Dynamic Epistemic Logic of Resource Bounded Information Mining Agents

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Motivation

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- Different epistemic logics treat resources as various constraints on agents' rationality: non-omniscient agents, reasoners who take time to derive consequences of their knowledge, DEL-style logics with inferential actions that require spending resources, etc.
- Logics of strategic abilities, in which agents' actions are associated with costs. And thus state-to-state transitions require spending some resources.

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Examples include medical tests, scientific experiments, database queries, etc.

Other Modelling Choices

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- The budget of each agent may also be different in different states, agents may be unaware of their own and others' budgets.

Language

Let $\mathbb{AG} = \{a_1, \ldots, a_k\}$ be a finite set of agents. We fix a set of terms

$$\text{Terms} = \{c_{(A,i)} \mid A \in \mathcal{L}_{PL}, i \in \mathbb{AG}\} \cup \{b_i \mid i \in \mathbb{AG}\}$$

Syntax

$$\varphi ::= p \mid (z_1 t_1 + \cdots + z_n t_n) \geq z) \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_i\varphi \mid C_G\varphi \mid [?^A_G]\varphi,$$

where $p \in \text{Prop}$, $t_1, \ldots, t_n \in \text{Terms}$, $z_1, \ldots, z_n, z \in \mathbb{Z}$[a], $i \in \mathbb{AG}$, $G \subseteq \mathbb{AG}$, and $A \in \mathcal{L}_{PL}$.

[a] A formula of the form $t \geq \frac{1}{2}$ can be viewed as an abbreviation for $2t \geq 1$, so we allow rational numbers to appear in our syntax.

We interpret the dynamic operator $[?^A_G]\varphi$ as *"after a group query by G whether formula A is true, φ is true"*. Or, alternatively, *"if G performs query A, they can achieve φ"*.

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- $K_a(b_i + b_j) \geq c_i(p \lor q)$ for *"agent a knows that the joint budget of i and j is higher than the cost of $p \lor q$ for agent i"*,
- $[?_G^{(p \lor q)}]C_H\psi$ meaning that *"after a joint query about $p \lor q$ by G it is common knowledge among H that ψ"*.

Models

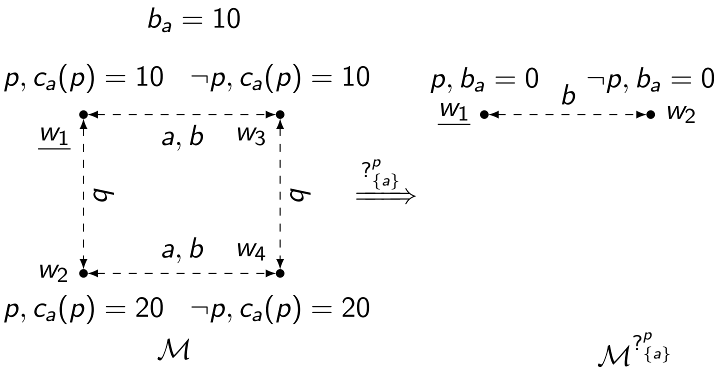
A *model* is a tuple $\mathcal{M} = (W, (\sim_i)_{i \in \mathbb{AG}}, \text{Cost}, \text{Bdg}, V)$, where

- W is a non-empty set of *states*,
- $\sim_i \subseteq (W \times W)$ is an equivalence relation for each $i \in \mathbb{AG}$,
- Cost: $\mathbb{AG} \times W \times \mathcal{L}_{PL} \longrightarrow \mathbb{Q}^+ \cup \{0\}$ assigns the (non-negative) *cost* to propositional formulas for each agent in each state, s.t.
  - $\text{Cost}_i(w, \top) = 0$,
  - $A \approx B \Rightarrow \text{Cost}_i(w, A) = \text{Cost}_i(w, B)$, where $A \approx B$ iff $A \equiv B$ or $A \equiv \neg B$.
- Bdg: $\mathbb{AG} \times W \longrightarrow \mathbb{Q}^+ \cup \{0\}$ is the (non-negative) *bugdet* of each agent at each state,
- V: Prop $\longrightarrow 2^W$ is a *valuation* of propositional variables.

Single-agent Case

Example

Alice needs test herself for COVID. It normally costs 20 resources and Alice has only 10. Luckily, she has a premium membership with 50% discount. But another agent Billy does not know about it (though he knows about the membership discount).



Updated Model

Given a model $\mathcal{M}$, an agent $i \in \mathbb{AG}$ and a formula $A \in \mathcal{L}_{PL}$, an *updated model* $\mathcal{M}'$ is a tuple $\mathcal{M}' = (W', (\sim'_j)_{j \in \mathbb{AG}}, \text{Cost}', \text{Bdg}', V')$, where

- $W' = \{w \in W \mid \mathcal{M}, w \models b_i \geq c_i(A)\}$;
- $\sim'_j = (W' \times W') \cap \sim^*_j$ with

$$\sim^*_j = \begin{cases} \sim_j & \text{if } j \neq i, \\ \sim_j \bigcap \Big(([A]_{\mathcal{M}} \times [A]_{\mathcal{M}}) \bigcup ([\neg A]_{\mathcal{M}} \times [\neg A]_{\mathcal{M}}) \Big) & \text{if } j = i; \end{cases}$$

- $\text{Cost}'_j(w, B) = \text{Cost}_j(w, B)$, for all $B \in \mathcal{L}_{PL}, j \in \mathbb{AG}$;
- $\text{Bdg}'_j(w) = \begin{cases} \text{Bdg}_j(w) - c_i(A), & \text{if } j = i, \\ \text{Bdg}_j(w), & \text{if } j \neq i, \end{cases}$
- $V'(p) = V(p) \cap W'$ for all $p \in \text{Prop}$.

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Since the access to the information is non-symmetric (different costs for different agents), it may be rational for agents to cooperate and optimize the amount of resources they need to obtain certain information.

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Group queries are performed by the following procedure: identify $i \in G$ with the lowest cost of A, let each member of G transfer $\frac{Cost_i(w,A)}{|G|}$ resources to i, then let i ask whether A is true and tell the answer to all agents in G.

Example

Telescope example

Three countries n, m and l are seeking to know a certain fact p about the universe. If any of them build a very expensive telescope, it will give them a correct answer. Country n is the richest among others having 15 abstract resources ($b_n = 15$). But due to some reasons, e.g. higher labour costs, it requires the highest amount of resources $c_n(p) = 30$ to build a telescope there. Country m has only 10 resources ($b_m = 10$), but it can build a telescope for $c_m(p) = 20$, for example due to better logistics. Country l is the poorest country with only $b_l = 5$ resources, while the cost of a telescope is the same as for n, so $c_l(p) = 30$. Finally, we assume that the costs of the telescope are known to all agents, n and m know the budgets of each other as well as l's budget. But l is unaware of the exact budget of m: it considers both options $b_m = 10$ and $b_m = 9$ as possible ones.

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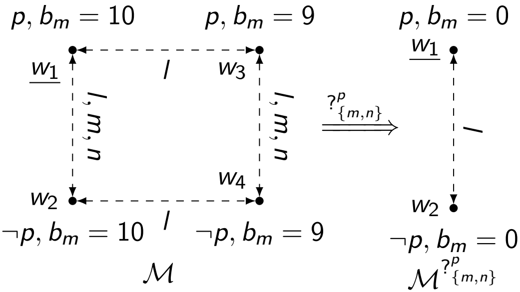

Formulas $b_n = 15, b_l = 5, c_n(p) = 30, c_m(p) = 20$ and $c_l(p) = 30$ hold in all four states of $\mathcal{M}$.

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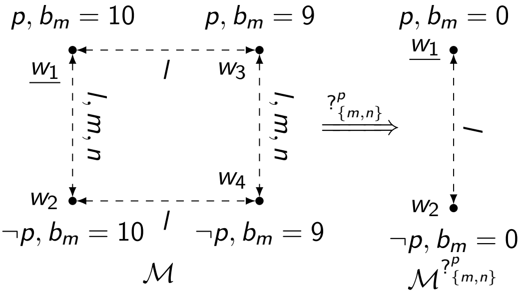

Formulas $b_n = 15, b_l = 5, c_n(p) = 30, c_m(p) = 20$ and $c_l(p) = 30$ hold in all four states of $\mathcal{M}$.

After the update, formulas $b_l = 5, c_n(p) = 30, c_m(p) = 20$ and $c_l(p) = 30$ hold in both w_1 and w_2 as before. But $b_n = 15$ no longer holds, since n's budget is decreased after update: $b_n - \frac{c_m(p)}{|\{n,m\}|} = 5$

Some Syntactic Sugar

Let's abbreviate *"the Budget Constraint of agent $i \in G$ for the G's query A"* as

$$\mathbf{BC}_i(G, A) \equiv \frac{\min_{j \in G}(c_j(A))}{|G|}$$

Then, denote the fact that *"the Budget Constraint for the query A for G is Satisfied"* as

$$\mathrm{BCS}(G, A) \equiv \bigwedge_{i \in G} (b_i \geq \mathbf{BC}_i(G, A))$$

If $\mathrm{BCS}(G, A)$ holds, we say that the query $[?^A_G]$ is realisable. Note that $\mathrm{BCS}(G, A)$ is in fact a formula of SPQ:

$$\mathrm{BCS}(G, A) \equiv \bigvee_{j \in G} \left(\bigwedge_{i \in G} \left(c_j(A) \leq c_i(A) \land b_i \geq \frac{c_j(A)}{|G|} \right) \right)$$

Updated Model

Given a model $\mathcal{M}$, a group $G \subseteq \mathbb{AG}$ and a formula $A \in \mathcal{L}_{PL}$, an *updated model* $\mathcal{M}'$ is a tuple $\mathcal{M}' = (W', (\sim_j')_{j \in \mathbb{AG}}, \text{Cost}', \text{Bdg}', V')$, where

- $W' = \{w \in W \mid \mathcal{M}, w \models \text{BCS}(G, A)\}$;
- $\sim_j' = (W' \times W') \cap \sim_j^*$ with

$$\sim_j^* = \begin{cases} \sim_j & \text{if } j \notin G, \\ \sim_j \bigcap \Big(([A]_\mathcal{M} \times [A]_\mathcal{M}) \bigcup ([\neg A]_\mathcal{M} \times [\neg A]_\mathcal{M}) \Big) & \text{if } j \in G; \end{cases}$$

- $\text{Cost}_j'(w, B) = \text{Cost}_j(w, B)$, for all $B \in \mathcal{L}_{PL}, j \in \mathbb{AG}$;
- $\text{Bdg}_j'(w) = \begin{cases} \text{Bdg}_j(w) - \frac{\min_{i \in G} \text{Cost}_i(w, A)}{|G|}, & \text{if } j \in G, \\ \text{Bdg}_j(w), & \text{if } j \notin G, \end{cases}$
- $V'(p) = V(p) \cap W'$ for all $p \in \text{Prop}$.

Results

Theorem 1.

SPQ is complete.

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Reduction-style axioms (except $C_G\varphi$):

(r_p) $[?^A_G]p \leftrightarrow (\mathrm{BCS}(G,A) \to p)$

($r_\geq$) $[?^A_G](\sum_{i=1}^{k} a_i t_i \geq z) \leftrightarrow \Big(\mathrm{BCS}(G,A) \to (\sum_{i=1}^{k} a_i t_i \geq z)^{(G,A)}\Big)$

($r_\neg$) $[?^A_G]\neg\varphi \leftrightarrow \mathrm{BCS}(G,A) \to \neg[?^A_G]\varphi$

($r_\wedge$) $[?^A_G](\varphi \wedge \psi) \leftrightarrow [?^A_G]\varphi \wedge [?^A_G]\psi$

(r_{K1}) $[?^A_G]K_j\varphi \leftrightarrow \mathrm{BCS}(G,A) \to K_j[?^A_G]\varphi$, for $j \notin G$

(r_{K2}) $[?^A_G]K_i\varphi \leftrightarrow \mathrm{BCS}(G,A) \to \bigwedge_{A' \in \{A, \neg A\}} \Big((A' \to K_i(A' \to [?^A_G]\varphi))\Big)$,

$i \in G$

(RC2) from $\chi \to [?^A_G]\psi$ and $(\chi \wedge \mathrm{BCS}(G,A)) \to$

$\to \bigwedge_{A' \in \{A, \neg A\}} (A' \to E_{H\cap G}(A' \to \chi)) \wedge E_{H\setminus G}\chi$, infer $\chi \to [?^A_G]C_H\psi$

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Theorem 3.

Model checking SPQ is in P.

MC algorithm mimics the definitions of semantics.

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- Costs can be represented as vectors $(r_1, \dots, r_k)$, where each $r_l \in (r_1, \dots, r_k)$ represents a specific resource
- How much should composed formulas cost?
- Future work: Quantified queries