Quantification in Dynamic Epistemic Logic
Day 5

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ESSLLI 2023

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- These generally increase complexity of a model.

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- Well, then it wouldn't be a public announcement anymore, now would it.
- Instead, it would be an action model.

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- A single such event can have multiple different outcomes.
- Example: tossing a coin, with outcomes “heads” and “tails”.
- Notation: event E has outcomes $o_1, \cdots, o_n$.

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- This gives accessibility relations (one per agent) on the outcomes.

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- Similarly, o_2 has precondition $\neg r_1$.

Action models

- Putting the three things together: $\mathsf{E} = (\mathsf{O}, \mathsf{R}, \mathsf{Pre})$ where
  - O is a set of outcomes,
  - for every $a \in A$, $\mathsf{R}_a \subseteq \mathsf{O} \times \mathsf{O}$ is an accessibility relation and
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- Variant that does change basic facts exists ("postconditions").
- Note the different font to distinguish from normal models.

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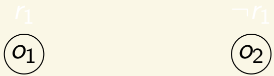

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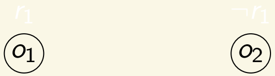

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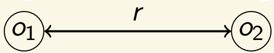

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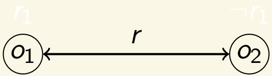

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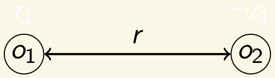

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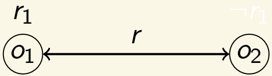

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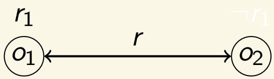

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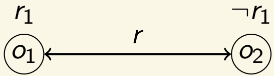

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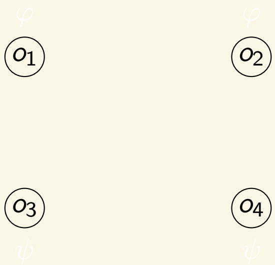

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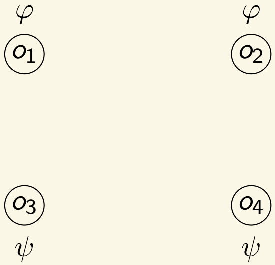

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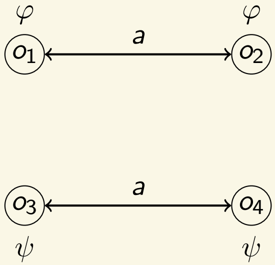

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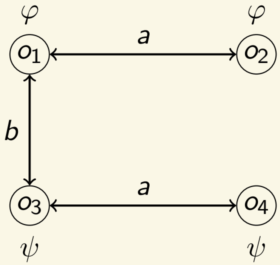

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- A pair (s, o) only results in a world in the new model if $M, s \models \mathsf{Pre}(o)$.
- So: new set of worlds given by

$$W * \mathsf{E} = \{(s, o) \in S \times \mathsf{O} \mid M, s \models \mathsf{Pre}(o)\}.$$

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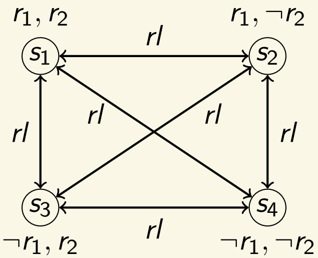
- Therefore: $(s_1, o_1) R * \mathsf{E}_a (s_2, o_2)$ iff $s_1 R_a s_2$ and $o_1 \mathsf{R}_a o_2$.
- Finally, valuation doesn't change: $(s, o) \in V * \mathsf{E}(p)$ iff $s \in V(p)$.

Example

- We return to our good old friend: me looking at the color of the first card.



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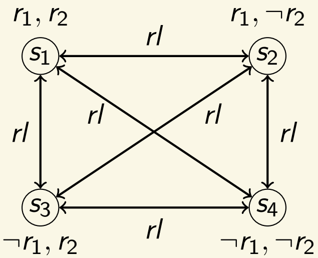

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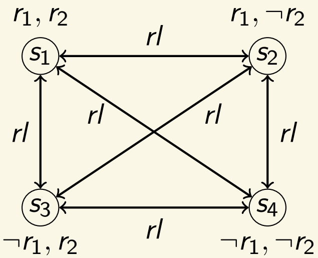

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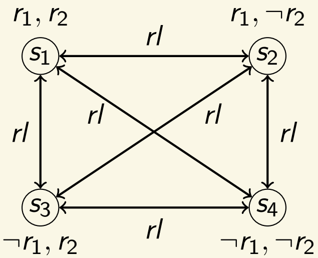

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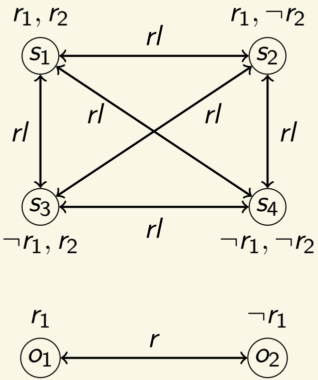

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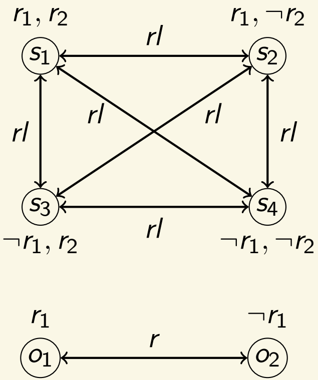

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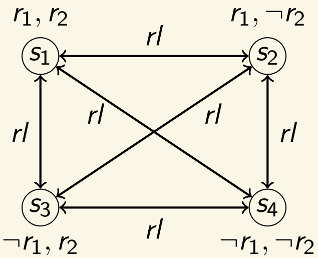

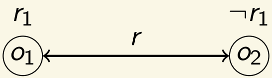

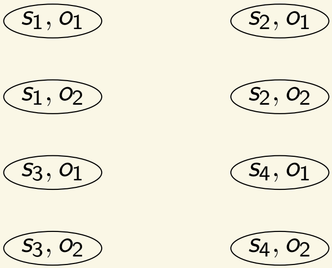

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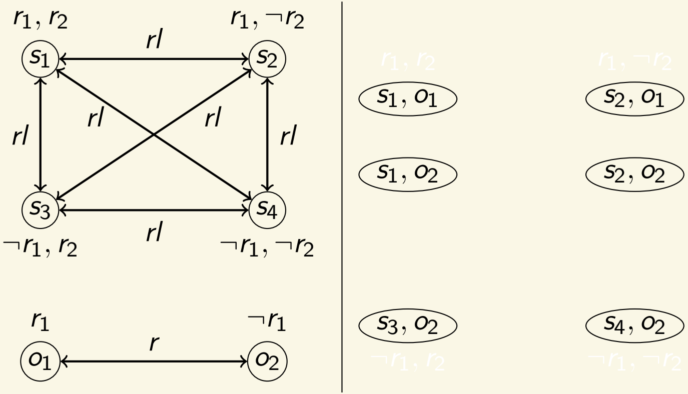

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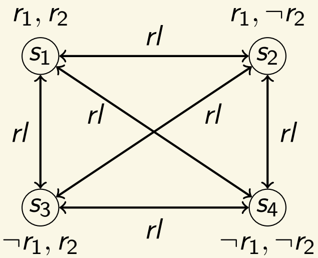

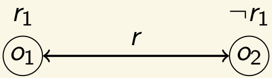

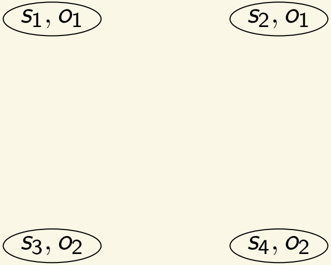

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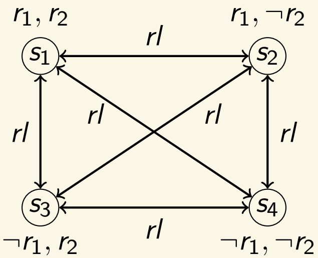

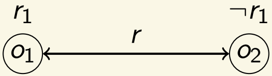

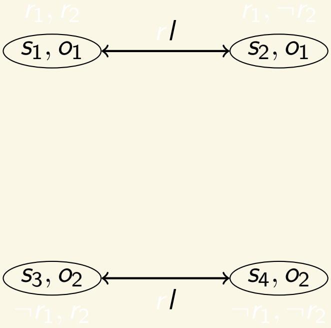

Example

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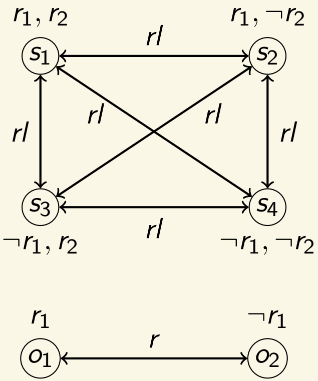

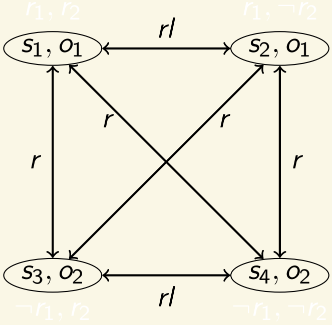

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- Action models generalize public announcements.

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- $(o_1, \varphi) \overset{a}{\mapsto} (o_2, \psi)$ is read as "if φ is true in s_1 and ψ is true in s_2, then o_1 happening in s_1 is indistinguishable from o_2 happening in s_2 for a."

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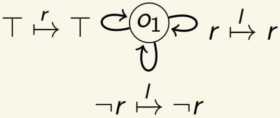

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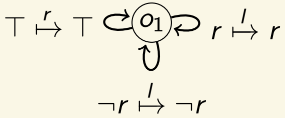

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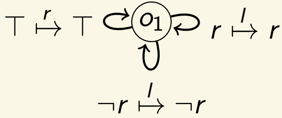

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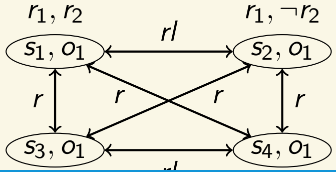

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The language of action model logic (AML) is given by

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \Box_a \varphi \mid [\mathsf{E}, X]\varphi,$$

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- Left to do: reduction axioms for $[\mathsf{E}, o]\Box_a\varphi$ and $[\mathsf{U}, o]\Box_a\varphi$.

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- Putting it all together:

$$[\mathsf{E}, o]\Box_a\varphi \leftrightarrow (\mathsf{Pre}(o) \to \bigwedge_{o' \in \mathsf{R}_a(o)} \Box_a[\mathsf{E}, o']\varphi)$$

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- It is more than sufficient to (1) know that they exist and (2) know more or less how to derive them.

Reduction

- Remember: reduction axioms give us free
  - completeness
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  - decidability

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  - completeness
  - expressivity results
  - decidability (but computationally expensive)

Update expressivity

- Recall definition of update expressivity:

Definition

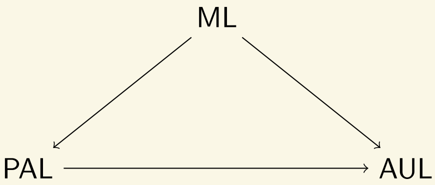
Let $e_1 : \mathfrak{M} \to \mathfrak{M}$ and $e_2 : \mathfrak{M} \to \mathfrak{M}$ be given. We say that e_2 dominates e_1, denoted $e_1 \rightsquigarrow e_2$ if for all M, s, if $e_1(M, s)$ exists, then $e_2(M, s)$ exists and the two pointed models are bisimilar.

Definition

Let $\mathcal{L}_1$ and $\mathcal{L}_2$ be languages with associated sets $\mathcal{E}_1$ and $\mathcal{E}_2$ of updates. We say that the *update expressivity* of $\mathcal{L}_1$ is at least as great as that of $\mathcal{L}_2$, denoted $\mathcal{L}_1 \preceq \mathcal{L}_2$ if:

For every $e_1 \in \mathcal{E}_1$ there is an $e_2 \in \mathcal{E}_2$ such that $e_1 \rightsquigarrow e_2$.

Update expressivity, single pointed



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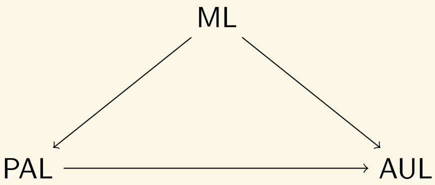

AML (1-pointed)

AUML (1-pointed)

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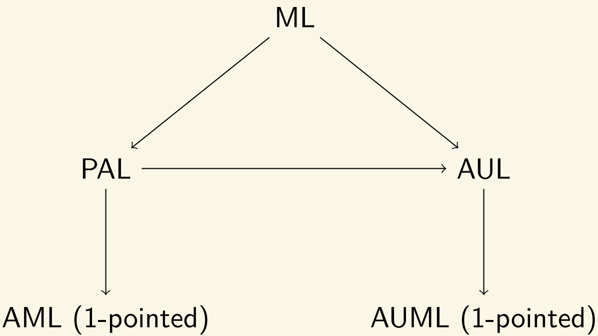

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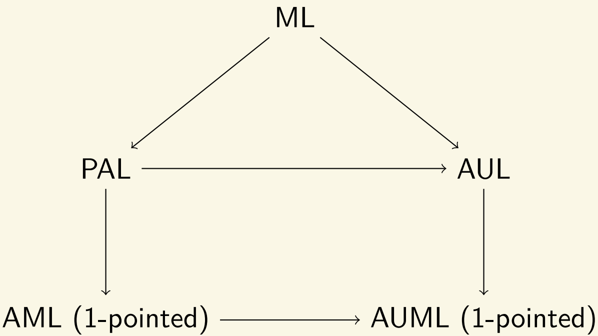

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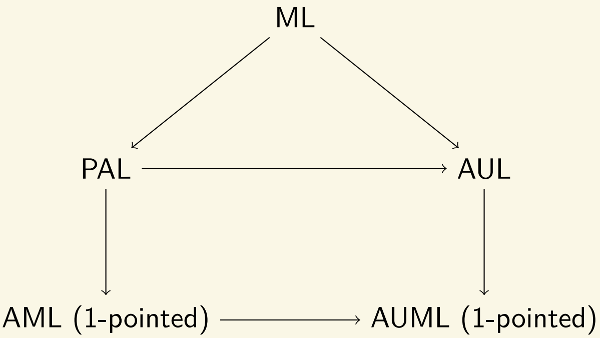

(Transitive arrows omitted)

Adapting the definition

- Previously used definition of update expressivity does not apply for one-on-many relations.
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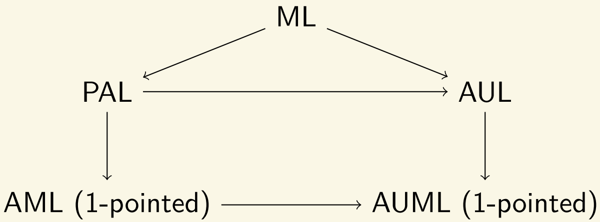
- Technical details not very important. (Included only for completeness.)
- What is important: general view of how poweful different updates are.



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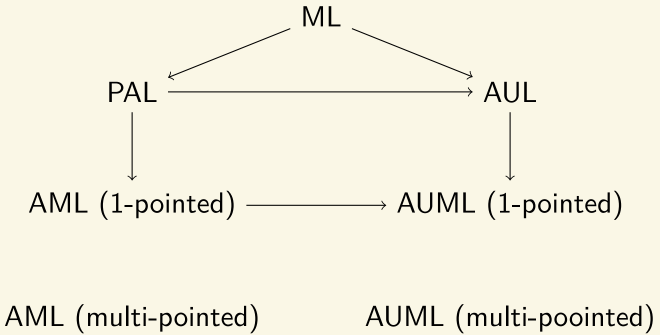

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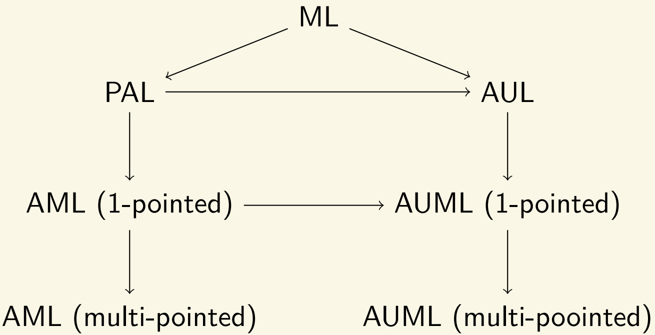

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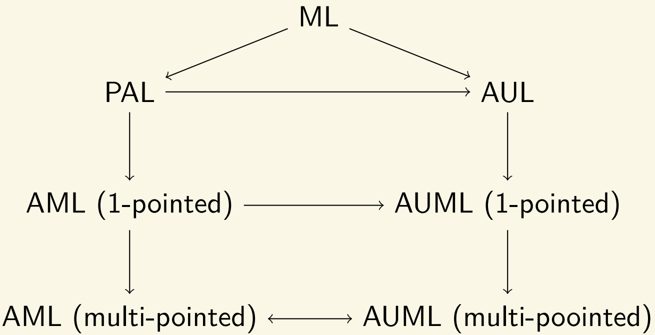

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- Details (e.g., restriction on domain of quantification) same as with APAL/AAUL.

Synthesis

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- But there is one important difference.
- In AAML/AAUML we can do *(global) synthesis*, while in APAL/AAUL we cannot.

Synthesis (II)

Definition

The *synthesis* problem for AAML is given as follows:

- **Input** A goal formula φ.
- **Output** An action model E, X such that $\models \langle \otimes \rangle \varphi \leftrightarrow \langle \mathsf{E}, X \rangle \varphi$.

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- Yet other words: there is a *uniform* strategy that achieves φ (whenever possible).

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- Synthesis for AAML is similar. So we don't discuss it in detail.

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- First thing to note: we can't always make φ true.
  - If $M, s \not\models p$, then $M, s \not\models \langle \mathrm{U}, X\rangle p$ for every U, X.
  - If $M, s \not\models \Diamond_a(q \wedge r)$, then $M, s \not\models \langle \mathrm{U}, X\rangle \Diamond_a q$ or $M, s \not\models \langle \mathrm{U}, X\rangle \Box_a r$ for every U, X.

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- This is fine. We don't need to make φ true everywhere, just everywhere *possible*.
- Possible in this case means: $M, s \models p \wedge \Diamond_a(q \wedge r)$.

Synthesis: example (part 2)

- $\varphi = p \land \Diamond_a q \land \Box_a r$.
- In $M * \mathsf{U}$ we need two worlds: (1) original world where p is true and (2) a-successor where q is true.
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- Arrow $(o_1, \top) \xrightarrow{a} (o_2, r)$.

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  4. The world (w, o_1) satisfies $\top$ and (s', o_2) satisfies r. Therefore: (s', o_2) is an a-successor of (s, o_1).
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- Putting it together: $M * \mathsf{U}, (s, o_1) \models \varphi$.

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- Example: $\varphi = (p \wedge \Diamond_a p \wedge \Box_a \top \wedge \Diamond_b q \wedge \Box_b r) \vee (p \wedge q \wedge \Diamond_a r \wedge \Box_a r \wedge \Box_b \top)$

Synthesis: conjunctive part

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- Let U_j, o_j be the arrow update model synthesized for $\chi_j \wedge \xi$.

Synthesis: conjunctive part

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- Now, let U be disjoint union of $\mathsf{U}_1,\cdots,\mathsf{U}_k$ plus one extra outcome o.
- Add arrows $(o,\top)\overset{a}{\mapsto}(o_j,\langle\mathsf{U}_j,o_j\rangle\xi)$.

Synthesis: conjunctive part (cont.)

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- Furthermore: if ξ and χ_j can be made true simultaneously in s', then they are true in (s', o_j).
- Hence: U, o always satisfies $\Box_a \xi$ and satisfies $\Diamond_a \chi_j$ whenever possible.

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- Two ways to do this: easy and hard.
- We start with easy.

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- We just have to do a little more work.

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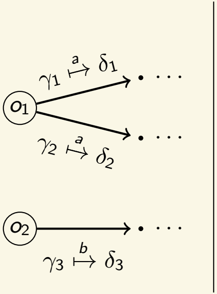

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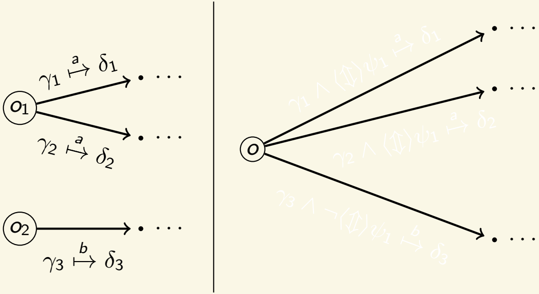

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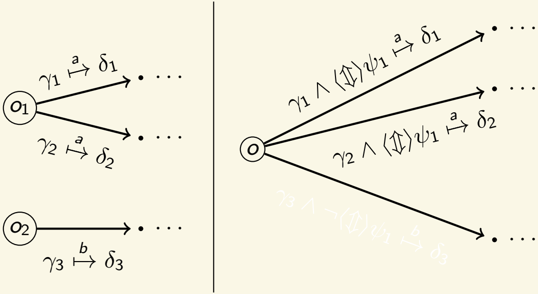

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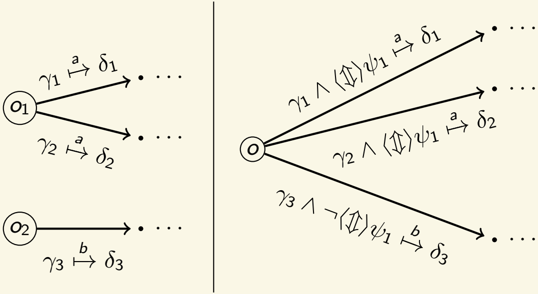

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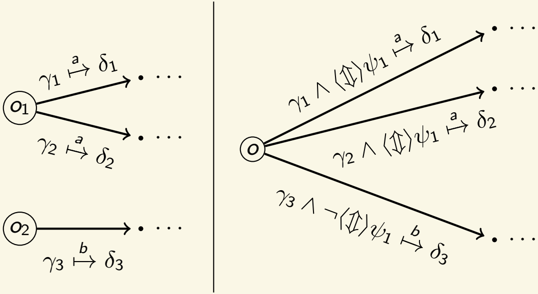

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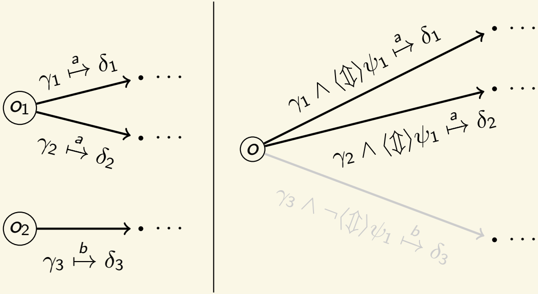

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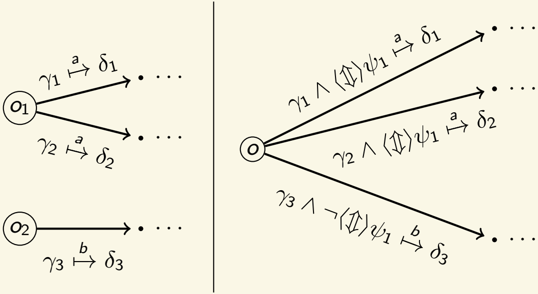

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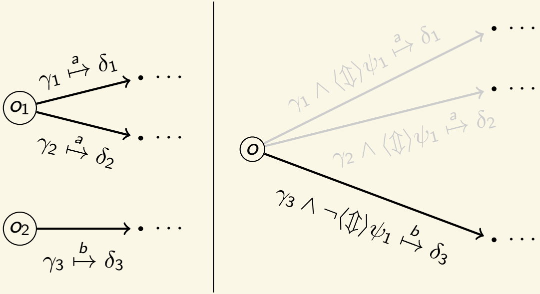

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- So we find $\mathsf{E}_\varphi, X_\varphi$ such that
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Table of Contents

Synthesis and reduction

- We have

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- Note: these are reduction axioms!
- This means we get all the goodies:
  - Sound and complete axiomatizations for AAML and AAUML!
  - AAML and AAUML are decidable!
  - AAML and AAUML are no more expressive than EL!

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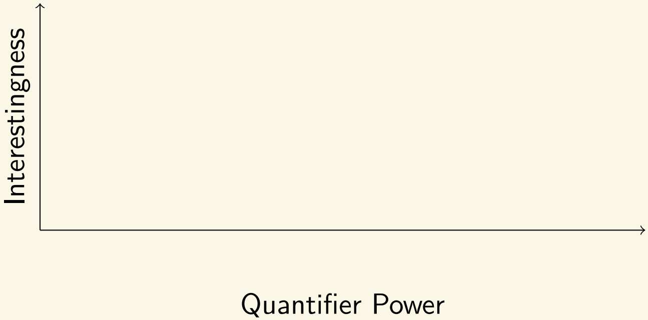
- What makes AAML and AAUML so different?
- Answer: they are a bit too powerful.

Power curve

- Suppose an agent is completely powerless. The only available action is ”do nothing”.



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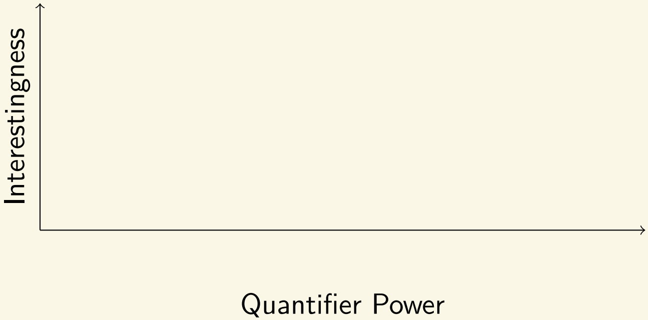

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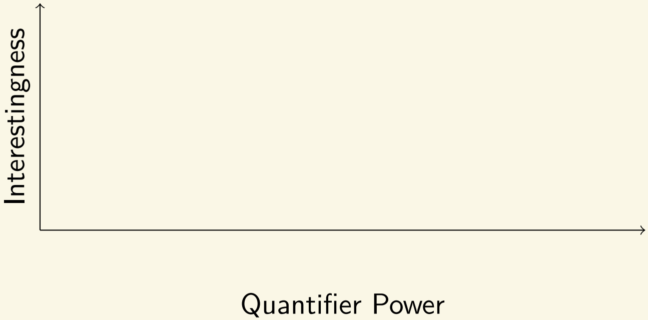

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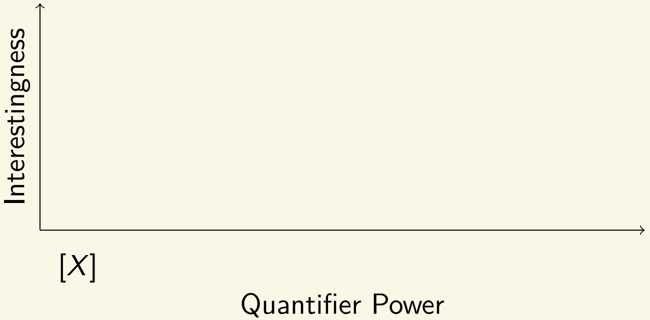

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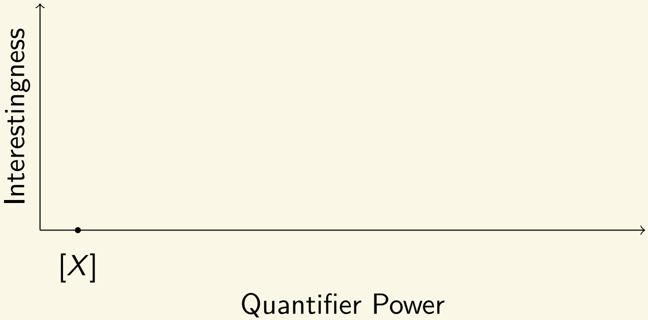

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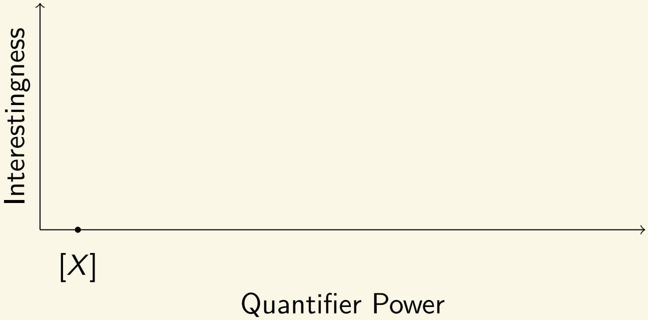

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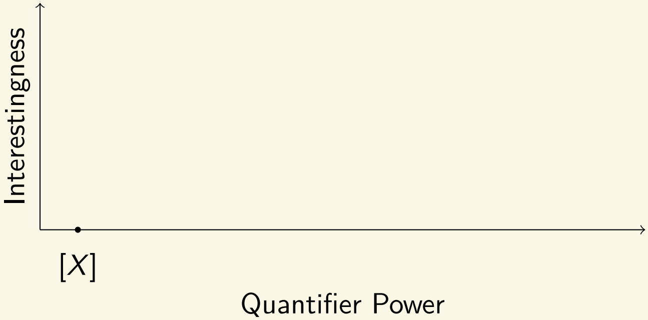

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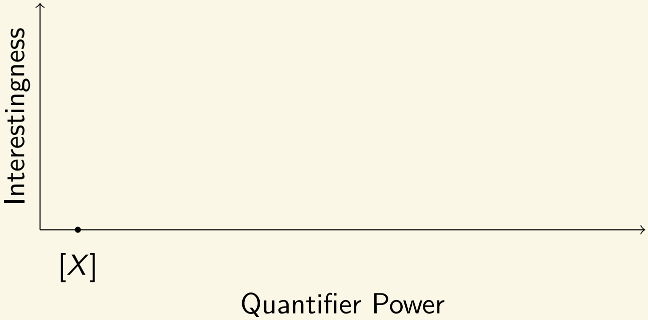

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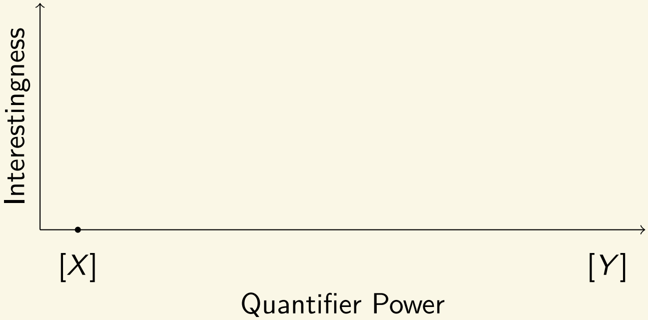

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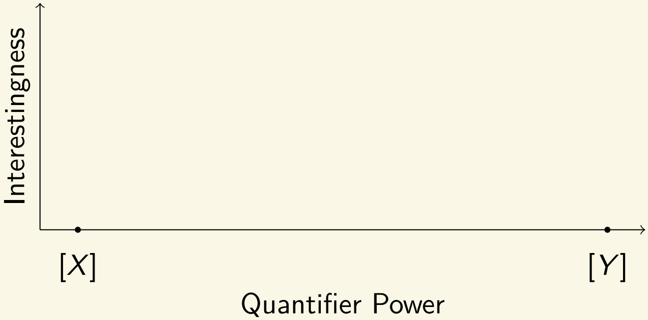

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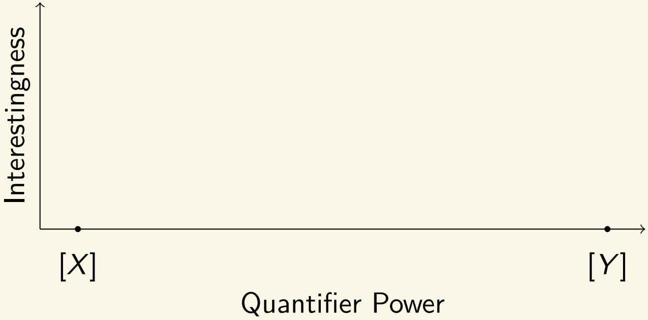

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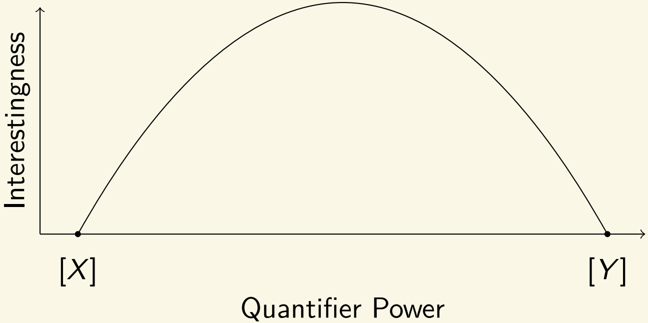

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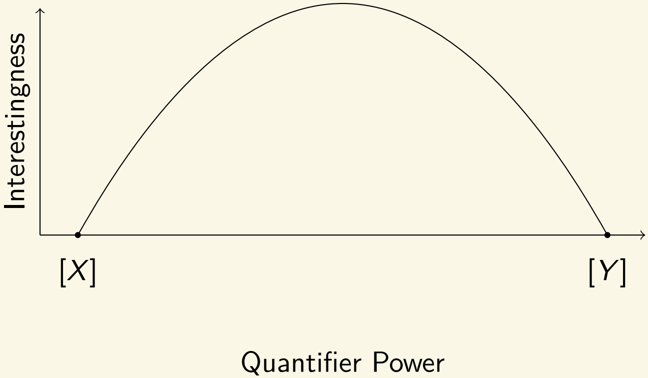

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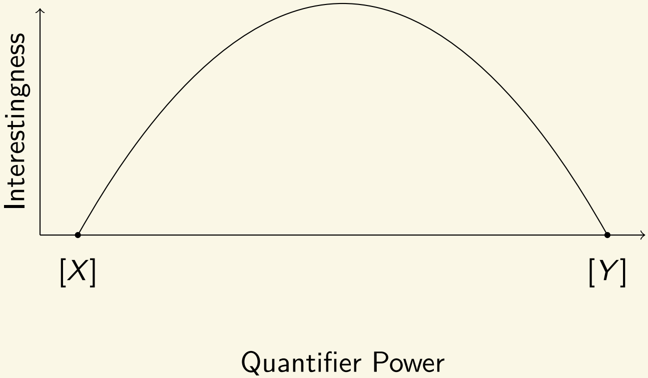

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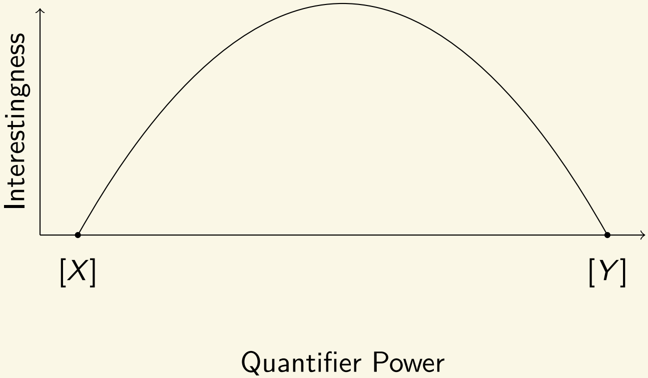

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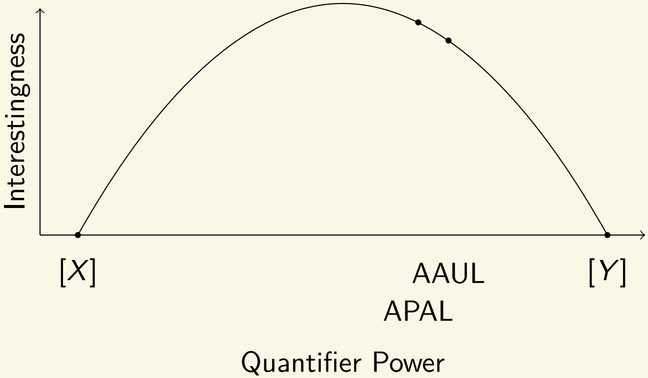

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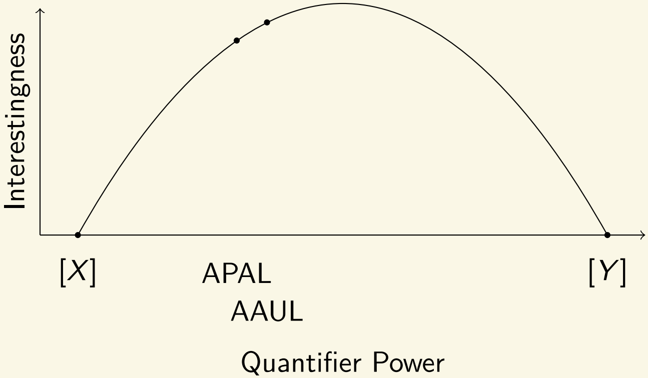

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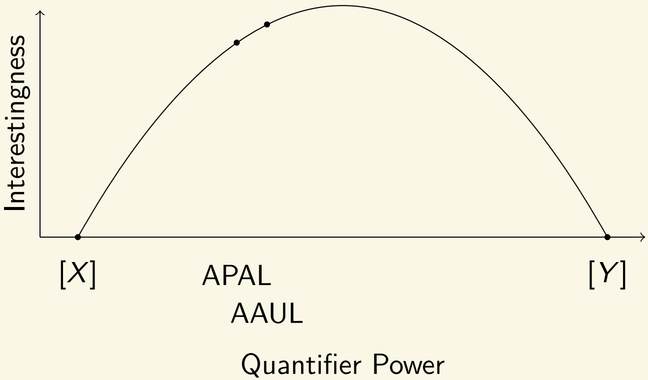

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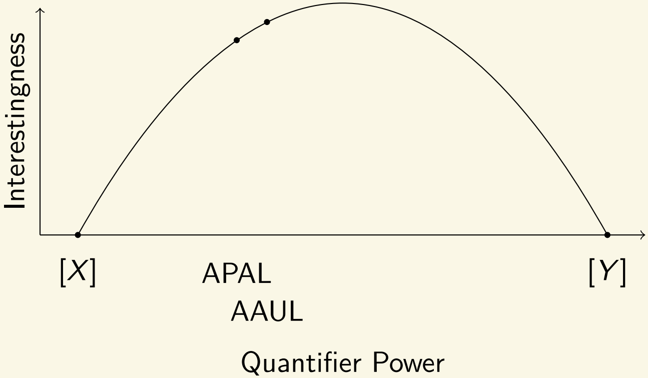

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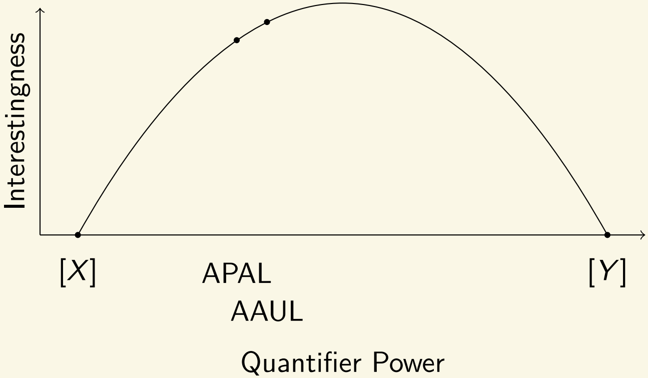

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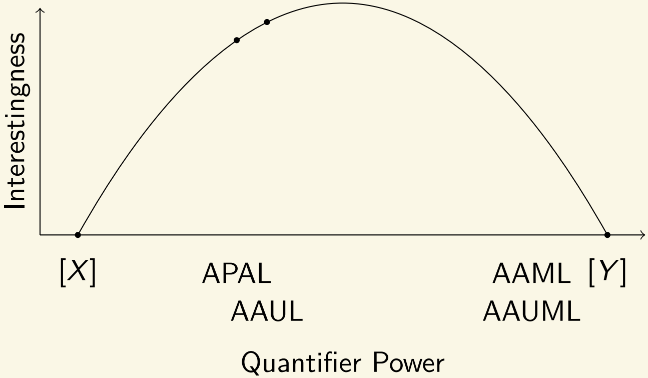

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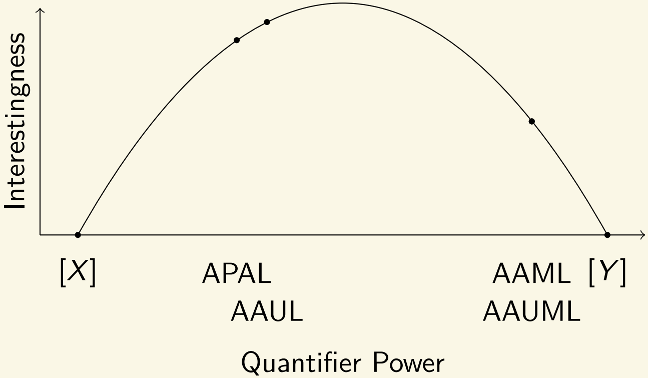

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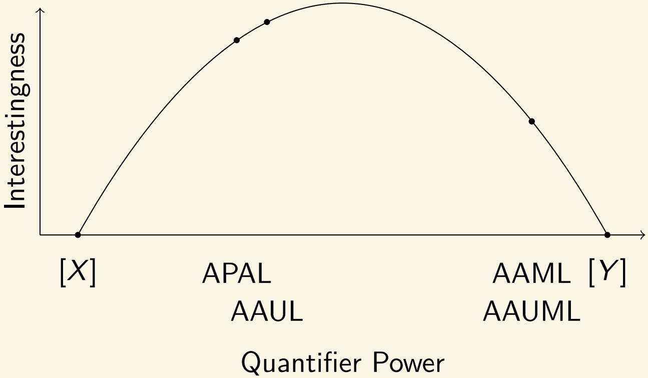

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- Note that group announcement and coalition announcements fall in this category.

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- As a result: both logics have the same expressivity as EL.
- This suggests: we should look for interesting updates that are less powerful than action models/arrow update models, not more powerful.