Groups versus Coalitions
On the Relative Expressivity of GAL and CAL

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What this talk is about

- Public Announcement Logic (PAL) can be extended with quantification over announcements
- Resulting formalisms are Group Announcement Logic (GAL)[1], and Coalition Announcement Logic (CAL)[2]
- We prove that GAL can express properties that CAL cannot express

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Models

Definition (Epistemic Model)

An epistemic model is a triple $M = (S, \sim, V)$, where

- S is a non-empty set of states,
- $\sim_i \subseteq S^2$ is an equivalence relation to each agent i,
- $V : P \to 2^S$ is the valuation function.

A pair M_s with $s \in S$ is called a pointed model.

An announcement of α in a pointed model M_s results in an updated pointed model $M_s^{\|\alpha\|_M}$ containing only α-states:

- $S^{\|\alpha\|_M} = \|\alpha\|_M$,
- $\sim_i^{\|\alpha\|_M} = \sim_i \cap (\|\alpha\|_M \times \|\alpha\|_M)$,
- $V^{\|\alpha\|_M}(p) = V(p) \cap \|\alpha\|_M$.

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Example

Alice buys ice creams for herself and her brother Bob. They know that Alice has chocolate ice cream ($choc_a$), and Bob has strawberry ice cream (str_b), but the ice cream seller Charles does not know who is having which ice cream.

$M_s \models choc_a \wedge str_b$, $M_s \models \neg K_c choc_a$, $M_s \models K_c(choc_a \rightarrow str_b)$

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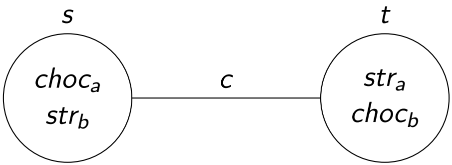

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Alice says that she is going to have chocolate ice cream ($K_a choc_a$).

$M_s^{||K_a choc_a||} \models K_c(choc_a \wedge str_b)$, $M_s^{||K_a choc_a||} \models K_a K_c(choc_a \wedge str_b)$

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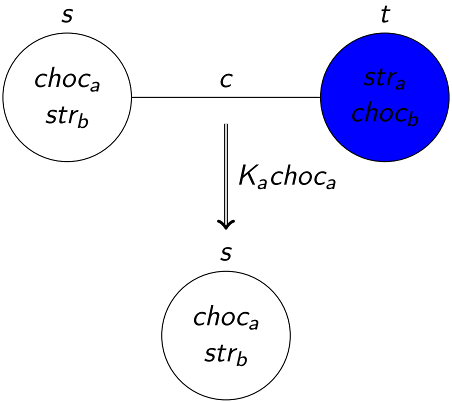

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Semantics of PAL

Definition (Semantics)

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\begin{array}{lll}
M_s \models p & \text{iff} & s \in V(p) \\
M_s \models \neg\phi & \text{iff} & M_s \not\models \phi \\
M_s \models \phi \wedge \psi & \text{iff} & M_s \models \phi \text{ and } M_s \models \psi \\
M_s \models K_i\phi & \text{iff} & \forall t \in S : s \sim_i t \text{ implies } M_t \models \phi \\
M_s \models [\alpha]\phi & \text{iff} & M_s \not\models \alpha \text{ or } M_s^{\|\alpha\|_M} \models \phi
\end{array}
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Formula $[\alpha]\phi$ is read as

after a public announcement of α, ϕ holds in the resulting model.

Dual of $[\alpha]\phi$

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Announcements by Groups

We are interested in the following restrictions on announcements:

- Announcements are made by agents
- Agents can only announce what they know
- Groups of agents can announce conjunctions of formulas, where each conjunct is a formula known by an agent in the group
- Coalitional ability wrt announcements: can a group make an announcement such that whatever agents outside the group announce, the goal formula holds?

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GAL and CAL

Group Announcement Logic (GAL) $=$ PAL $+ \{[\![G]\!]\phi, \langle\!\langle G\rangle\!\rangle\phi\}$

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Coalition Announcement Logic (CAL) $=$ PAL $+ \{[\langle G\rangle]\phi, \langle[G]\rangle\phi\}$

$\langle[G]\rangle\phi$: 'there is an announcement by agents from G such that whatever agents $A \setminus G$ outside of the coalition announce, ϕ holds'

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Semantics of GAL and CAL

Let α_G be a shorthand for a formula of the type $K_i\phi_i \wedge \ldots \wedge K_j\phi_j$, where $i, \ldots, j \in G$, and $\phi_i, \ldots, \phi_j$ are formulas of epistemic logic.

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Alice has an announcement after which Charles does not know who is having which ice cream: $\langle\!\langle a \rangle\!\rangle \neg K_c(choc_a \wedge str_b)$ is true in s (she can just say 'Thank you' to Charles).

However, Alice does not have a strategy to make sure that Charles does not know who is having which ice cream: $\langle\![G]\!\rangle \neg K_c(choc_A \wedge str_B)$ is false in s (if Alice says 'Thank you', Bob might still simultaneously announce 'I am having strawberry ice cream!')

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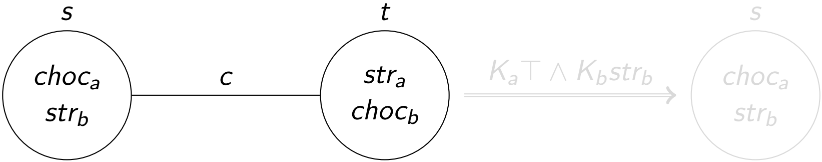

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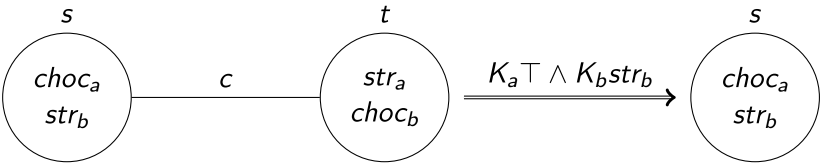

Expressivity of GAL and CAL: Introduction

Definition (Expressive power)

Let $\mathcal{L}_1$ and $\mathcal{L}_2$ be two languages. $\mathcal{L}_1$ is at least as expressive as $\mathcal{L}_2$ ($\mathcal{L}_1 \geq \mathcal{L}_2$) iff for all $\phi \in \mathcal{L}_2$ there is an equivalent $\psi \in \mathcal{L}_1$.

Some known results:

- EL $\geq$ PAL and PAL $\geq$ EL[3]
- GAL $\geq$ PAL and PAL $\not\geq$ GAL[4]
- CAL $\geq$ PAL and PAL $\not\geq$ CAL[5]

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Expressivity of GAL and CAL: why is it difficult

- Assume that there are two finite models M_s and N_t such that for some $\phi \in \mathcal{L}_{GAL}$, $M_s \models \phi$ and $N_t \not\models \phi$.
- Can we possibly prove that for all $\psi \in \mathcal{L}_{CAL}$, $M_s \models \psi$ iff $N_t \models \psi$
- No
- For a fixed model, all $\langle G \rangle$ and $[G]$ in ϕ can be replaced by *particular* $\langle \psi_G \rangle$ (since CAL and GAL both contain PAL)
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Expressivity of GAL and CAL: Chain Models

Class $\mathcal{M}_A$ of a-models. Solid line — a's relation, dashed line — b's relation, dotted rectangles — c's relation, and p is true at black states.

Class $\mathcal{M}_B$ of b-models.

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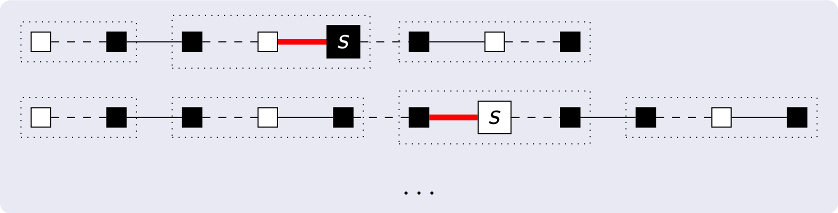

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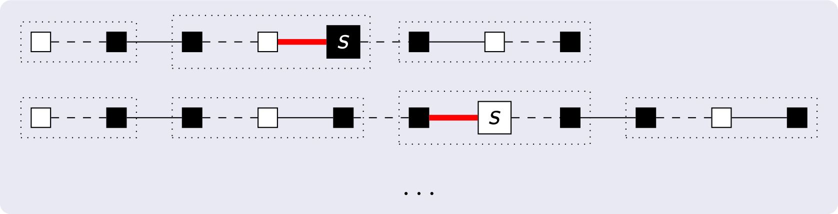

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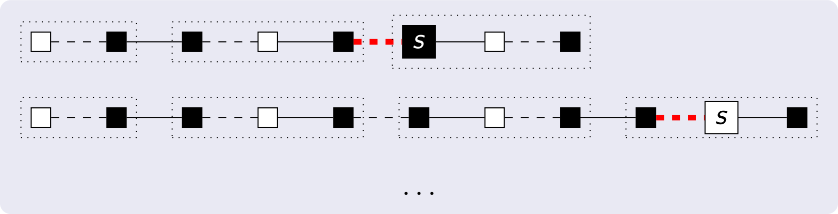

Expressivity of GAL and CAL: Formula Games

- A game is between the $\forall$-player and the $\exists$-player
- If either player cannot move, she loses
- Coalition announcements $[\langle G\rangle]\phi$ and $\langle[G]\rangle\phi$ are split into announcements by the coalition and announcements by the anti-coalition using half-coalition announcements $\rangle A \setminus G, \psi_G \langle$ and $\langle A \setminus G, \psi_G \rangle$

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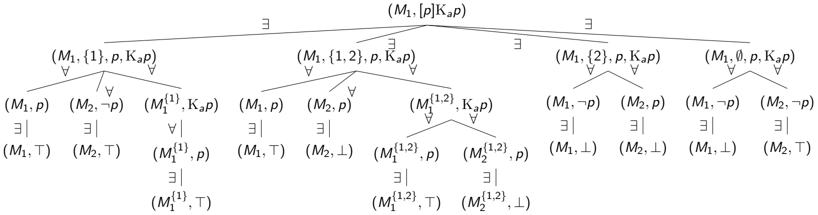

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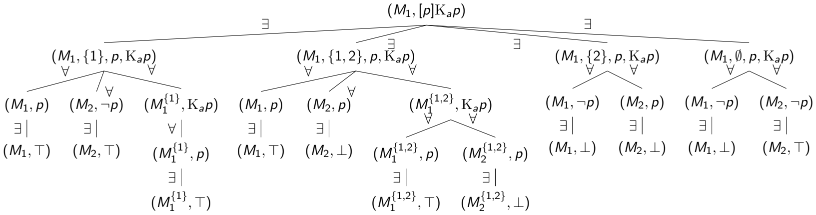

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Expressivity of GAL and CAL: General Idea

- Two infinite classes of models $\mathcal{M}_A$ and $\mathcal{M}_B$
- We define the property Φ of being an *a*-model expressible in GAL, i.e. for all $M_s \in \mathcal{M}_A$: $M_s \models \Phi$ and for all $N_t \in \mathcal{M}_B$: $N_t \not\models \Phi$
- Using formula games, we argue that for all $\Psi \in \mathcal{L}_{CAL}$, if for all $M_s \in \mathcal{M}_A$: $M_s \models \Psi$, then there is an $N_t \in \mathcal{M}_B$: $N_t \models \Psi$
- Handwavy argument: CAL operators require all agents to make a simultaneous announcement. For chain models, the intersection of all agents' relations is the identity, and hence if a coalition can force some configuration of an *a*-model, then it can replicate the same configuration in a *b*-model. GAL operators, however, do not require all agents to participate in an announcement, and relations of individual agents are not discerning enough to force isomorphic submodels of given *a*- and *b*-models.

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Conclusions and Further Research

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We have shown that some properties of models are expressible in GAL but not in CAL. The other direction is future work.

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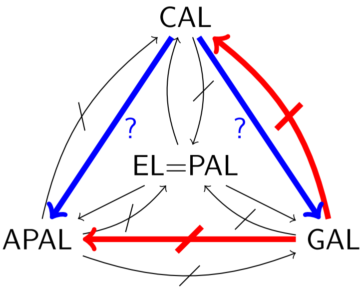

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Thank you for attention!