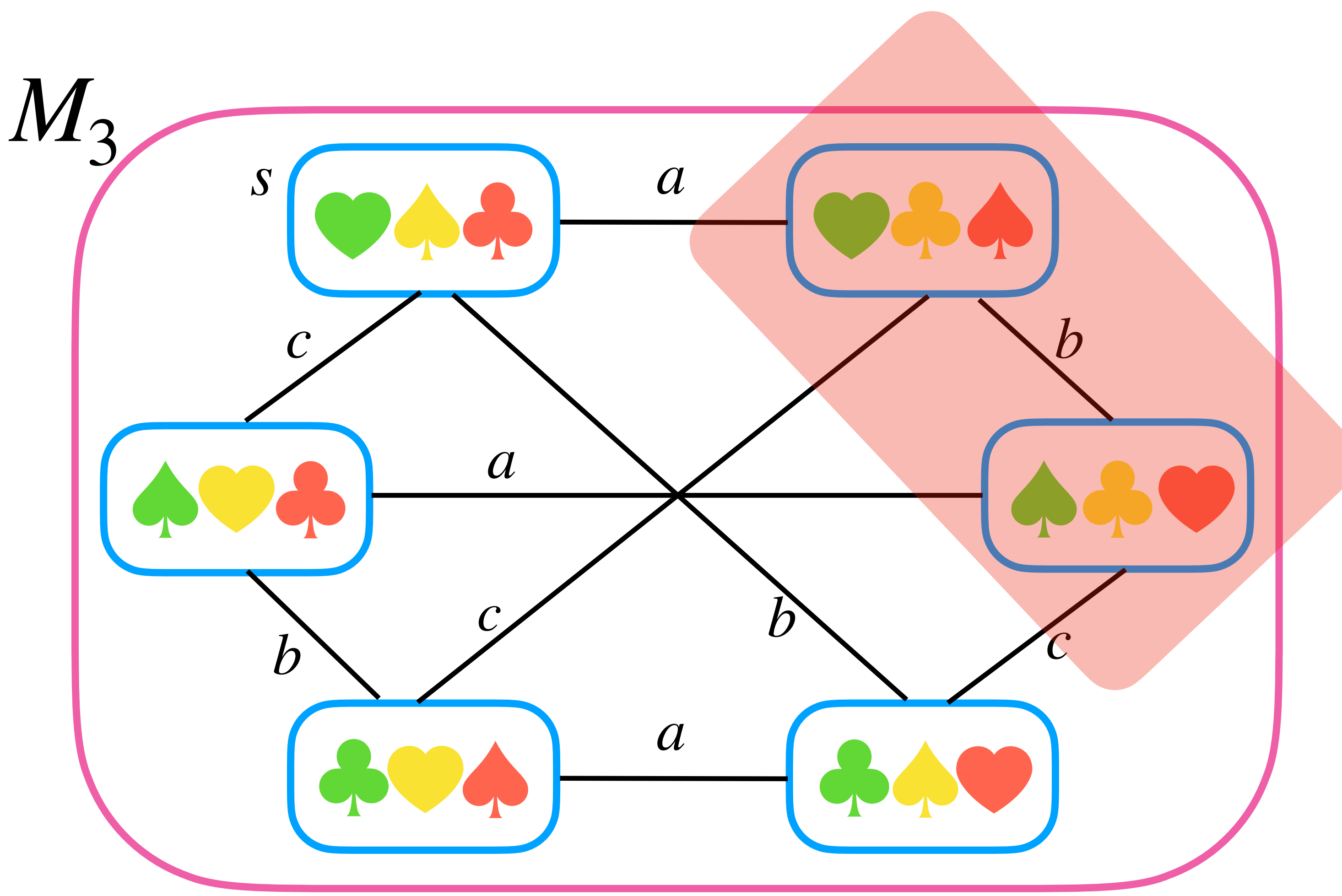

$$M, s \models [\,!\,](\heartsuit_a \vee \clubsuit_a \vee \spadesuit_a)$$

Arbitrary PAL

Language of APAL

$$\mathscr{APAL} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\varphi]\varphi \mid [!]\varphi$$

Semantics

$$M, s \models [!]\varphi \text{ iff } \forall \psi \in \mathscr{PAL} : M, s \models [\psi]\varphi$$
$$M, s \models \langle ! \rangle\varphi \text{ iff } \exists \psi \in \mathscr{PAL} : M, s \models \langle\psi\rangle\varphi$$

Do you notice anything interesting in the definition of semantics?

Balbiani et al. *'Knowable' as 'Known After an Announcement'*, 2008.

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$$M, s \vDash [!]\varphi \text{ iff } \forall \psi \in \mathscr{APAL} : M_s \vDash [\psi]\varphi$$

$$[p]\varphi, [\Box_a \Diamond_b (p \to q)]\varphi, [[!]\varphi]\varphi$$

Why would we restrict the scope of quantification?

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Quantification is restricted to formulas of PAL in order to avoid circularity

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Some validities

$$\langle \psi \rangle \varphi \to \langle ! \rangle \varphi \qquad [!]\varphi \to \varphi$$

$$\langle ! \rangle \varphi \leftrightarrow \langle ! \rangle \langle ! \rangle \varphi \qquad \langle ! \rangle [!] \varphi \leftrightarrow [!] \langle ! \rangle \varphi$$

Quantification is restricted to formulas of PAL in order to avoid circularity

Balbiani et al. *'Knowable' as 'Known After an Announcement'*, 2008.

APAL versus PAL

Theorem. PAL and EL are equally expressive

What do you think about APAL versus PAL?

The easy direction. $\mathscr{PAL} \subseteq \mathscr{APAL}$: APAL subsumes PAL

The not so easy direction. $\mathscr{APAL} \subseteq \mathscr{PAL}$?

$[!]\varphi$ is quite powerful as it quantifies over formulas with all propositional variables (even those not explicitly present in φ) and over formulas of arbitrary finite modal depth

APAL versus PAL

Theorem. PAL and EL are equally expressive

The not so easy direction. $\mathscr{APAL} \subseteq \mathscr{PAL}$?

$[!]\varphi$ is quite powerful as it quantifies over formulas with all propositional variables (even those not explicitly present in φ) and over formulas of arbitrary finite modal depth

Since PAL = EL, we provide a proof for the case of EL

Consider $\langle ! \rangle(\Box_a p \wedge \neg \Box_b \Box_a p)$

There is a public announcement such that a learns p and b does not know that a has learned p

APAL versus PAL

Consider $\langle ! \rangle (\Box_a p \wedge \neg \Box_b \Box_a p)$

Assume that there is a $\psi \in \mathcal{EL}$ which is equivalent to the given APAL formula

Since ψ is finite, there must be a $q \in P$ that does not appear in ψ

We will exploit the feature that $\langle ! \rangle$ still quantifies over formulas with q

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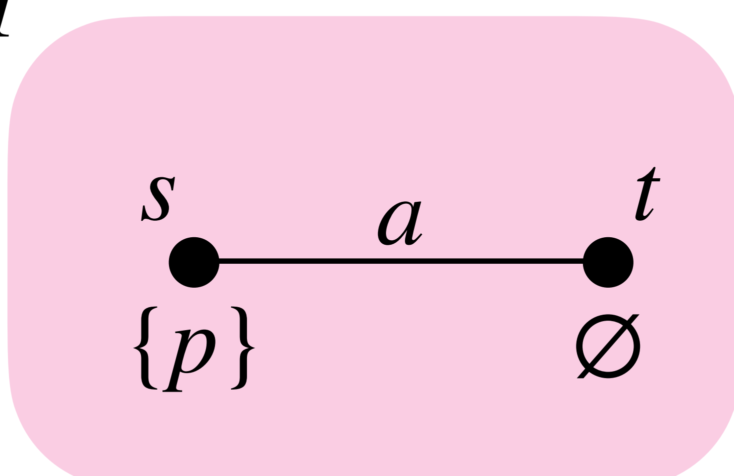
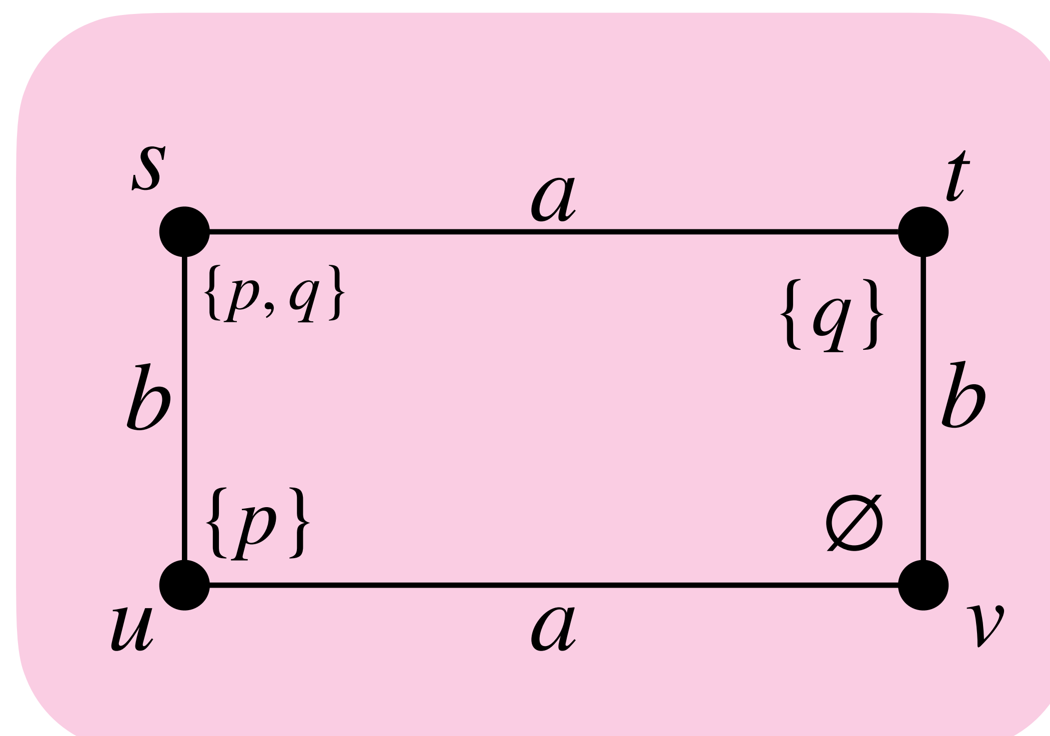
M



N



$M, s \models \langle ! \rangle(\Box_a p \land \neg \Box_b \Box_a p)$?

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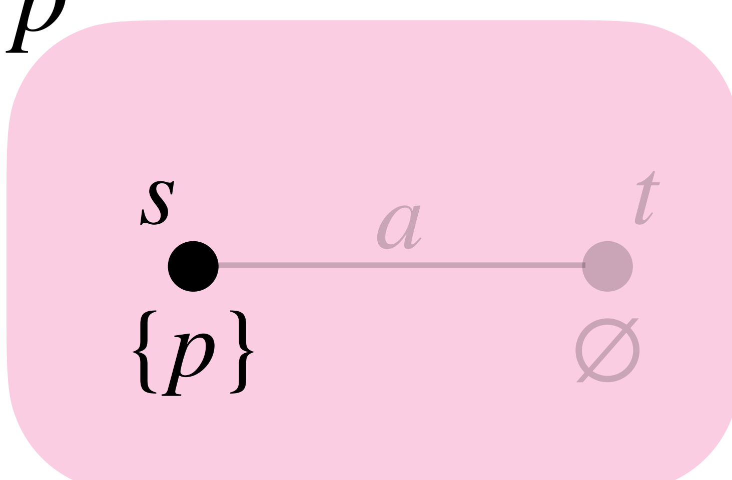
$M * p$



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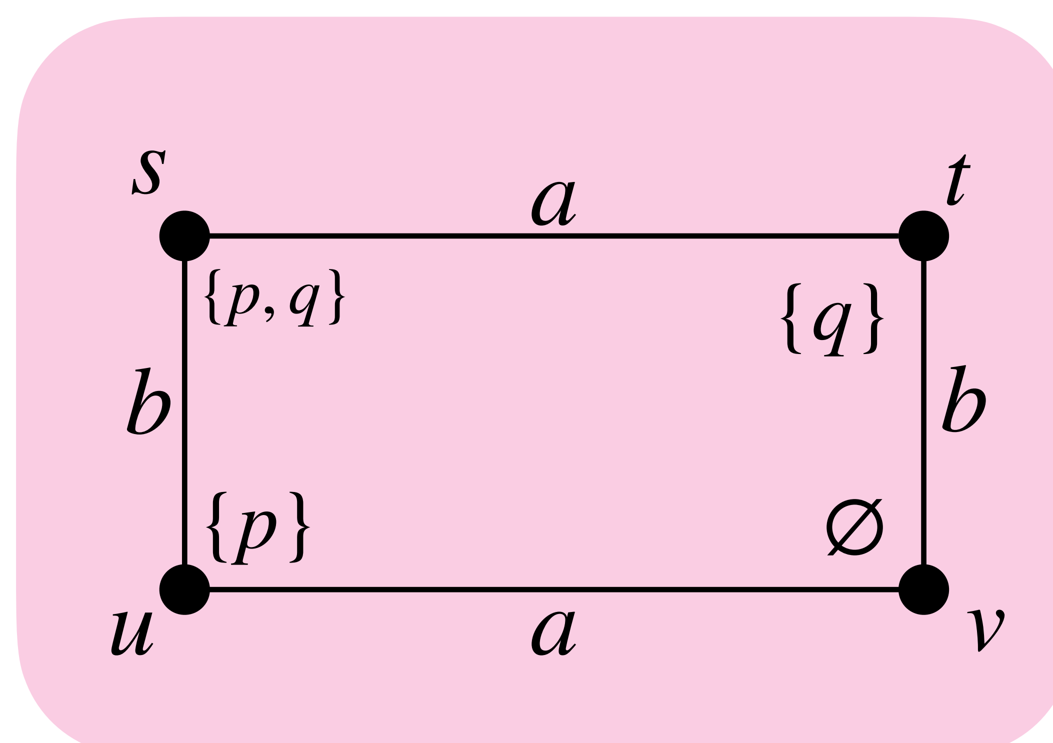
N



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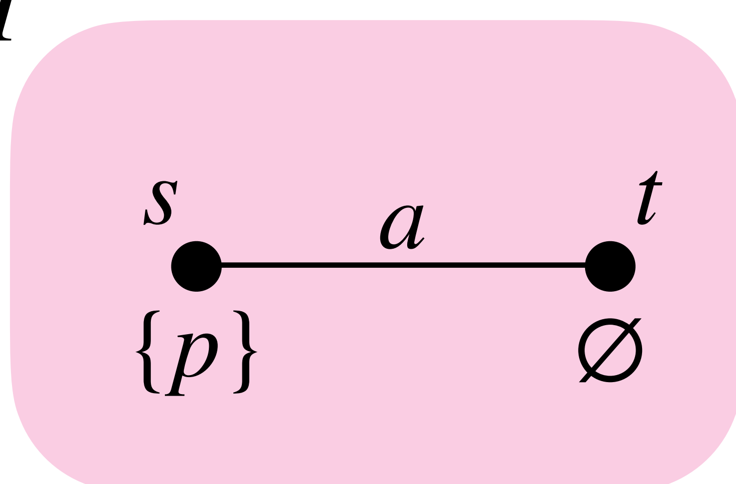
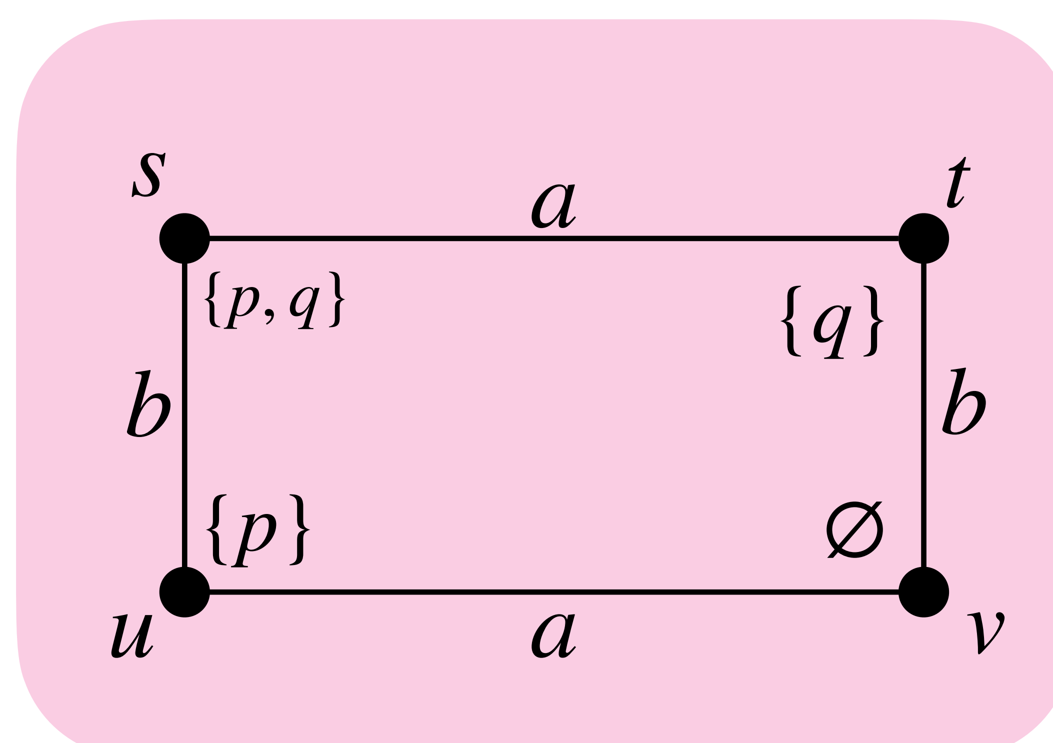
M



$M, s \not\models \langle ! \rangle (\Box_a p \wedge \neg \Box_b \Box_a p)$

N



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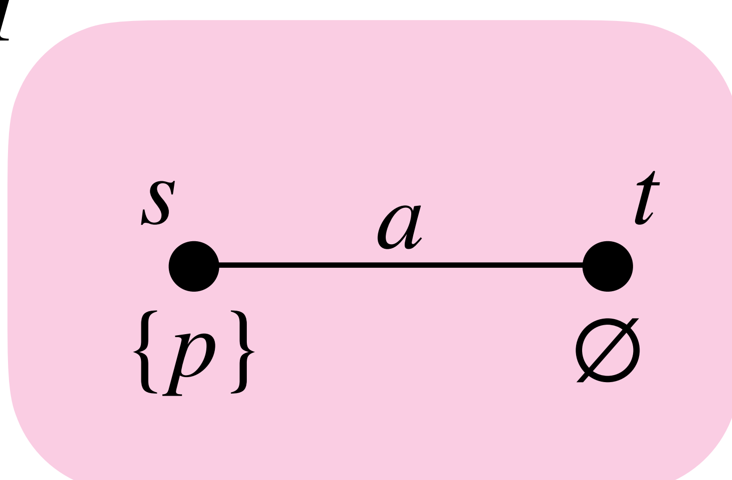

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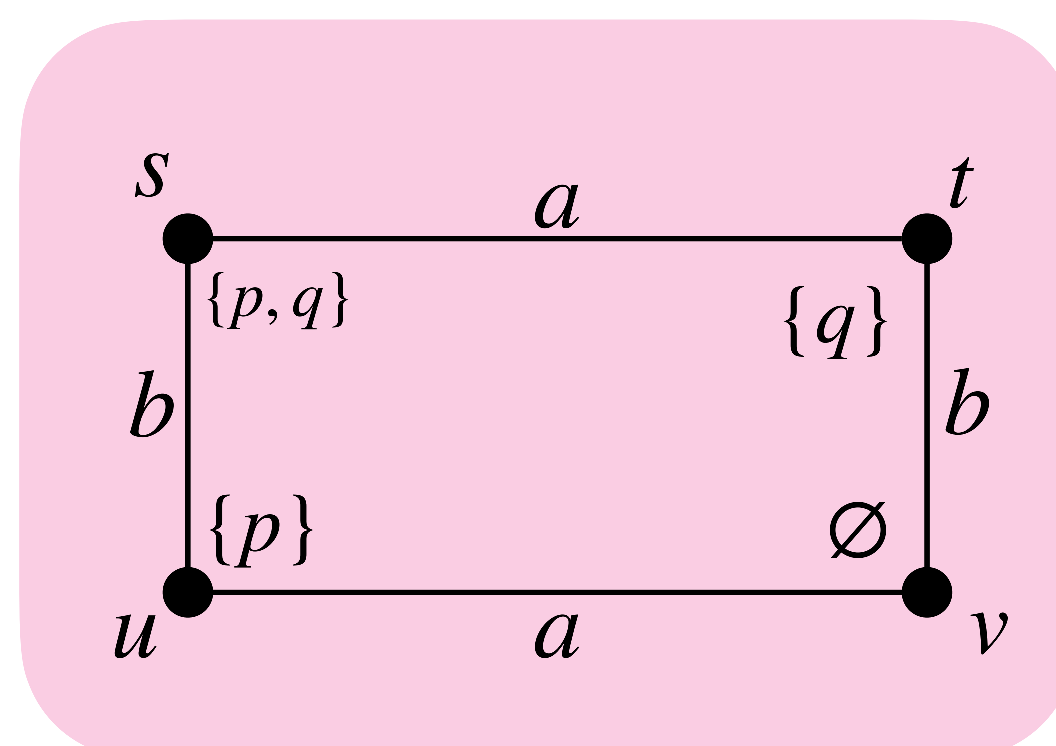

$N, s \vDash \langle ! \rangle (\Box_a p \land \neg \Box_b \Box_a p)$?

APAL versus PAL

Consider $\langle ! \rangle (\Box_a p \wedge \neg \Box_b \Box_a p)$

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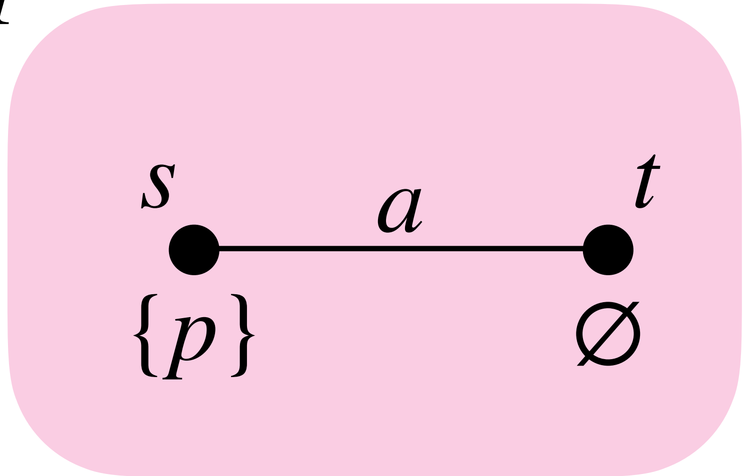
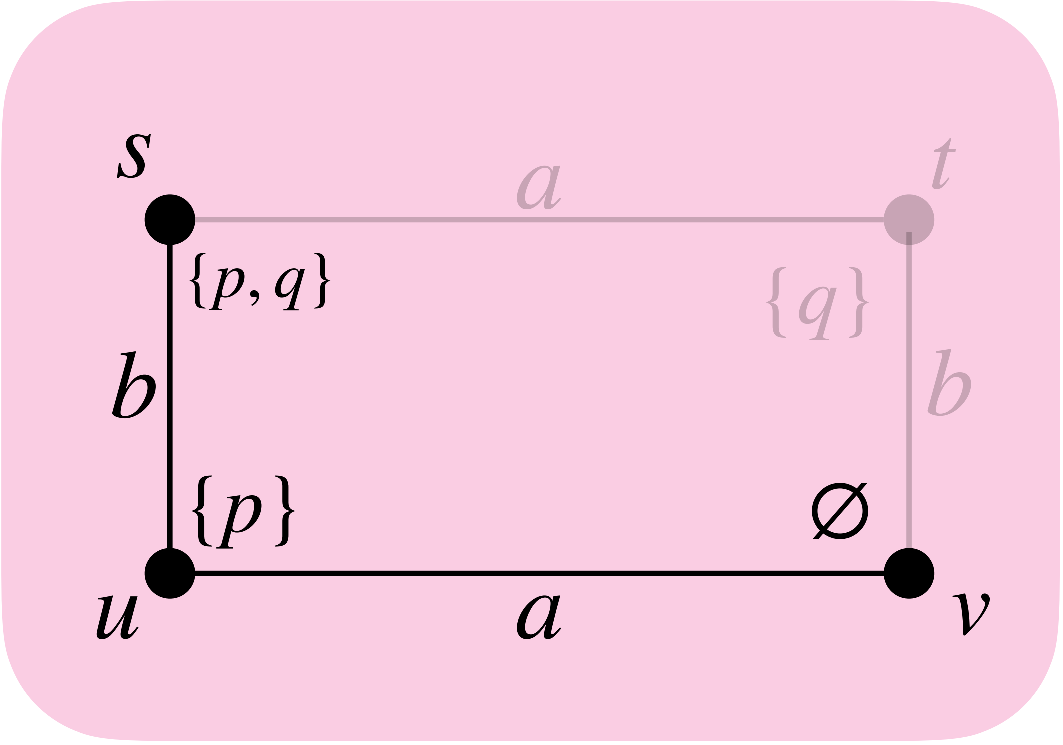


$N * (q \to p)$



$N, s \models \langle ! \rangle (\Box_a p \wedge \neg \Box_b \Box_a p)$?

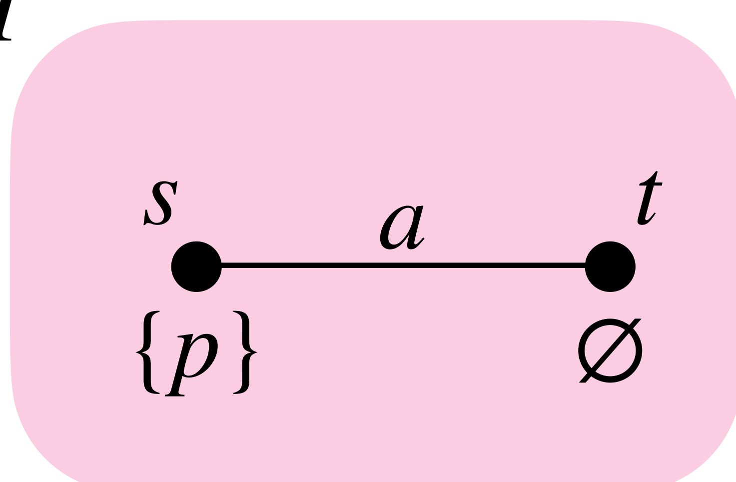
APAL versus PAL

Consider $\langle ! \rangle (\Box_a p \wedge \neg \Box_b \Box_a p)$

Assume that there is a $\psi \in \mathscr{E}\mathscr{L}$ which is equivalent to the given APAL formula

Since ψ is finite, there must be a $q \in P$ that does not appear in ψ

M



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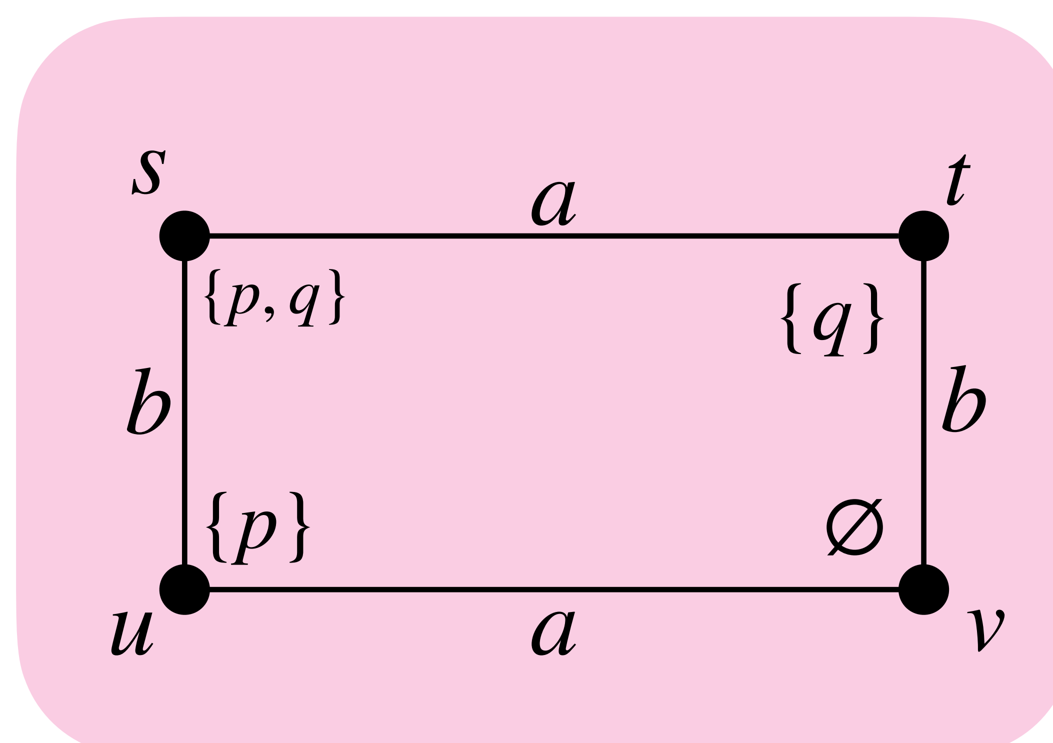

$N, s \models \langle ! \rangle (\Box_a p \wedge \neg \Box_b \Box_a p)$

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Consider $\langle ! \rangle (\Box_a p \wedge \neg \Box_b \Box_a p)$

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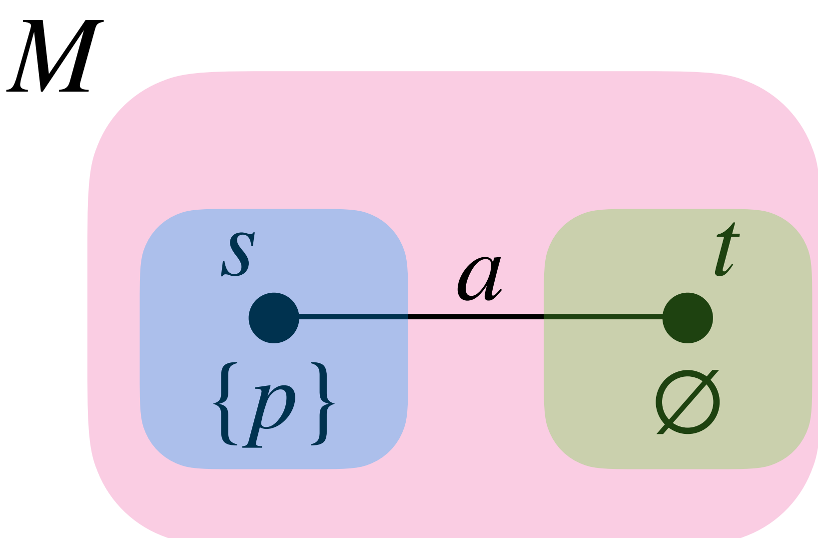
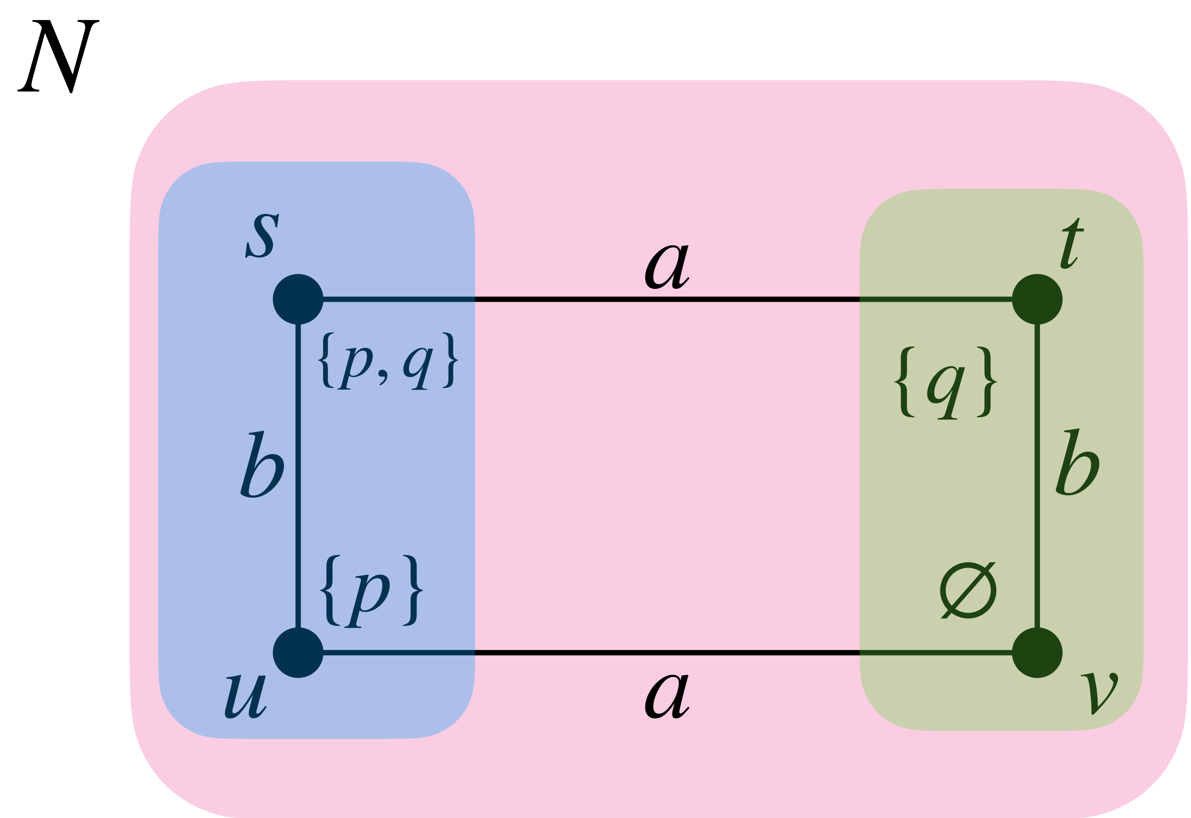
Since ψ is finite, there must be a $q \in P$ that does not appear in ψ



What about ψ?



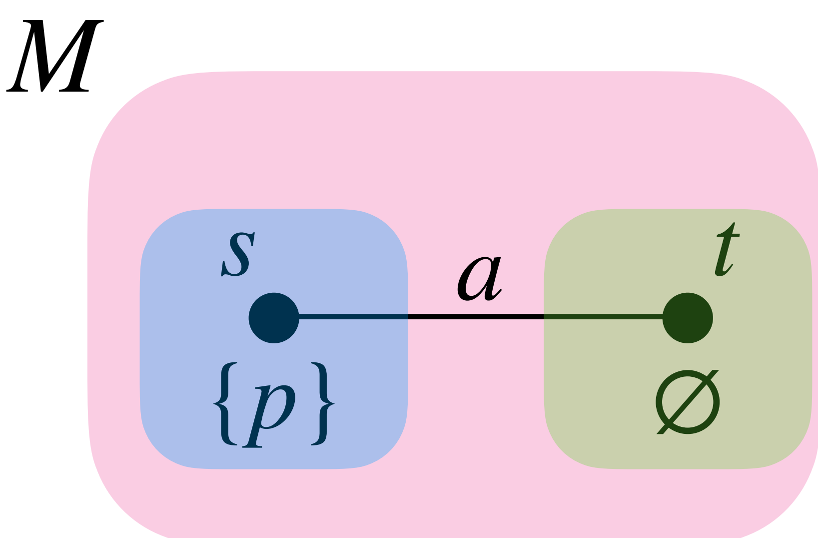
APAL versus PAL

Consider $\langle ! \rangle (\Box_a p \wedge \neg \Box_b \Box_a p)$

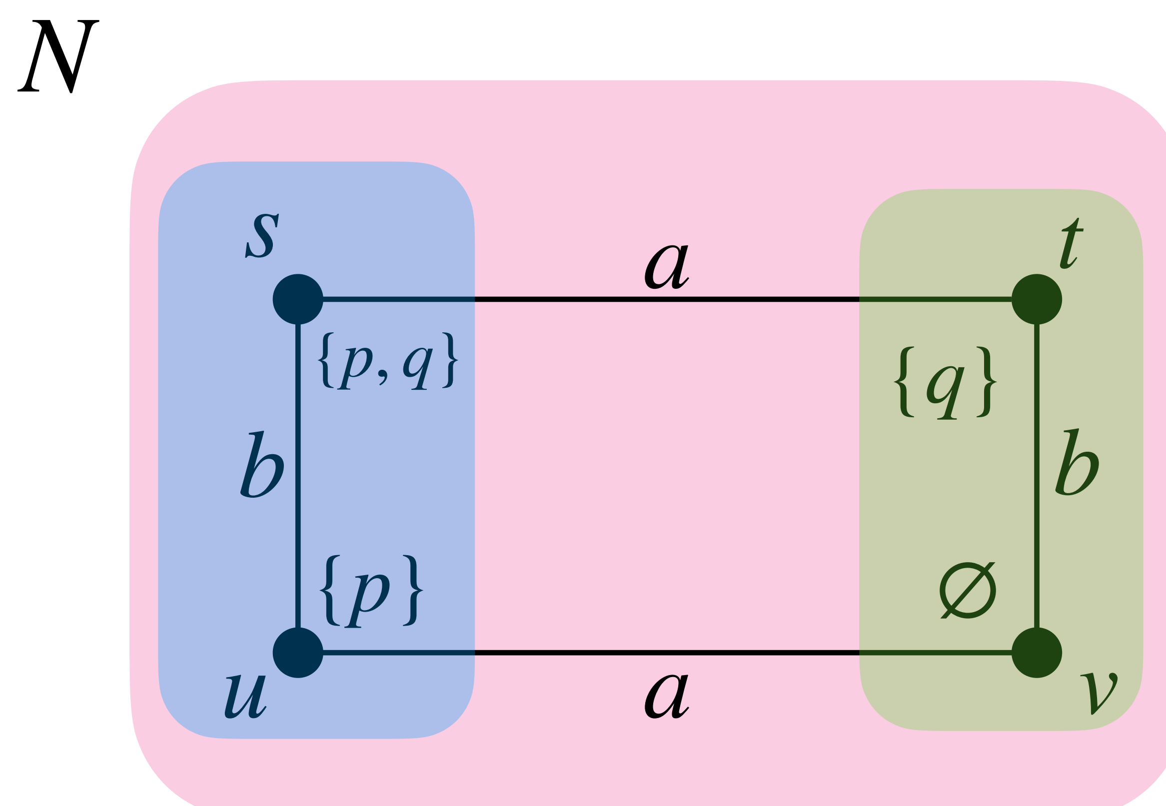
Assume that there is a $\psi \in \mathscr{EL}$ which is equivalent to the given APAL formula

Contradiction!

Since ψ is finite, there must be a $q \in P$ that does not appear in ψ



ψ can not tell the difference between M and N



APAL versus PAL: Encore

In the presented proof, we exploited the feature that $\langle ! \rangle$ quantifies over all propositional variables

Recall that $\langle ! \rangle$ quantifies over formulas of arbitrary finite modal depth. We will exploit this feature now

Consider $\langle ! \rangle(\Box_a \neg p \land \neg \Box_b \Box_a \neg p)$

Assume that there is a $\psi \in \mathscr{EL}$ which is equivalent to the given APAL formula

Since ψ is finite, it has some finite modal depth n

APAL versus PAL: Encore

Consider $\langle ! \rangle (\Box_a \neg p \wedge \neg \Box_b \Box_a \neg p)$

Assume that there is a $\psi \in \mathcal{EL}$ which is equivalent to the given APAL formula

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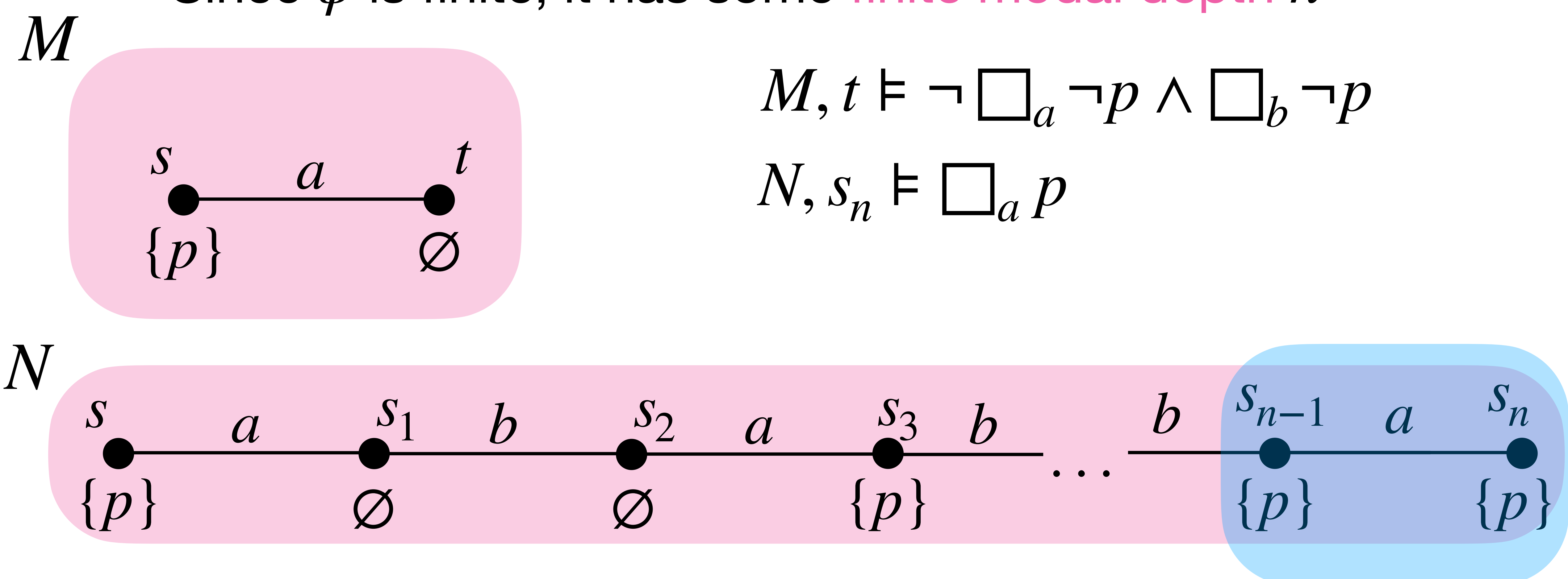
$M, t \models \neg \Box_a \neg p \wedge \Box_b \neg p$

$N, s_n \models \Box_a p$



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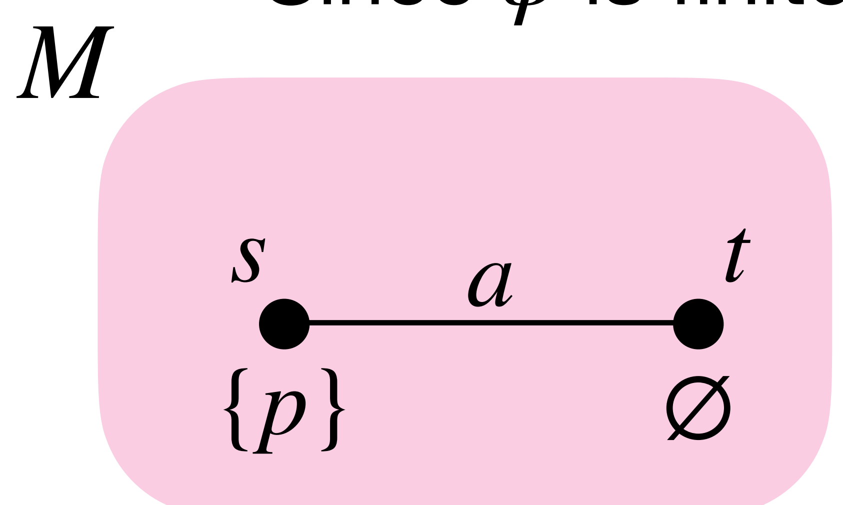
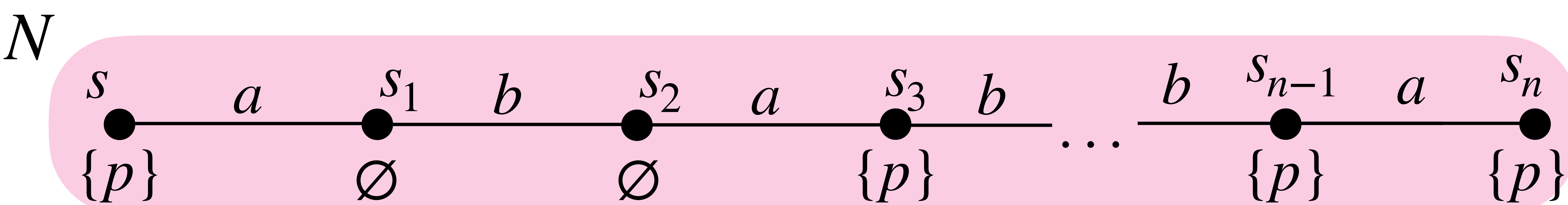


$M, t \models \langle ! \rangle (\Box_a \neg p \land \neg \Box_b \Box_a \neg p)$?

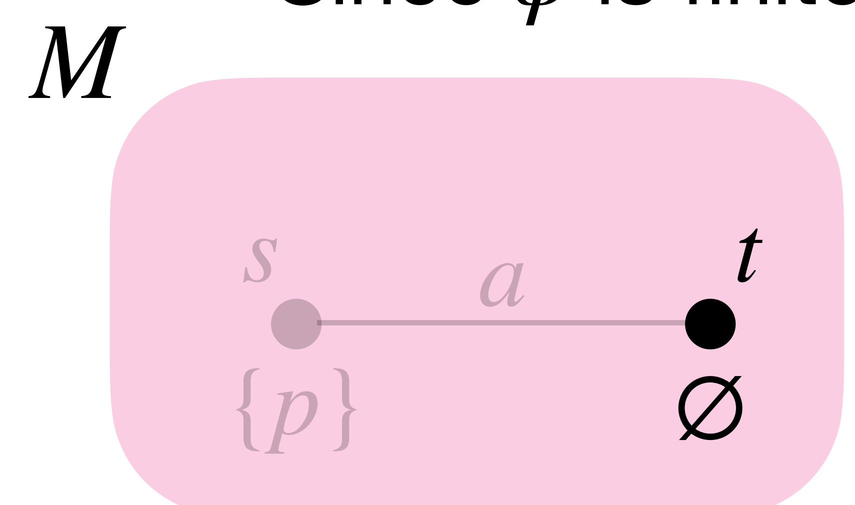


APAL versus PAL: Encore

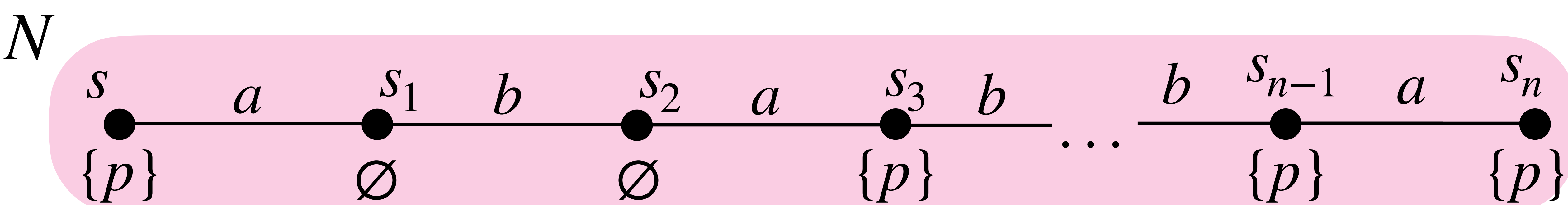
Consider $\langle ! \rangle (\Box_a \neg p \wedge \neg \Box_b \Box_a \neg p)$

Assume that there is a $\psi \in \mathscr{EL}$ which is equivalent to the given APAL formula

Since ψ is finite, it has some finite modal depth n



$M, t \models \langle ! \rangle (\Box_a \neg p \wedge \neg \Box_b \Box_a \neg p)$?



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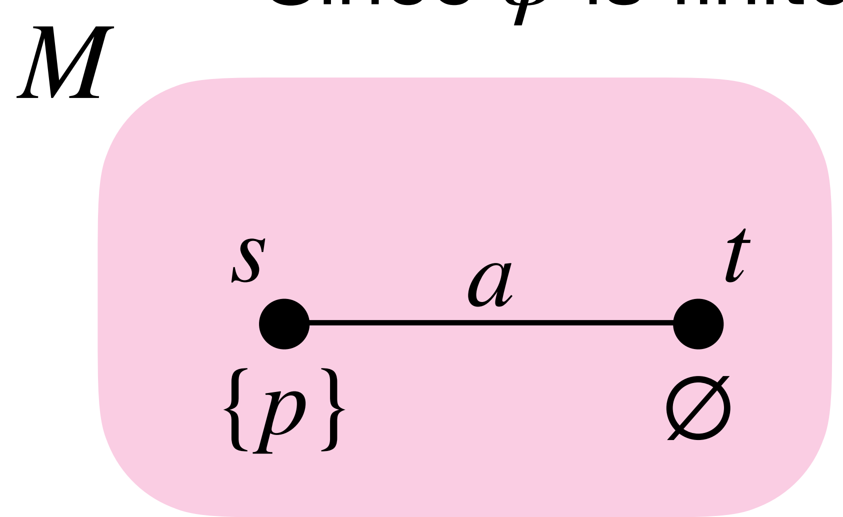
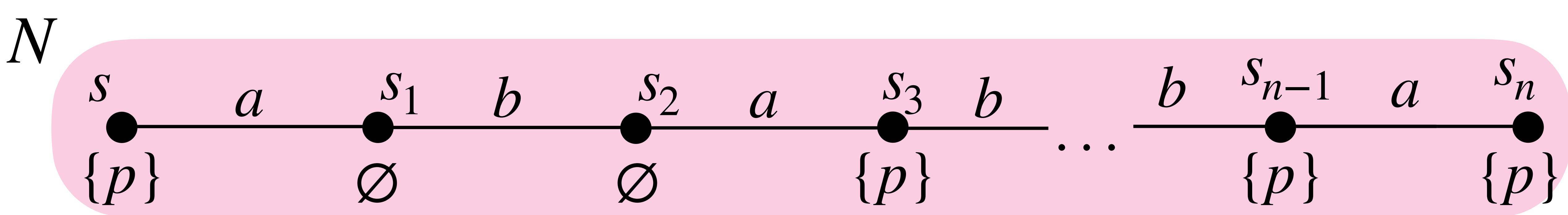


$$M, t \not\models \langle ! \rangle (\Box_a \neg p \wedge \neg \Box_b \Box_a \neg p)$$

$$N, s_1 \models \langle \psi \rangle (\Box_a \neg p \wedge \neg \Box_b \Box_a \neg p)$$



State s_n is unique and allows us to specify uniquely other states

APAL versus PAL: Encore

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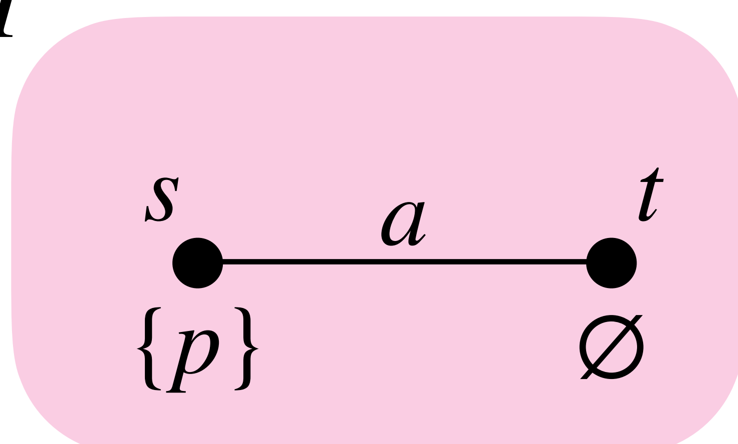
M



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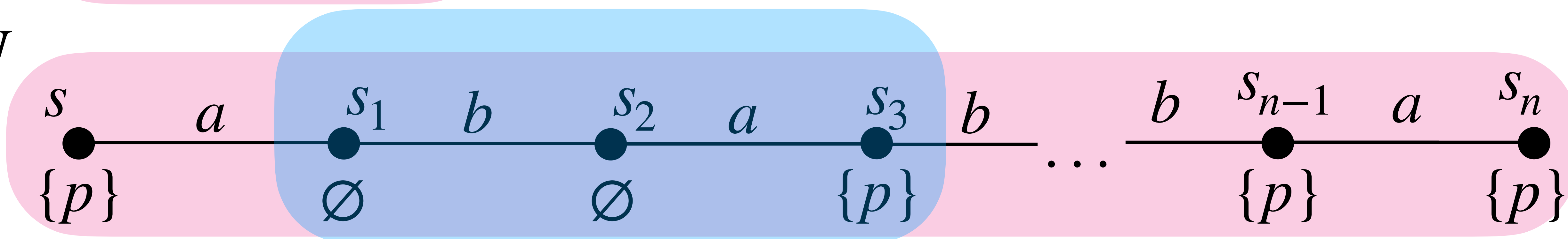
N



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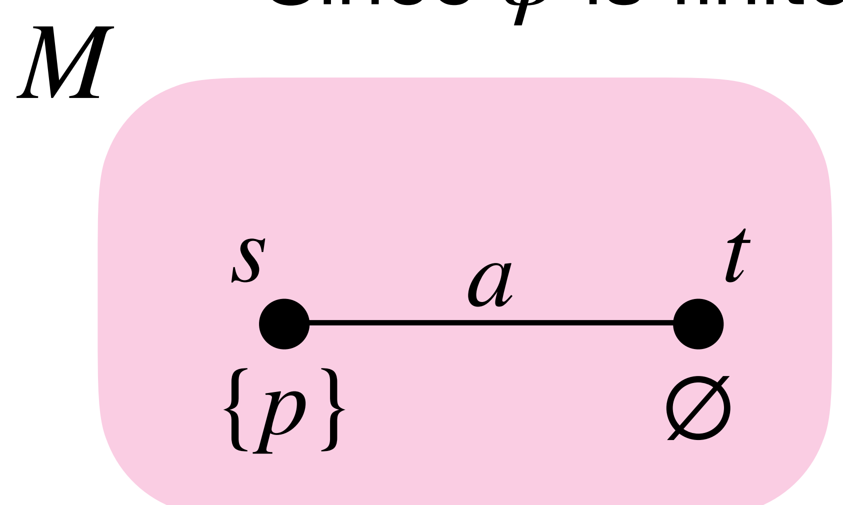
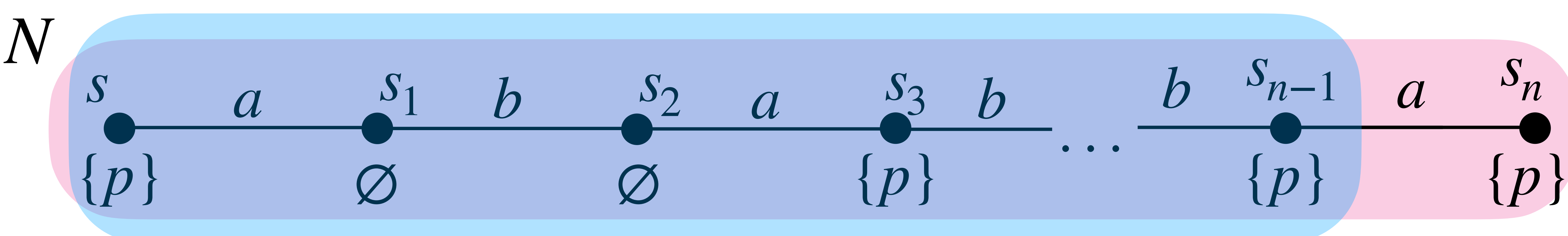


M and N are 'the same' up to n steps



Cannot find the difference with ψ!

APAL versus PAL

Theorem. PAL and EL are equally expressive

$[!]\varphi$ is quite powerful as it quantifies over formulas with all propositional variables (even those not explicitly present in φ) and over formulas of arbitrary finite modal depth

Theorem. APAL is more expressive than PAL and EL

There are no reduction axioms for APAL, hence we have to find a proper axiomatisation…

Axiomatisation of APAL

Language of APAL $\mathscr{APAL} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\varphi]\varphi \mid [!]\varphi$

Semantics $M, s \models [!]\varphi$ iff $\forall \psi \in \mathscr{PAL} : M, s \models [\psi]\varphi$

Axioms of EL and PAL

$[!]\varphi \rightarrow [\psi]\varphi$ with $\psi \in \mathscr{PAL}$

From $\{\eta([\psi]\varphi) \mid \psi \in \mathscr{PAL}\}$ infer $\eta([!]\varphi)$

Infinite number of premises

$$\frac{\eta([\psi_1]\varphi)\ \eta([\psi_2]\varphi)\ \eta([\psi_3]\varphi)\ \cdots}{\eta([!]\varphi)}$$

We call such a rule infinitary

Balbiani, Van Ditmarsch. *A simple proof of the completeness of APAL*, 2015.

Completeness of APAL

We can prove completeness using the canonical model construction and a Lindenbaum type lemma

Recall APAL

$$M, s \models [!]\varphi \text{ iff } \forall \psi \in \mathscr{PAL} : M, s \models [\psi]\varphi$$

$$M, s \models \langle ! \rangle\varphi \text{ iff } \exists \psi \in \mathscr{PAL} : M, s \models \langle \psi \rangle\varphi$$

$[!]\varphi$ →

MCS

$$[!]\varphi \rightarrow [\psi_1]\varphi$$
$$[!]\varphi \rightarrow [\psi_2]\varphi$$
$$[!]\varphi \rightarrow [\psi_3]\varphi$$
$$\dots$$

Instances of an axiom schema

Balbiani, Van Ditmarsch. *A simple proof of the completeness of APAL*, 2015.

Completeness of APAL

We can prove completeness using the canonical model construction and a Lindenbaum type lemma

Recall APAL

$$M, s \models [!]\varphi \text{ iff } \forall \psi \in \mathscr{PAL} : M, s \models [\psi]\varphi$$

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MCS $[!]\varphi$

$[\psi_1]\varphi$

$[\psi_2]\varphi$

$[\psi_3]\varphi$

$\dots$

By closure under MP

Balbiani, Van Ditmarsch. *A simple proof of the completeness of APAL*, 2015.

Completeness of APAL

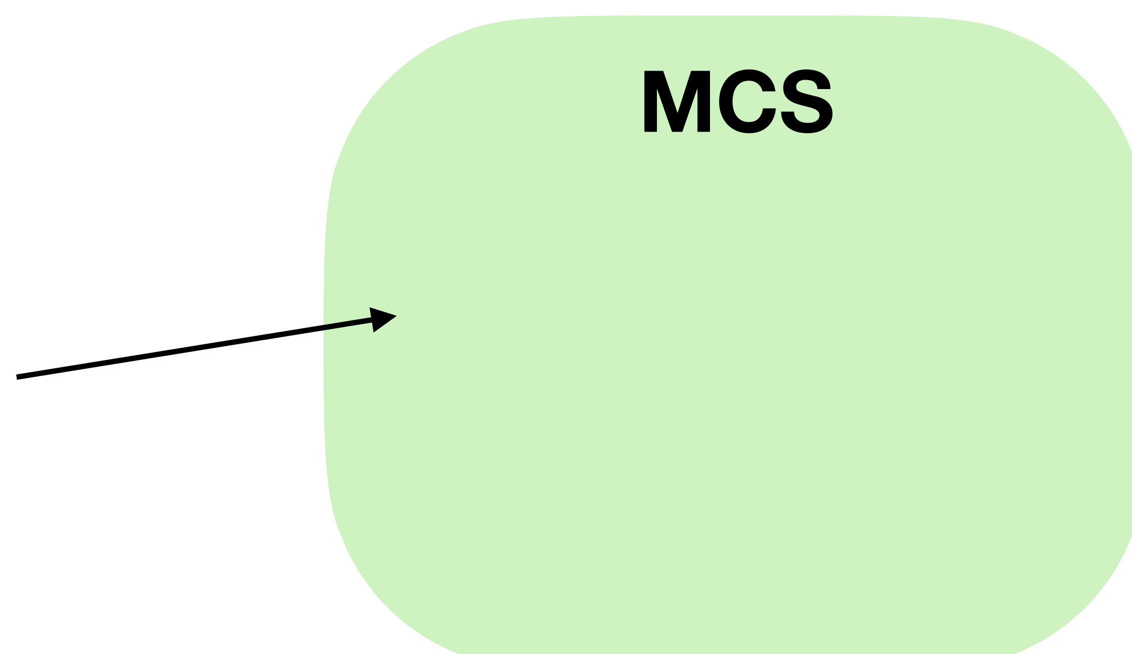
We can prove completeness using the canonical model construction and a Lindenbaum type lemma

Recall APAL

$$M, s \vDash [\,!\,]\varphi \;\text{ iff }\; \forall \psi \in \mathscr{PAL} : M, s \vDash [\psi]\varphi$$

$$M, s \vDash \langle\,!\,\rangle\varphi \;\text{ iff }\; \exists \psi \in \mathscr{PAL} : M, s \vDash \langle\psi\rangle\varphi$$

$$\neg[\,!\,]\varphi$$



Add a witness

Balbiani, Van Ditmarsch. *A simple proof of the completeness of APAL*, 2015.

Completeness of APAL

We can prove completeness using the canonical model construction and a Lindenbaum type lemma

Recall APAL

$$M, s \models [!]\varphi \text{ iff } \forall \psi \in \mathscr{PAL} : M, s \models [\psi]\varphi$$

$$M, s \models \langle ! \rangle\varphi \text{ iff } \exists \psi \in \mathscr{PAL} : M, s \models \langle \psi \rangle\varphi$$

MCS

$$\neg[!]\varphi$$

$$\neg[\psi_n]\varphi$$

Add a witness

Balbiani, Van Ditmarsch. *A simple proof of the completeness of APAL*, 2015.

Axiomatisation of APAL

Axioms of EL and PAL

$[!]\varphi \rightarrow [\psi]\varphi$ with $\psi \in \mathscr{PAL}$

From $\{\eta([\psi]\varphi) \mid \psi \in \mathscr{PAL}\}$

infer $\eta([!]\varphi)$

Theorem. There is a sound and complete infinitary axiomatisation of APAL

Open Problem. Is there a finitary axiomatisation of APAL?

Balbiani, Van Ditmarsch. *A simple proof of the completeness of APAL*, 2015.

Backstabbing the OP

A logic has the finite model property (FMP) iff every formula of the logic that is true in some model is also true in a finite model

Finitary axiomatisation $\wedge$ FMP $\rightarrow$ Decidability

$$\varphi$$

Finitary axiomatisation

Finding the proof of $\neg\varphi$

If successful, φ is not satisfiable

FMP

Looking for a finite model of φ

If successful, φ is satisfiable

Urquhart. *Decidability and the Finite Model Property*, 1981.

Backstabbing the OP

A logic has the finite model property (FMP) iff every formula of the logic that is true in some model is also true in a finite model

$$\text{Finitary axiomatisation} \wedge \text{FMP} \rightarrow \text{Decidability}$$

$$\equiv$$

$$\neg\text{Decidability} \rightarrow \neg\text{Finitary axiomatisation} \vee \neg\text{FMP}$$

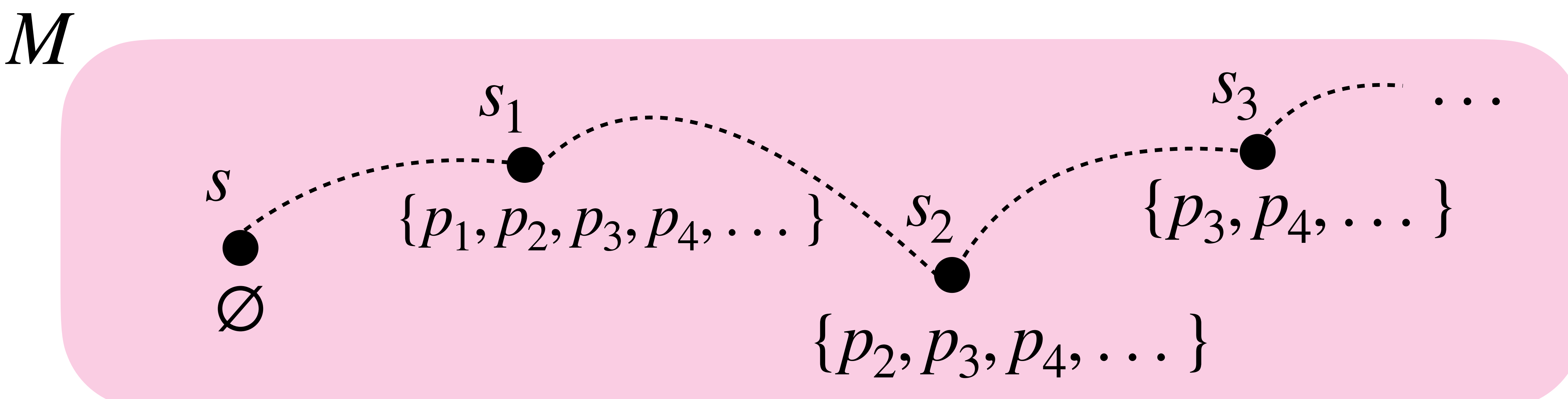
APAL is undecidable. If we show that APAL has the FMP, then we will know that it is not finitely axiomatisable…

Urquhart. *Decidability and the Finite Model Property*, 1981.

No FMP for APAL

$[!]\varphi$ is quite powerful as it quantifies over formulas with all propositional variables (even those not explicitly present in φ) and over formulas of arbitrary finite modal depth

However, it is not powerful enough to pick out all interesting submodels of a model



Example. Try removing all states apart from s using only propositional announcements

French, Van Ditmarsch, RG. *No Finite Model Property for Logics of Quantified Announcements*, 2021.

Back to the OP

$\neg$Decidability $\rightarrow$ $\neg$Finitary axiomatisation $\vee$ $\neg$FMP

One can also show the lack of the FMP via the arbitrary modal depth way

Open Problem. Is there a finitary axiomatisation of APAL?

Kuijer. *Expressivity of Logics of Knowledge and Action,* 2014
French, Van Ditmarsch. *Undecidability for arbitrary public announcement logic*, 2008.
Urquhart. *Decidability and the Finite Model Property*, 1981.
French, Van Ditmarsch, RG. *No Finite Model Property for Logics of Quantified Announcements*, 2021.

Overview of APAL

Axioms of EL and PAL

$[!]\varphi \to [\psi]\varphi$ with $\psi \in \mathscr{PAL}$

From $\{\eta([\psi]\varphi) \mid \psi \in \mathscr{PAL}\}$ infer $\eta([!]\varphi)$

Infinite number of premises

Open Problem. Is there a finitary axiomatisation of APAL?

Theorem. APAL is more expressive than PAL

Theorem. APAL is sound and complete

Theorem. SAT-APAL is undecidable

Theorem. Complexity of MC-APAL is PSPACE-complete

French, Van Ditmarsch. *Undecidability for arbitrary public announcement logic*, 2008.

Balbiani, Van Ditmarsch. *A simple proof of the completeness of APAL*, 2015.

Take-home message

- Quantifying is fun
- Quantifying in DEL (usually) yields unexpected results
- APAL quantifies over PAL formulas that may include any propositional variables and can be of any arbitrary finite depth

Open Problem. Is there a finitary axiomatisation of APAL?