Formal Verification of Diffusion Auctions

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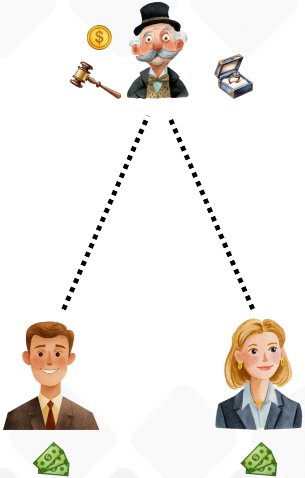
Motivation (1/2)



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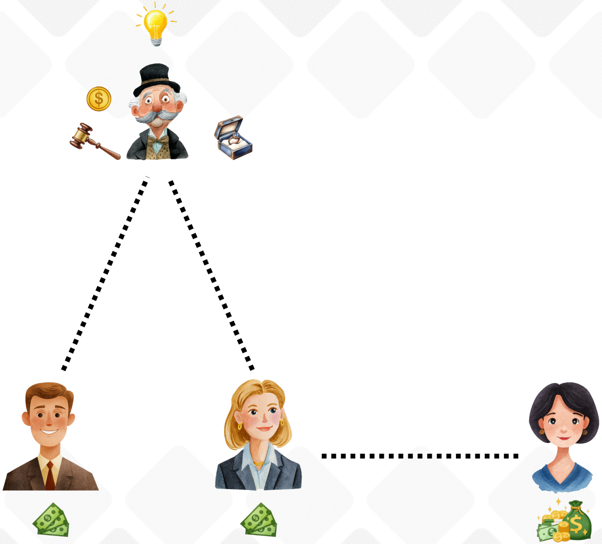

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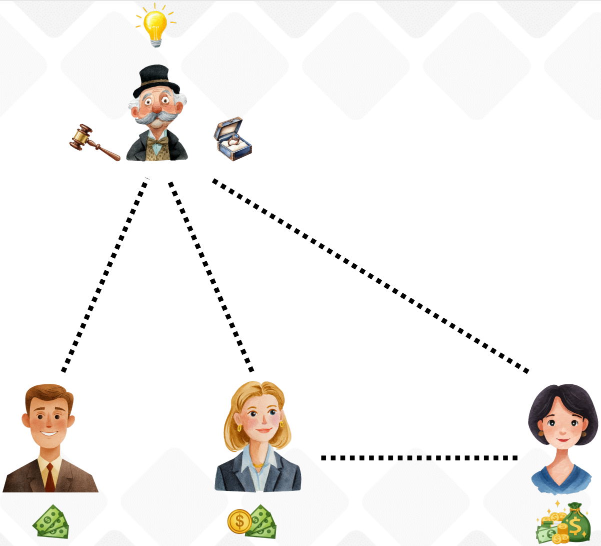

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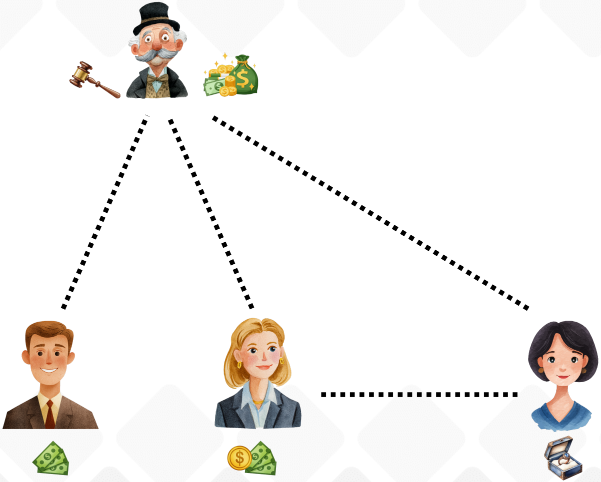

Motivation (2/2)

Using buyers' connections to promote the auction: potential increase in sellers' revenue

Diffusion auctions (Zhao et al. 2018; Li et al. 2022)

Sellers propose incentives to buyers so that they can benefit from inviting their neighbors

Scope

Formally reasoning about the diffusion of auctions :

- Representing properties
- Verification through model checking

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Scope

Formally reasoning about the diffusion of auctions

Contribution

- Logic-based approach to verification of diffusion auctions
  - Multi-seller
  - Strategic dimension for sellers
  - Nash equilibrium
- Model checking and strategy existence problems
- Expressivity

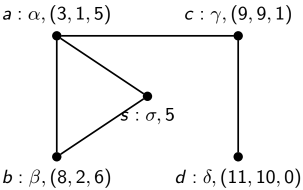
Model

An n-seller Diffusion Auction Mechanism is composed of

- Agents Agt, divided into *buyers* B and n *sellers* S
- *Friendship* relation F, naming function N, and budget Bdg for each agent;
- Buyers' *valuation* function V
- Incentive function I
- Allocation P, payment Pay, and utility U functions for each agent

Seller s is named σ and her budget is 5. Buyer a is named α, $Bdg(a) = 3$, $V(a) = 1$, and $I(a, s) = 5$, ...

P and Pay are defined as in a first price auction



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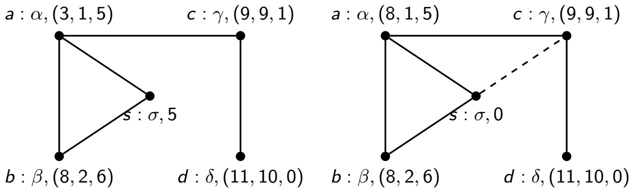


Initially, the seller's utility would be 7, and the winner would be β, but if the seller incentivizes α, the seller's utility would be 9, and the winner would be γ.

Formally, $(ut_\sigma = 7) \land \heartsuit\beta \land \langle \sigma : \alpha \rangle (ut_\sigma = 9 \land \heartsuit\gamma)$

n-seller logic for diffusion incentives

Syntax of $\mathcal{L}^n$

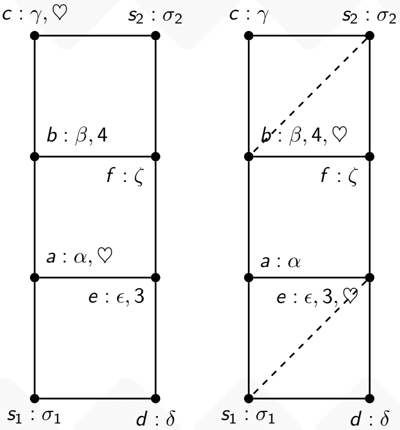
$$\varphi := \alpha \mid (z_1 t_1 + \ldots + z_m t_m) \geqslant z \mid \neg\varphi \mid (\varphi \land \varphi) \mid \Box\varphi \mid [\sigma_1 : \beta_1, \ldots, \sigma_n : \beta_n]\varphi \mid \heartsuit\alpha$$

where α is a name or $\bullet$, σ_i is a seller name, β_i is a is a buyer name, $z \in \mathbb{Z}$, t_i is a term

Semantics

- $\Box\varphi$: all friends of the current agent satisfy φ (and $\Diamond\varphi \stackrel{\text{def}}{=} \neg\Box\neg\varphi$)
- $\heartsuit\alpha$: the agent named α gets an item in the current configuration of a mechanism
- $[\sigma_1 : \beta_1, \sigma_2 : \beta_2, \ldots, \sigma_n : \beta_n]\varphi$ (or $[\overline{\sigma} : \overline{\beta}]\varphi$): if each seller σ_i successfully incentivizes the corresponding buyer β_i to invite all their friends to join her auction, then φ holds
- $\langle\overline{\sigma} : \overline{\beta}\rangle\varphi \stackrel{\text{def}}{=} \neg[\overline{\sigma} : \overline{\beta}]\neg\varphi$: each seller σ_i can successfully incentivize the corresponding buyer β_i and φ holds

Example



Assume a first-price auction M, where both sellers have budgets of 1 and all incentives are 1. Buyers' valuations are 1, except for b and e, whose valuations are 4 and 3, resp.

$$M \models ut_{\sigma_1} = 2 \wedge ut_{\sigma_2} = 2 \wedge \\ [\sigma_1 : \delta, \sigma_2 : \gamma](ut_{\sigma_1} > 2 \wedge ut_{\sigma_2} > 2)$$

$$M \models [\sigma_1 : \delta, \sigma_2 : \gamma]\left(\bigwedge_{i \in \mathrm{Nom}} i \to (\heartsuit \bullet \vee \Diamond \heartsuit \bullet)\right)$$

Results

Assume finite models with polynomially computable functions, then

Theorem 1

Model checking $\mathcal{L}^n$ is in P for the class of finite models with polynomially computable placement, payment, and utility functions.

Strategy existence problem

Given a mechanism M and a goal $\varphi \in \mathcal{L}^n$, determine whether there is a finite sequence of concurrent incentivisations $\langle \overline{\sigma} : \overline{\beta} \rangle^* = \langle \overline{\sigma} : \overline{\alpha} \rangle \ldots \langle \overline{\sigma} : \overline{\gamma} \rangle$ such that $M, s \models \langle \overline{\sigma} : \overline{\beta} \rangle^* \varphi$

Theorem 2

The strategy existence problem is NP-complete

Strategic extension

Logic $\mathcal{SL}^n$

The logic extends $\mathcal{L}^n$ with $\langle\!\langle C \rangle\!\rangle\varphi$ for $C \subseteq S$, meaning that there is a (one-step) strategy for the coalition of sellers C such that no matter what other sellers do, φ holds.

Abbreviation $[\![C]\!]\varphi := \neg\langle\!\langle C \rangle\!\rangle\neg\varphi$

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$\mathcal{SL}^n$ captures, for instance,

- $\langle\!\langle \sigma_1, \sigma_2 \rangle\!\rangle [\![\sigma_3]\!](ut_{\sigma_1} > ut_{\sigma_3})$: the first two sellers can prevent the third seller from having a utility equal to or higher than that of the first seller (in two-incentive rounds)
- Nash equilibria over bounded number of incentive rounds

Results

Theorem 3

For the class of finite models with functions computable in polynomial space, model checking $\mathcal{SL}^n$ is in PSPACE-complete

Theorem 4

$\mathcal{SL}^n$ is strictly **more expressive** than $\mathcal{L}^n$

We also provide model-checking algorithms for both logics

Conclusion

- Capturing properties of diffusion auctions, like item allocations, utility increase, and Nash equilibrium.
- Model checking algorithms allow the verification of such properties
- Future work
  - Probabilistic extension
  - Strategic dimension for buyers

References

Li, Bin, Dong Hao, Hui Gao, and Dengji Zhao (2022). "Diffusion auction design". In: *Artificial Intelligence* 303, p. 103631. DOI: 10.1016/J.ARTINT.2021.103631. URL: https://doi.org/10.1016/j.artint.2021.103631.

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