Varieties of Distributed Knowledge

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AiML 2024

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- Before we dive into details: general overview.

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- But what they *would* know if they pooled their knowledge.
- Or perhaps: *could* know?
- Main question: how do they pool knowledge?

Profit!

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  Step 2 ???
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- We're focusing on Step 2.
- What kind of communication is used to establish distributed knowledge?

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- Multiple authors studied the step.
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  - "Intersection."
  - "Full communication."
- (Note: terminology is not standardized, these are the terms we'll use.)

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  1. $(w, w') \in R_a$ and
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- Then φ is distributed knowledge in w iff φ holds in all w' that are still accessible after information sharing.

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- Meaning: shared information determines which worlds w' are considered.
- But shared information is *not* taken into account when evaluating φ in w'.
- Effectively: $D_G\varphi$ holds iff after sharing information, group G could determine that φ was the case before the information sharing.

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- Effectively, $R'_a = \bigcap_{b \in G} R_b$.
- Formally:

$$w \models D_G\varphi \Leftrightarrow \forall w'(w, w') \in \bigcap_{b \in G} R_b : w' \models \varphi.$$

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- They simply appeal to different intuitions about the type of communication.

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- Communication method doesn't matter. Agent b has the information to exclude w', and we assume this information reaches G.

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- Each agent contributes the set of formulas they know to the group discussion.
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- Formula φ is distributed knowledge if it follows from shared formulas.

Comparison

- Known result[1]: "full communication" is strictly stronger than "intersection", i.e.,

[1]W. van der Hoek, B. van Linder and J.-J. Meyer, Group knowledge is not always distributed (neither is it always implicit), Mathematical Social Sciences 38 (1999), pp. 215–240.

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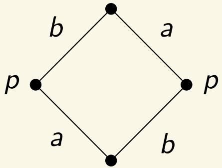
- Example: $D_{\{a,b\}}p$ in this model.



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More intuition

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- But we can draw more distinctions!

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- Question presents itself: what if agents can only share finite amount of information?

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- If a goes first and says "p", b can use this to determine that q holds.
- Once b's turn comes, they can then contribute their newly-learned "q".
- With simultaneous sharing, such newly-learned information can't be used.

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- I.e., is there an "infinity plus 1" step in the order?

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- Or does it suffice for there to be some $b \in G$ that learns φ?

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  - Some $b \in G$ learn φ ($\exists$) vs. all $b \in G$ learn φ ($\forall$).

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- We use ϵ to indicate non-answers to impossible questions.

Types

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- Hence if one agent knows φ after communication, then all do.
- But some variants are non-equivalent.

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- In this presentation, no details of definitions.

Contribution

- Our technical contribution:
  - Formally define each variant.
    More involved than you'd think, since distributed knowledge is a “hidden” second order quantifier.
  - Determine which variants are equivalent.
    And which imply which.
- In this presentation, no details of definitions.
- Instead, focus on (non)equivalences.

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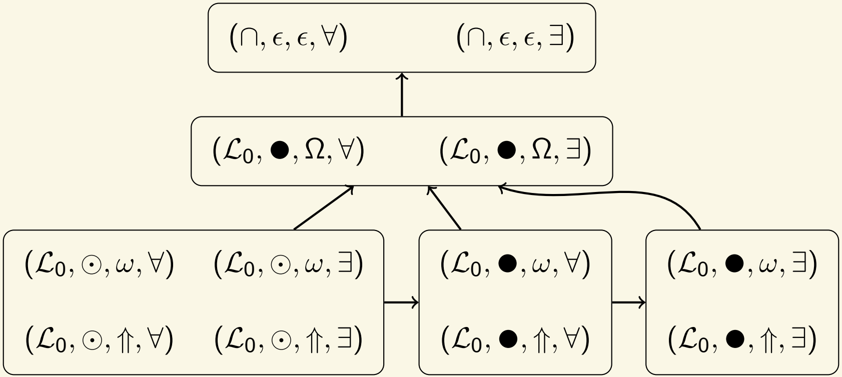
The results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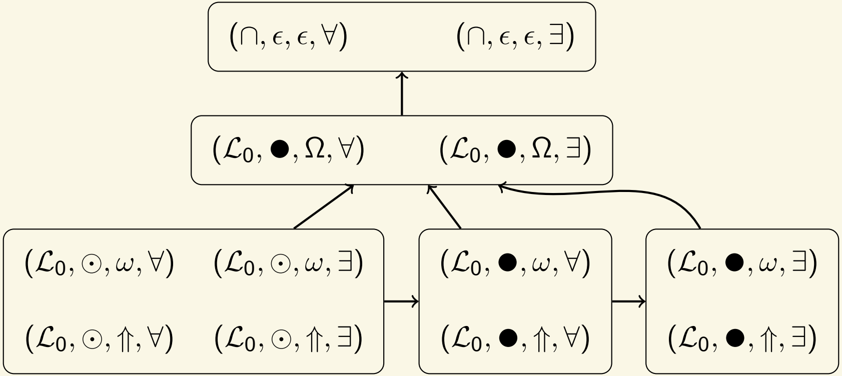
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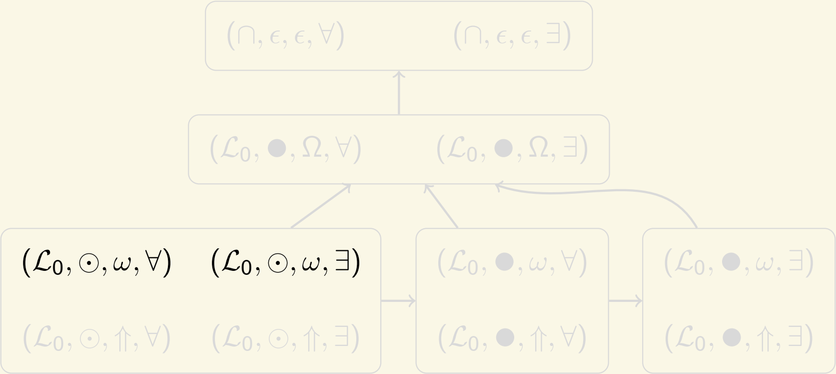

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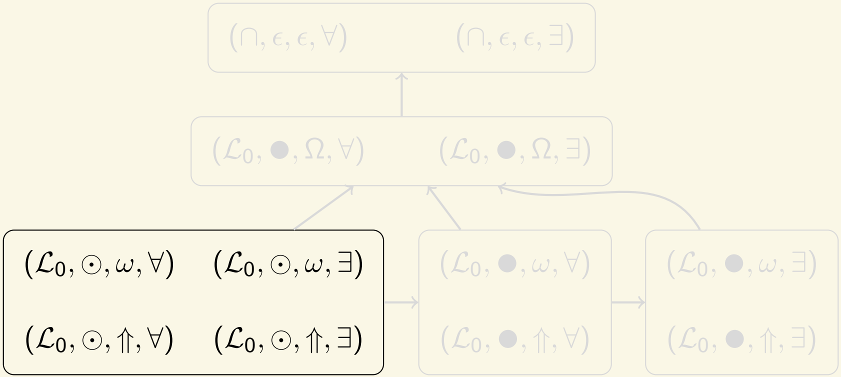
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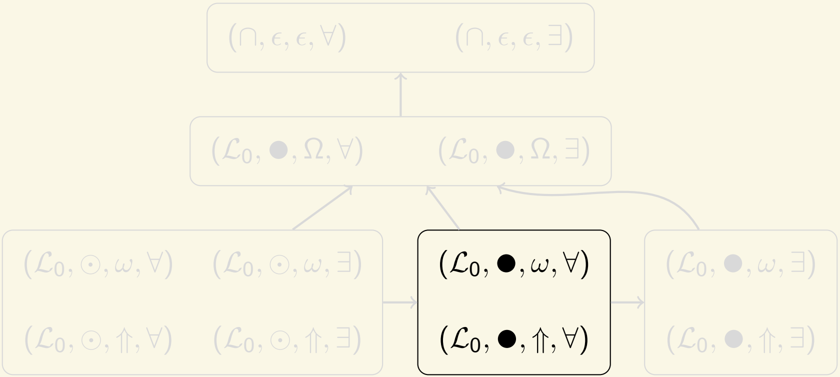
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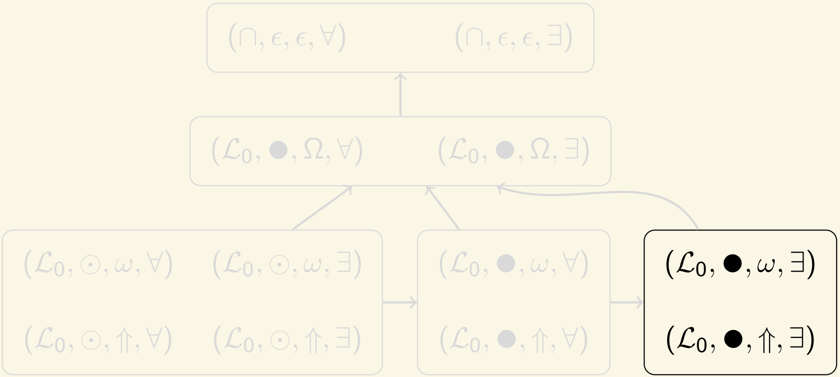

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- So that's single/finite set versus infinite set.

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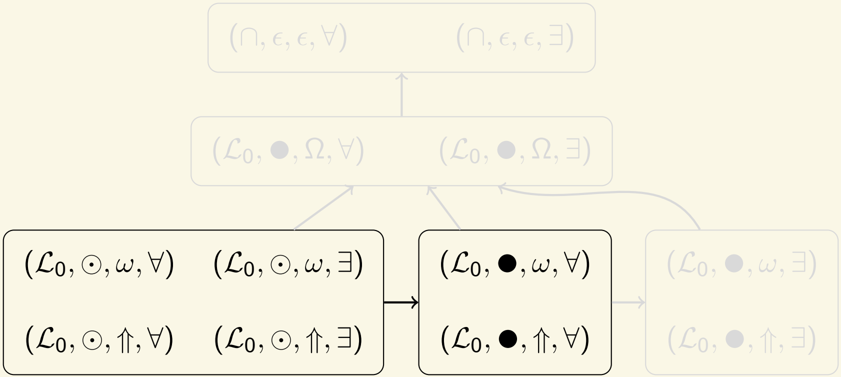
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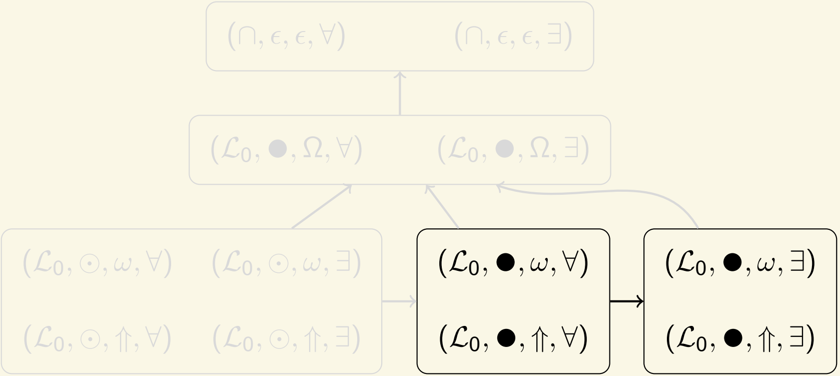
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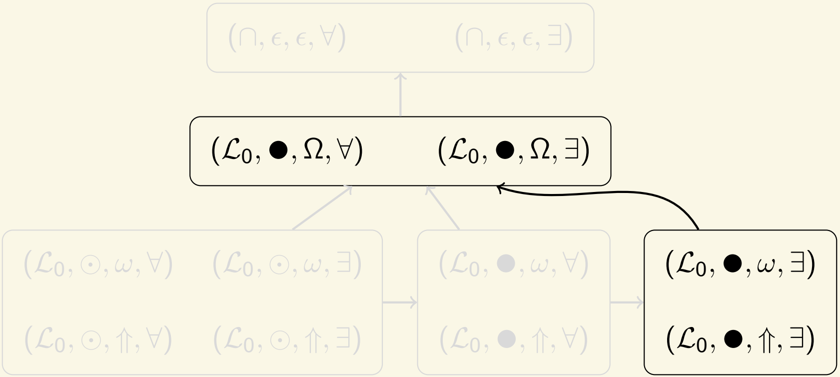
- Agents b and c then say which x_i and y_j hold, taking from $\omega + 1$ to $\omega + \omega$.
- Now a, b and c know that z holds.
- This process cannot be done in ω steps. So with $(\mathcal{L}_0, \bullet, \omega, \forall)$ we have $\neg D_{\{a,b,c\}}z$.

The results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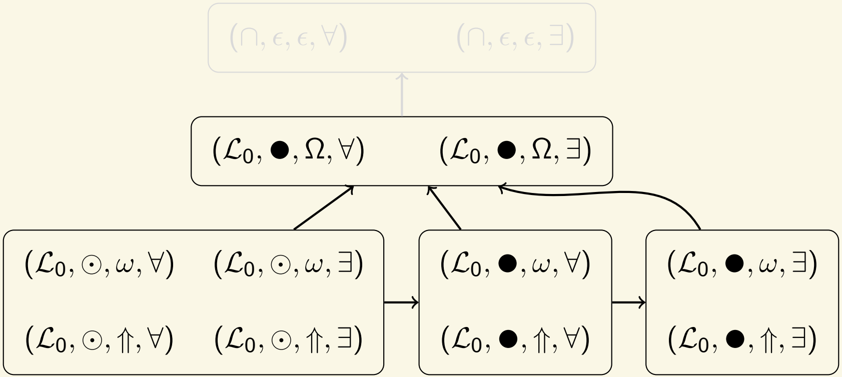

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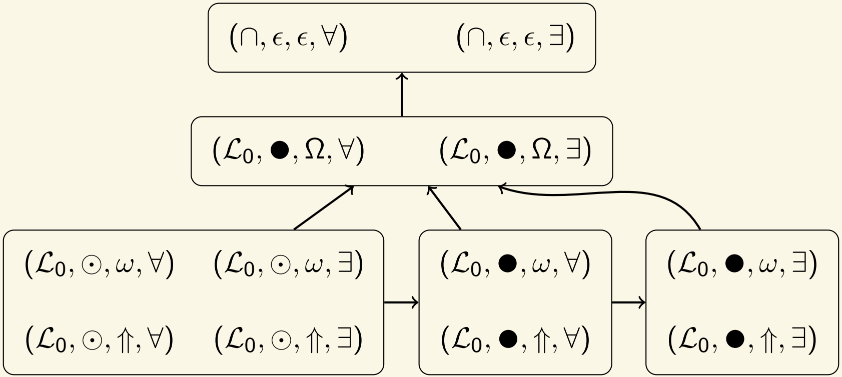
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Conclusion

- We have
  - Defined several new variants of distributed knowledge.
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- Future work:
  - Having axiomatizations would be cool.
  - There may be yet more interesting variations of distributed knowledge that could be studied.