Some properties of epistemic models expressible in group announcement logic are not expressible in coalition announcement logic

Groups versus Coalitions: On the Relative Expressivity of GAL and CAL

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1. Preliminaries

Public announcement logic (PAL) [3] is used to reason about the effects of truthful public announcements on agents' knowledge. Group announcement logic (GAL) [1] and coalition announcement logic (CAL) [2] extend PAL with quantification over announcements. In our work, we show that there are some properties of models that are expressible in GAL and are not expressible in CAL.

Definition 1 (Epistemic Model) *An* epistemic model *is a triple* $M = (S, \sim, V)$*, where*

- S *is a non-empty set of states,*
- $\sim_i \subseteq S^2$ *is an equivalence relation to each agent* i*,*
- $V : P \to 2^S$ *is the valuation function.*

A pair M_s *with* $s \in S$ *is called a* pointed model.

An announcement of α *in a pointed model* M_s *results in an* updated pointed model $M_s^{\|\alpha\|_M}$ *containing only* α*-states:*

- $S^{\|\alpha\|_M} = \|\alpha\|_M$,
- $\sim_i^{\|\alpha\|_M} = \sim_i \cap (\|\alpha\|_M \times \|\alpha\|_M)$,
- $V^{\|\alpha\|_M}(p) = V(p) \cap \|\alpha\|_M$.

Definition 2 (Semantics) *Let* M_s *be a pointed epistemic model. The semantics for boolean cases is as usual.*

$$M_s \models K_i\phi \text{ iff } \forall t \in S : s \sim_i t \text{ implies } M_t \models \phi$$
$$M_s \models [\alpha]\phi \text{ iff } M_s \not\models \alpha \text{ or } M_s^{\|\alpha\|_M} \models \phi$$
$$M_s \models \langle\alpha\rangle\phi \text{ iff } M_s \models \phi \text{ and } M_s^{\|\alpha\|_M} \models \phi$$

$[\alpha]\phi$, $\langle\alpha\rangle\phi$: 'after public announcement of α, ϕ holds in the resulting model'.

2. Announcements by groups and coalitions

We are interested in the following restrictions on announcements:

- Announcements are made by agents
- Agents can only announce what they know
- Groups of agents can announce conjunctions of formulas, where each conjunct is a formula known by an agent in the group
- Coalitional ability wrt announcements: can a group make an announcement such that whatever agents outside the group announce, the goal formula holds?

Group Announcement Logic (GAL) = PAL + $\{[\![G]\!]\phi, \langle\!\langle G\rangle\!\rangle\phi\}$

$\langle\!\langle G\rangle\!\rangle\phi$: 'there is an announcement by agents from G such that ϕ holds in the resulting model'

$[\![G]\!]\phi$: 'whatever agents from G announce, ϕ holds in the resulting model'

Coalition Announcement Logic (CAL) = PAL + $\{[\langle G\rangle]\phi, \langle[G]\rangle\phi\}$

$\langle[G]\rangle\phi$: 'there is an announcement by agents from G such that whatever agents $A \setminus G$ outside of the coalition announce, ϕ holds'

$[\langle G\rangle]\varphi$: 'whatever agents from G announce, there is an announcement by the agents from the outside of the coalition, such that ϕ holds'

Let α_G be a shorthand for a formula of the type $K_i\phi_i \wedge \ldots \wedge K_j\phi_j$, where $i, \ldots, j \in G$, and $\phi_i, \ldots, \phi_j$ are formulas of epistemic logic.

$$M_s \models [\![G]\!]\phi \text{ iff } \forall\alpha_G : M_s \models [\alpha_G]\phi$$
$$M_s \models [\langle G\rangle]\phi \text{ iff } \forall\alpha_G\, \exists\beta_{A\setminus G} : M_s \models \alpha_G \to \langle\alpha_G \wedge \beta_{A\setminus G}\rangle\phi$$

$$M_s \models \langle\!\langle G\rangle\!\rangle\phi \text{ iff } \exists\alpha_G : M_s \models \langle\alpha_G\rangle\phi$$
$$M_s \models \langle[G]\rangle\phi \text{ iff } \exists\alpha_G\, \forall\beta_{A\setminus G} : M_s \models \alpha_G \wedge [\alpha_G \wedge \beta_{A\setminus G}]\phi$$

3. Groups versus coalitions

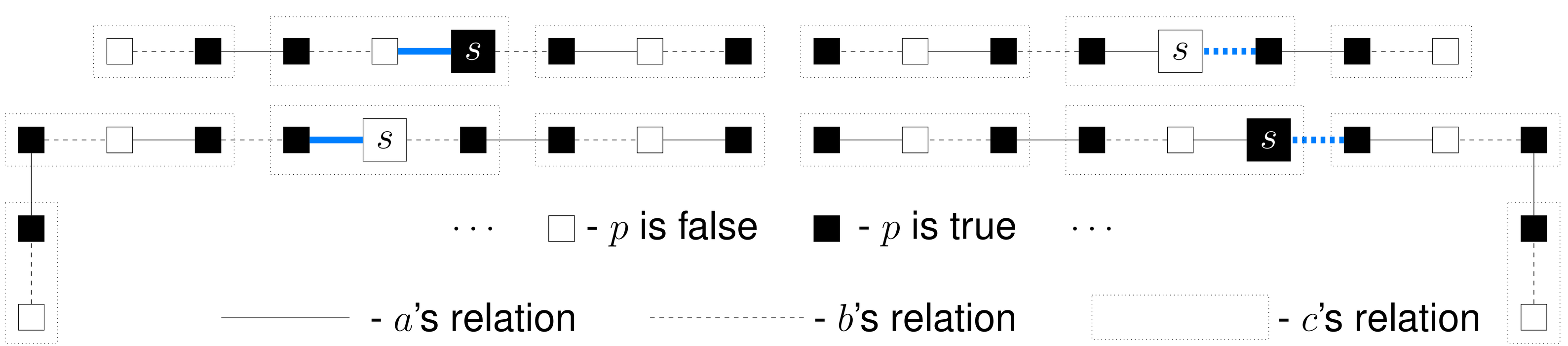
Definition 3 (Expressive power) *Let* $\mathcal{L}_1$ *and* $\mathcal{L}_2$ *be two languages.* $\mathcal{L}_1$ *is* at least as expressive *as* $\mathcal{L}_2$ *(*$\mathcal{L}_2 \preceq \mathcal{L}_1$*) iff for all* $\phi \in \mathcal{L}_2$ *there is a* $\psi \in \mathcal{L}_1$ *such that* ϕ *and* ψ *are satisfied in the same pointed models.*

Theorem 1 $GAL \not\preceq CAL$

One ingredient in the proof are *formula games*:

- A game is between the $\forall$-player and the $\exists$-player
- If either player cannot move, she loses
- Coalition announcements $[\langle G\rangle]\phi$ and $\langle[G]\rangle\phi$ are split into announcements by the coalition and announcements by the anti-coalition using half-coalition announcements $]\!\rangle A \setminus G, \psi_G \langle\![$ and $\langle\![A \setminus G, \psi_G]\!\rangle$

To show that a property ϕ is expressible in GAL but not CAL, we have to work with infinite classes of models (because both CAL and GAL include PAL, and in a given model a fixed PAL formula can be used for any announcement).



a-models and b-models

- Two infinite classes of models $\mathcal{M}_A$ and $\mathcal{M}_B$
- We define the property Φ of *being an a-model* expressible in GAL, i.e. for all $M_s \in \mathcal{M}_A$: $M_s \models \Phi$ and for all $N_t \in \mathcal{M}_B$: $N_t \not\models \Phi$
- Using formula games, we argue that for all $\Psi \in \mathcal{L}_{CAL}$, if for all $M_s \in \mathcal{M}_A$: $M_s \models \Psi$, then there is an $N_t \in \mathcal{M}_B$: $N_t \models \Psi$
- CAL operators require all agents to make a simultaneous announcement. For chain models, the intersection of all agents' relations is the identity, and hence if a coalition can force some configuration of an a-model, then it can replicate the same configuration in a b-model. GAL operators, however, do not require all agents to participate in an announcement, and relations of individual agents are not discerning enough to force isomorphic submodels of given a- and b-models.

References

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