Dynamic Epistemic Logic of Resource Bounded Information Mining Agents

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Introduction

We present a logic of Semi-Public (Group) Queries (SPQ) for reasoning about agents who may purchase a new piece of information from a trustworthy source. In this setting (groups of) agents can perform a public query (ask a question) about whether some formula is true and receive a private correct answer.

Modelling Choices

- Only propositional information can be 'purchased'. Hence, information about agents' knowledge and budgets remains private.
- Formulas' costs and agents' budgets are non-negative.
- Cost of the same formula may differ for different agents.
- Agents may be uncertain about their own and others' budgets and costs.
- Coalitions of agents may cooperate and optimize the amount of resources they need to obtain a certain piece of information.

Models

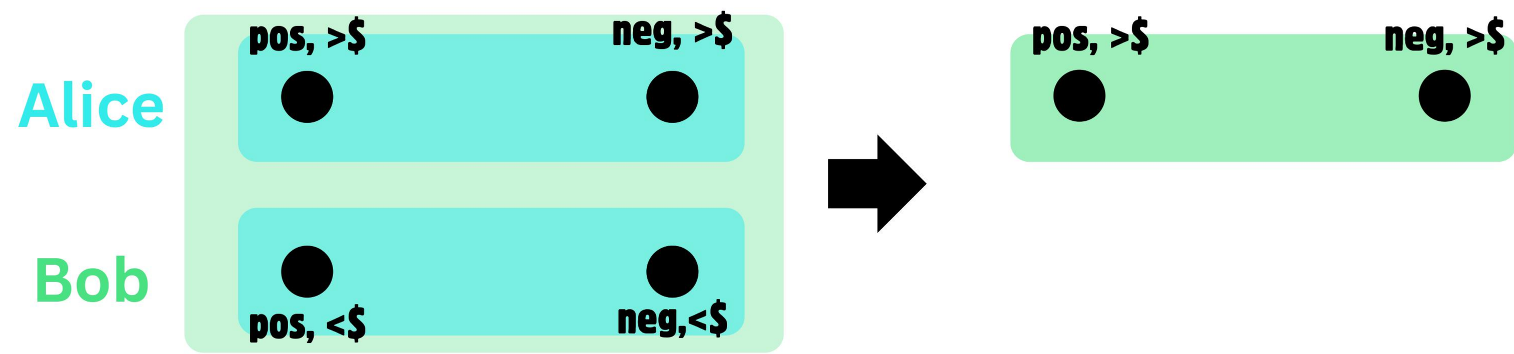
We use Kripke-style semantics endowed with Cost and Budget functions. A *model* is a tuple $\mathcal{M} = (W, (\sim_i)_{i\in\mathbb{AG}}, \mathsf{Cost}, \mathsf{Bdg}, V)$, where

- W is a non-empty set of *states*,
- $\sim_i \subseteq (W \times W)$ is an equivalence relation for each $i \in \mathbb{AG}$,
- $\mathsf{Cost} : \mathbb{AG} \times W \times \mathcal{L}_{PL} \longrightarrow \mathbb{Q}^+$ assigns the (non-negative) *cost* to propositional formulas for each agent in each state,
- $\mathsf{Bdg} : \mathbb{AG} \times W \longrightarrow \mathbb{Q}^+$ is the (non-negative) *bugdet* of each agent at each state,
- $V : \mathsf{Prop} \longrightarrow 2^W$ is a *valuation* of propositional variables.

Restrictions:
(C1) $\mathsf{Cost}_i(w, \top) = 0$,
(C2) $A \approx B$ implies $\mathsf{Cost}_i(w, A) = \mathsf{Cost}_i(w, B)$, where $A \approx B$ iff $A \equiv B$ or $A \equiv \neg B$.

Example

Alice and Bob suspect that Alice may be COVID-positive. The price of a test is common knowledge among them, but Bob is in doubts whether Alice has enough money to take it. Next day Bob observes that Alice takes the test, but its result is confidential and only available to Alice.



Update

We abbreviate the fact that *"the Budget Constraint for the query A for G is Satisfied"* as $\mathrm{BCS}(G, A)$. Note that $\mathrm{BCS}(G, A)$ is a syntactic construction:

$$\mathrm{BCS}(G, A) \equiv \bigvee_{j\in G}\Big(\bigwedge_{i\in G}\Big(c_j(A) \leq c_i(A) \wedge b_i \geq \frac{c_j(A)}{|G|}\Big)\Big).$$

Given a model $\mathcal{M}$, a group $G \subseteq \mathbb{AG}$ and a formula $A \in \mathcal{L}_{PL}$, an *updated model* $\mathcal{M}'$ is a tuple $\mathcal{M}' = (W', (\sim'_j)_{j\in\mathbb{AG}}, \mathsf{Cost}', \mathsf{Bdg}', V')$, where

- $W' = \{w \in W \mid \mathcal{M}, w \models \mathrm{BCS}(G, A)\}$;
- $\sim'_j = (W' \times W') \cap \sim^*_j$ with

$$\sim^*_j = \begin{cases} \sim_j & \text{if } j \notin G \\ \sim_j \cap \Big(([A]_\mathcal{M} \times [A]_\mathcal{M}) \cup ([\neg A]_\mathcal{M} \times [\neg A]_\mathcal{M})\Big) & \text{if } j \in G \end{cases}$$

- $\mathsf{Cost}'_j(w, B) = \mathsf{Cost}_j(w, B)$, for all $B \in \mathcal{L}_{PL}, j \in \mathbb{AG}$;
- $\mathsf{Bdg}'_j(w) = \begin{cases} \mathsf{Bdg}_j(w) - \frac{\min_{i\in G} \mathsf{Cost}_i(w,A)}{|G|}, & \text{if } j \in G \\ \mathsf{Bdg}_j(w), & \text{if } j \notin G \end{cases}$
- $V'(p) = V(p) \cap W'$ for all $p \in \mathsf{Prop}$.

where $[A]_\mathcal{M} = \{w \in W \mid \mathcal{M}, w \models A\}$.

For a group query $[?^A_G]$, the agent with the lowest cost of A is identified and each member of G spends an equal amount of resources. After that, every member of G receives a correct answer. However, we can also implement other resource distribution rules.

Results

Theorem 1. SPQ is complete.

Theorem 2. SAT problem for SPQ is decidable.

Theorem 3. Model checking SPQ is in PTIME.