

Quantification in Dynamic Epistemic Logic (ESSLLI 2023) Exercises

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These exercises are intended purely as “food for thought”, they are entirely optional. We will not provide solutions to the exercises (in fact, for some exercises there is no single right answer), nor will we discuss them during the lectures. But if you want to talk about them we will, of course, be happy to discuss the exercises before or after the lectures.

1 Day 1

1. Is the update expressivity relation \preceq transitive and symmetric?
2. In this course, we defined public announcements as model restrictions, i.e., $[\varphi]$ changes the set of worlds from W to $W \cap \llbracket \varphi \rrbracket_M$. There are also other ways to define public announcements, however. In particular, consider the updates $[\!|\varphi|]$ and $[?\varphi]$, where $[\!|\varphi|]$ deletes all arrows to $\neg\varphi$ worlds and $[?\varphi]$ deletes all arrows between worlds with different values of φ .

So $[\!|\varphi|]$ has the same effect as the arrow update $[(\top, A, \varphi)]$ and $[?\varphi]$ has the same effect as the arrow update $[(\varphi, A, \varphi), (\neg\varphi, A, \neg\varphi)]$.

- (a) Are there reduction axioms for $[\!|\varphi|]$ and $[?\varphi]$? If so, what are they?
- (b) How does the update expressivity of $[\!|\varphi|]$ and $[?\varphi]$ compare to that of public announcements and arrow updates?
- (c) How do the update expressivity of $[\!|\varphi|]$ and $[?\varphi]$ compare to each other?

2 Day 2

3. An early attempt to provide an axiomatization for APAL used the following rule:

from $\vdash \psi \rightarrow [\theta][p]\varphi$ infer $\vdash \psi \rightarrow [\theta][!]\varphi$, where p does not occur in φ, ψ, θ .

Unfortunately, things are not that easy: this rule is unsound. Find a counterexample to this rule, i.e., find φ, ψ and θ such that $\models \psi \rightarrow [\theta][p]\varphi$ but $\not\models \psi \rightarrow [\theta][!]\varphi$ while p does not occur in φ, ψ and θ .

3 Day 3

4. Verify whether multi-step strategies in GAL and CAL are reducible to single-step strategies. In other words, check whether the following are valid: $\langle G \rangle \varphi \leftrightarrow \langle G \rangle \langle G \rangle \varphi$ and $\langle\!\langle G \rangle\!\rangle \varphi \leftrightarrow \langle\!\langle G \rangle\!\rangle \langle\!\langle G \rangle\!\rangle \varphi$ (Warning: a full answer is not known to us.)
5. Can you define the semantics of $[!]\varphi$ and $\langle\!\langle ! \rangle\!\rangle \varphi$ in terms of relativised group announcements? If yes, provide the definition, if not, explain why.

4 Day 4

6. Show that $D_G \varphi \rightarrow \langle G \rangle E_G \varphi^+$, where φ^+ belongs to the positive fragment of \mathcal{EL} , is not valid by providing a counterexample.
7. Provide possible definitions of agent 'being able to achieve φ ' using various notions of group knowledge (or lack thereof) and group and coalition announcements.

5 Day 5

8. Action models are a generalization of public announcements, and arrow update models are a generalization of arrow updates.
 - (a) What would be the corresponding generalization of substitutions? That is to say, what would "substitution models" look like?
 - (b) What would be the expressivity and update expressivity of these "substitution models"?
 - (c) Is global synthesis possible for substitution models? (Warning: we don't know the answer to this exercise, so we don't know how hard it is.)