(Arbitrary) Partial Communication

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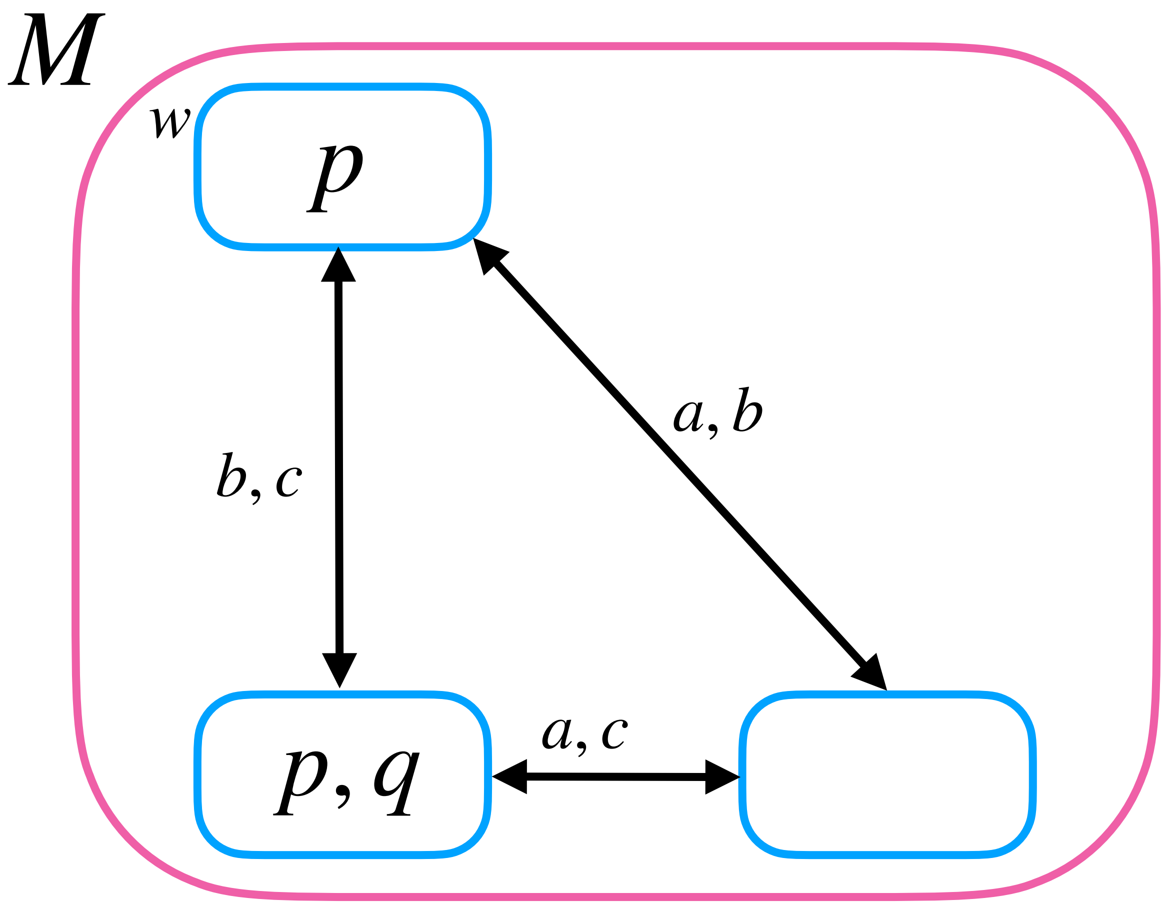
Communication

The underlying framework I. Models



A model M is a tuple $\langle W, R, V \rangle$, where $W \neq \varnothing$ is a set of worlds, $R : A \to 2^{W \times W}$ is an indistinguishability relation, and $V : P \to 2^W$ is the valuation function.

Relation R_G is $\bigcap_{i \in G} R_i$

$$(M, w) \Vdash p \wedge \neg q$$

$D_G\varphi$: group G (distributively) knows φ

Communication

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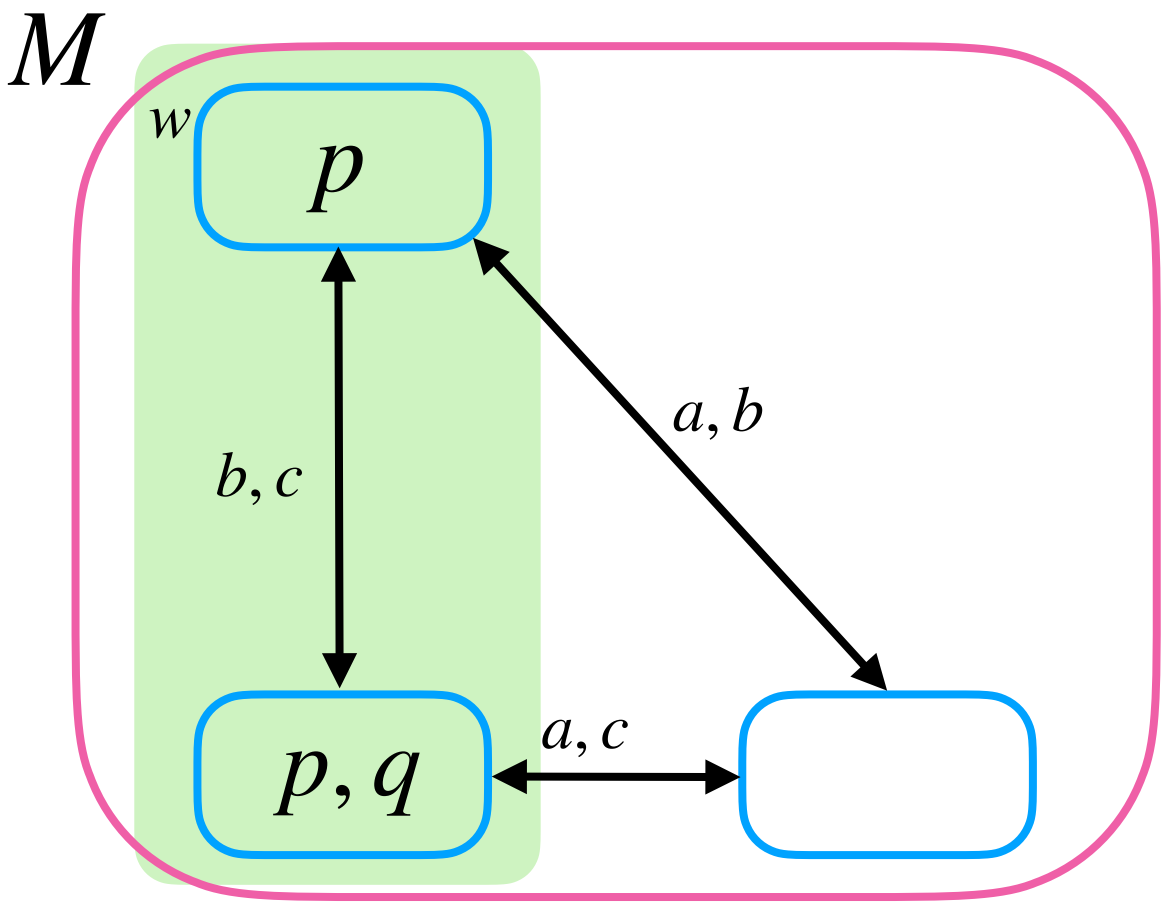

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$$(M, w) \Vdash p \wedge \neg q$$

$$(M, w) \Vdash D_{\{b,c\}} p \qquad (M, w) \Vdash D_{\{b,c\}} \neg D_{\{a\}} p$$

Communication

The underlying framework II. Language and semantics

$$\mathscr{L} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid D_G\varphi$$

$$(M, w) \Vdash p \text{ iff } w \in V(p)$$
$$(M, w) \Vdash \neg\varphi \text{ iff } (M, w) \nVdash \varphi$$
$$(M, w) \Vdash \varphi \wedge \psi \text{ iff } (M, w) \Vdash \varphi \text{ and } (M, w) \Vdash \psi$$
$$(M, w) \Vdash D_G\varphi \text{ iff } \forall v \in W : R_Gwv \text{ implies } (M, v) \Vdash \varphi$$

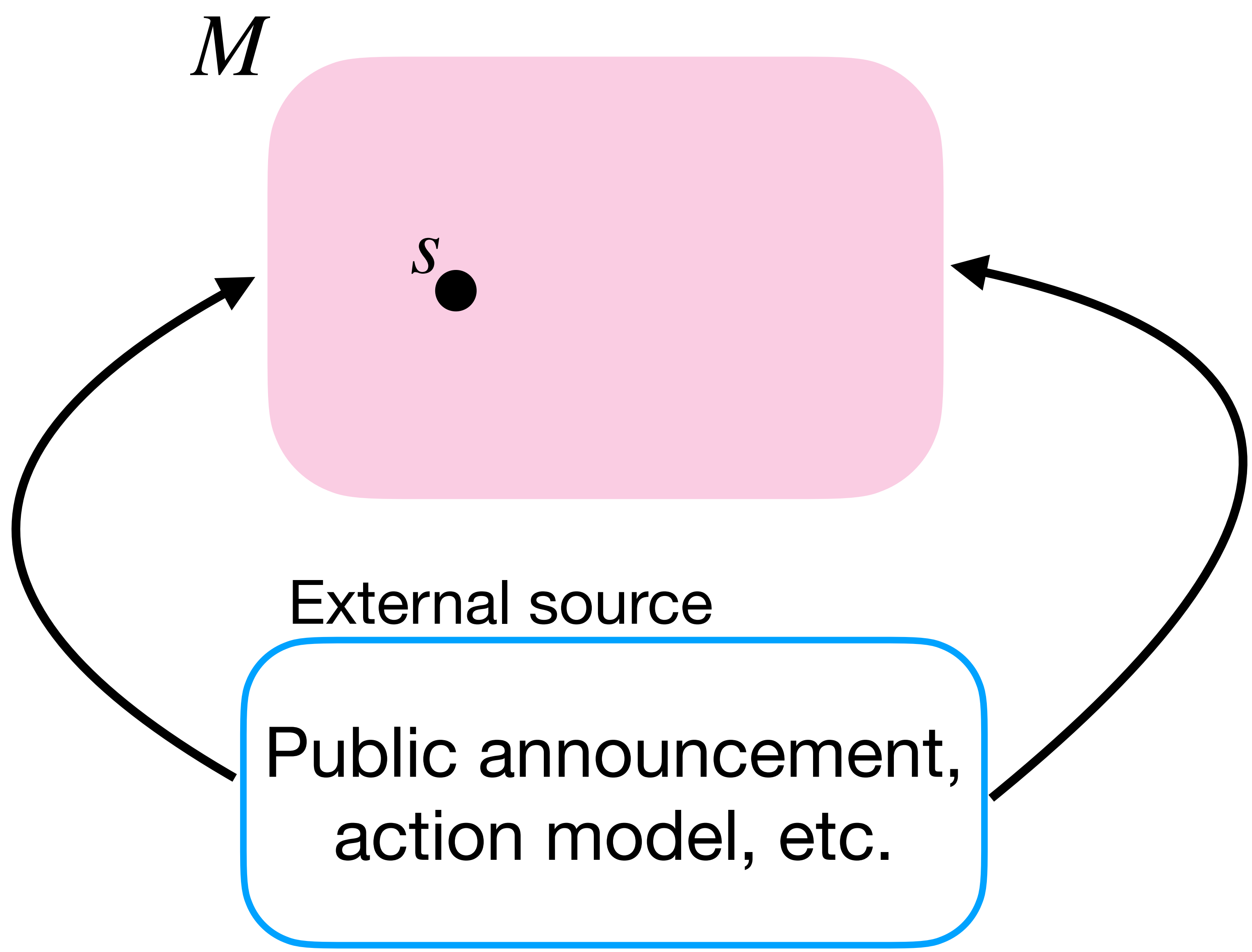
Theorem I. $\mathscr{L}$ has a sound and complete axiomatisation

Theorem II. Model checking $\mathscr{L}$ is in P

Halpern, Moses. *A Guide to Completeness and Complexity for Modal Logics of Knowledge and Belief*. 1992.

Communication

Dynamic epistemic logic (DEL) usually deals with exogenous sources of information



Communication

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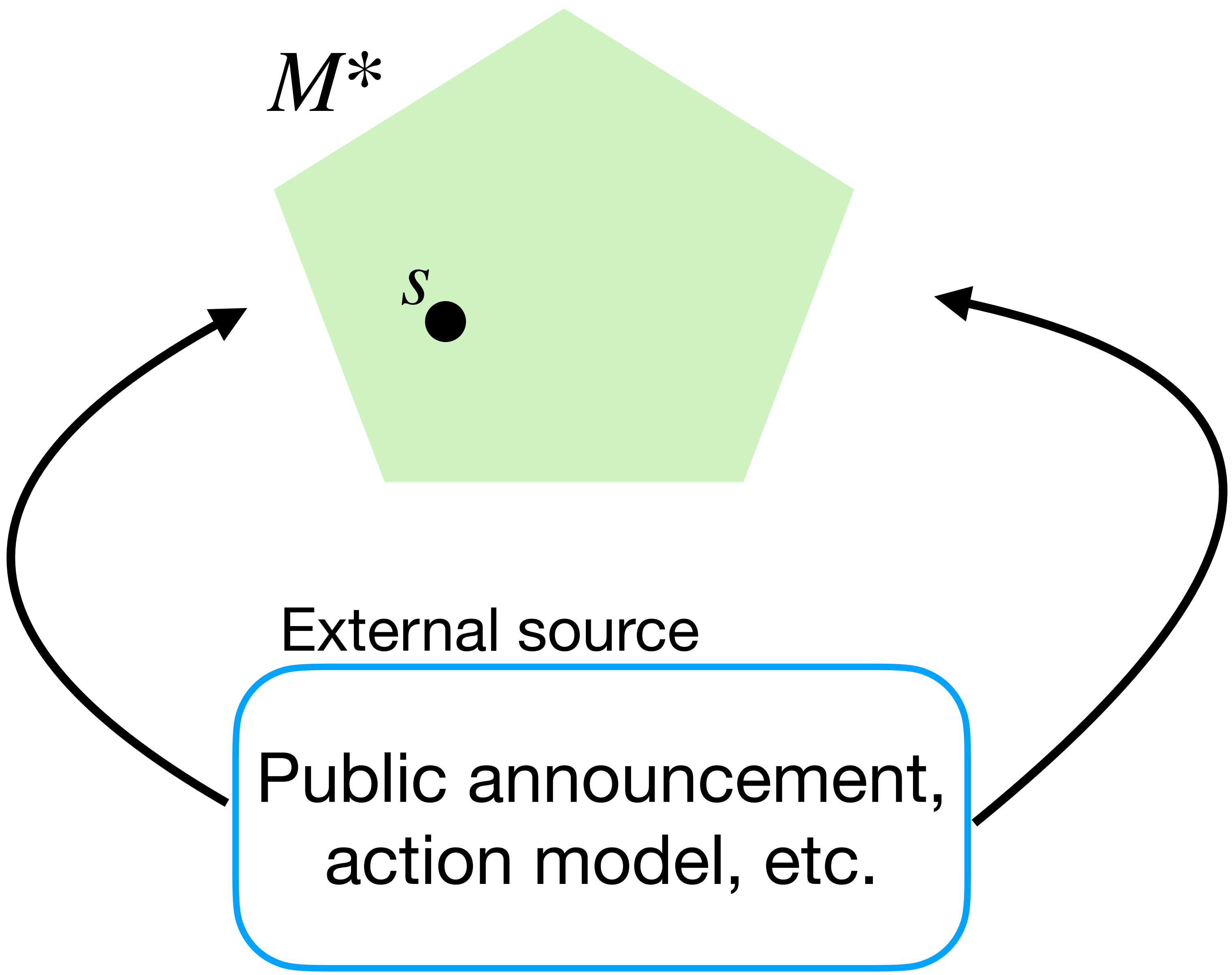

Communication

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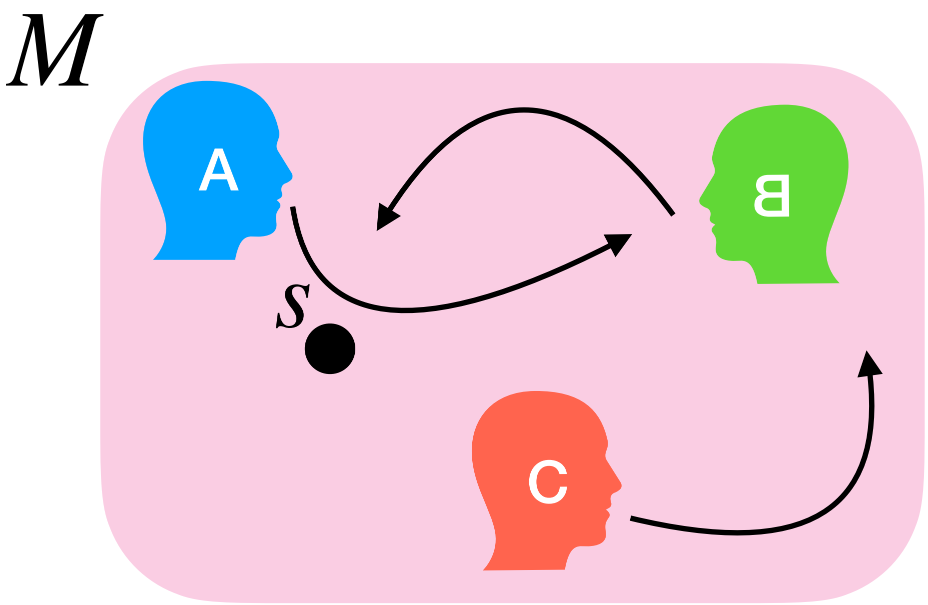

A recent trend in DEL is communication between agents

Communication

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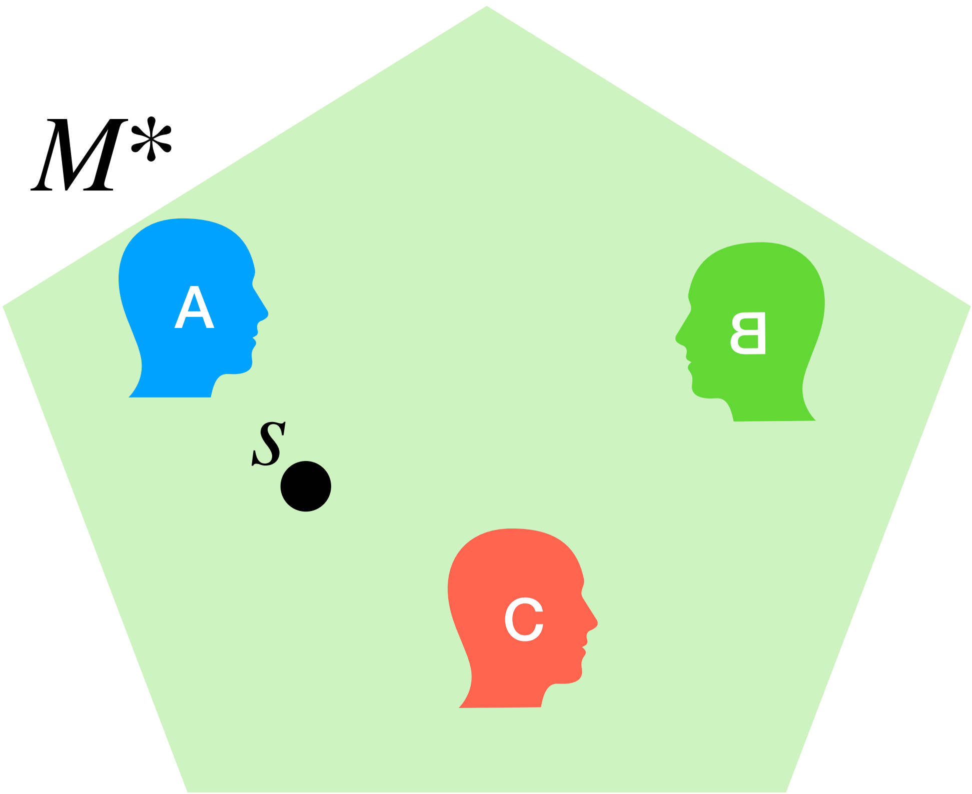


A recent trend in DEL is communication between agents

Communication

Dynamic epistemic logic (DEL) usually deals with exogenous sources of information

A recent trend in DEL is communication between agents

Examples

A single agent sharing all her information with everyone

A group of agents share all their information among themselves

An agent private reading all information of another agent

And so on and so on and so on…

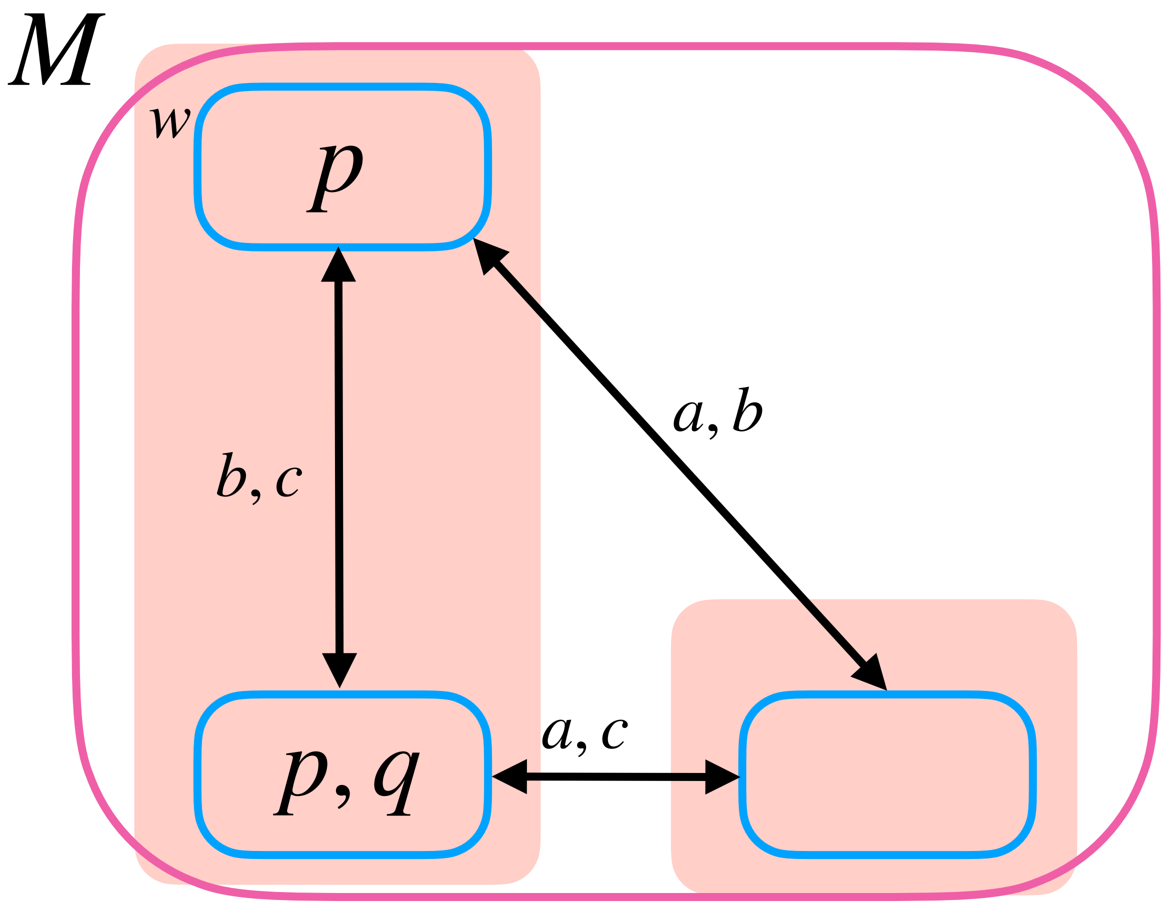
Baltag. *What DEL is good for?* 2010.

Ågotnes, Wáng. *Resolving distributed knowledge,* 2017.

Baltag, Smets. *Learning what others know,* 2021.

Partial Communication

We consider topic-based communication



Topic is represented by a formula of a logic

Agents b and c publicly share their information on p

Partial Communication

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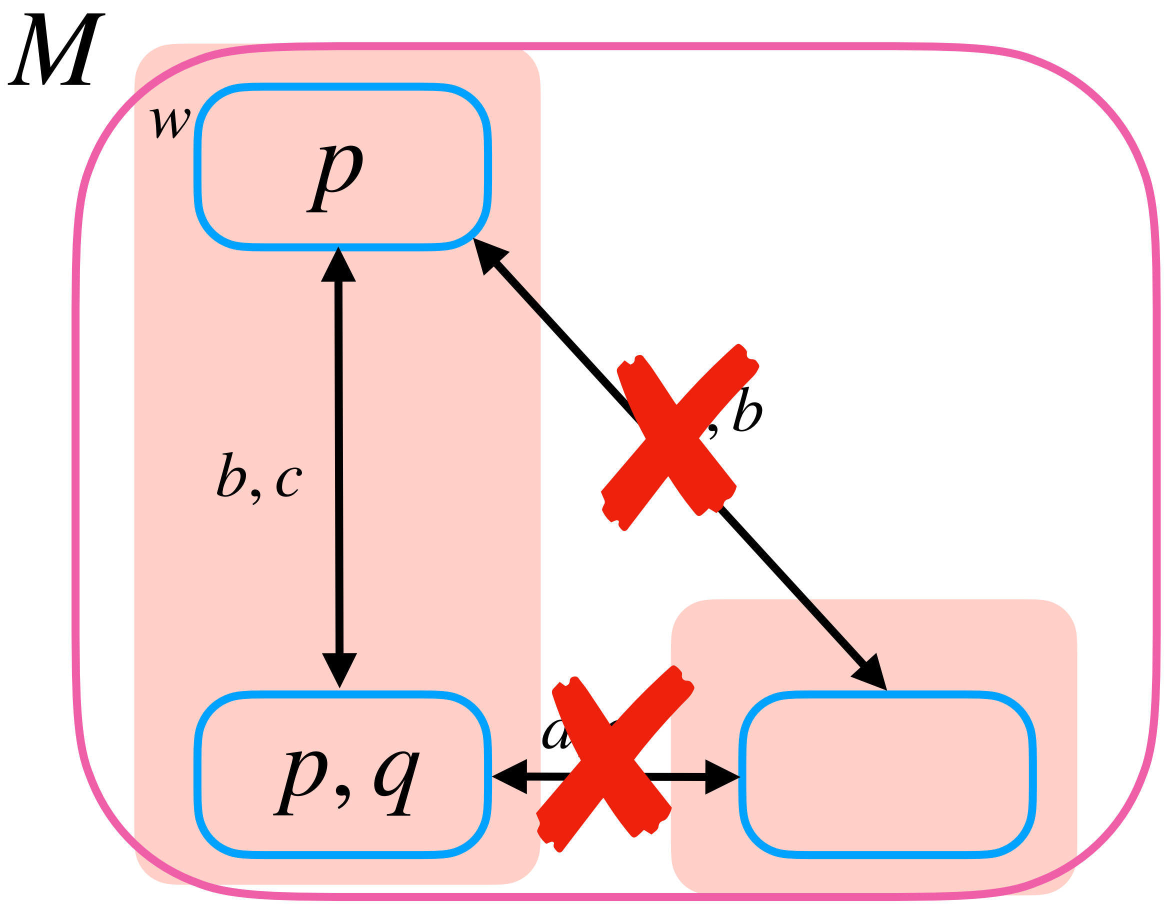

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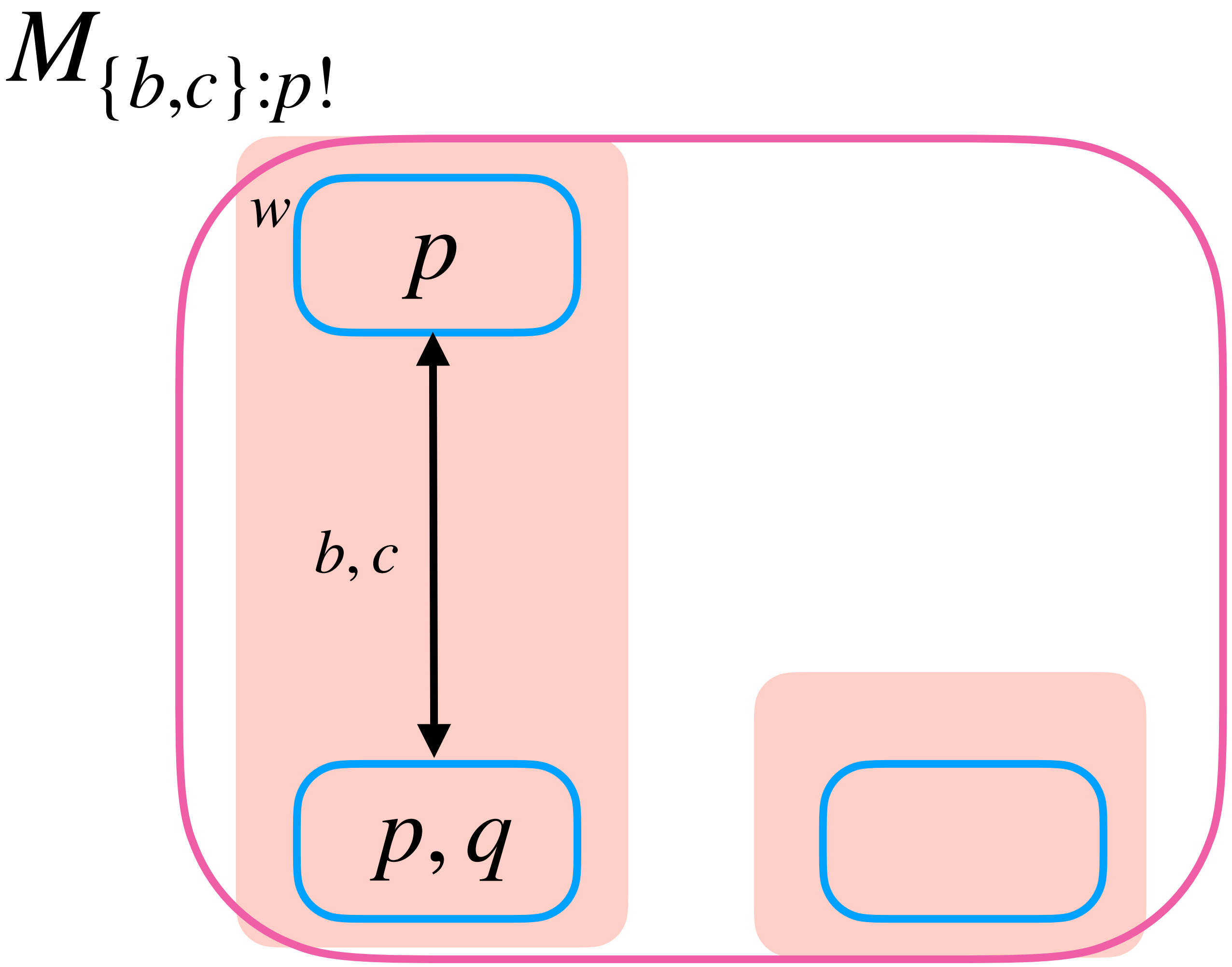

Topic is represented by a formula of a logic

Agents b and c publicly share their information on p

The result of communication is an updated model

$[S : \chi!]\varphi$: after agents in S publicly communicate on topic χ, φ is true

Partial Communication

$$\mathscr{L}_{S:\chi!} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid D_G\varphi \mid [S : \chi!]\varphi$$

$$\sim_{\varphi}^{M} := ([\![\chi]\!]^M \times [\![\chi]\!]^M) \cup ([\![\neg\chi]\!]^M \times [\![\neg\chi]\!]^M)$$

$$R_i^{S:\chi!} := R_i \cap (R_S \cup \sim_{\chi}^{M})$$

Given a model $M = \langle W, R, V \rangle$, an updated model $M_{S:\chi!}$ is $\langle W, R^{S:\chi!}, V \rangle$

$$(M, w) \Vdash [S : \chi!]\varphi \text{ iff } (M_{S:\chi!}, w) \Vdash \varphi$$

Partial Communication

$[S : \chi!]\varphi \leftrightarrow [S : \neg\chi!]\varphi$: communication on topic is equivalent to communication on its negation

Theorem I. $\mathscr{L}_{S:\chi!}$ has a sound and complete axiomatisation via reduction axioms

Theorem II. $\mathscr{L}_{S:\chi!}$ is as expressive as $\mathscr{L}$

Theorem III. Model checking $\mathscr{L}_{S:\chi!}$ is in P

(Arbitrary) Partial Communication

We shift the emphasis from the effects of particular communication to the (non-)existence of a communication achieving some goal

$\langle S : \star\ ! \rangle \varphi$: There is a topic on which agents in S can communicate to achieve φ

$[S : \star\ !]\varphi$: Whichever topic agents in S communicate on, they cannot avoid φ

$$(M, w) \Vdash [S : \star\ !]\varphi \text{ iff } \forall \chi \in \mathscr{L}_{S:\chi!}: (M_{S:\chi!}, w) \Vdash \varphi$$
$$(M, w) \Vdash \langle S : \star\ ! \rangle \varphi \text{ iff } \exists \chi \in \mathscr{L}_{S:\chi!}: (M_{S:\chi!}, w) \Vdash \varphi$$

(Arbitrary) Partial Communication

Theorem I. $\mathscr{L}^*_{S:\chi!}$ has a sound and complete infinitary axiomatisation

Theorem II. $\mathscr{L}^*_{S:\chi!}$ is strictly more expressive than $\mathscr{L}_{S:\chi!}$

Theorem III. Model checking $\mathscr{L}^*_{S:\chi!}$ is PSPACE-complete

Discussion

I. We studied topic-based communication between agents

II. We considered extension with quantification

III. We compared both approaches to DEL public communication (to public announcements)

?. Private communication on a topic

?. Take into account group notions of belief and knowledge

?. Alternative definitions of a topic of communication