FORMAL VERIFICATION OF DIFFUSION AUCTIONS

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INTRODUCTION

In an auction, a seller can leverage buyers' **social networks** for promoting the auction.

Increasing the number of participants leads to a potential increase in the sellers' revenue.

We provide a **logic-based framework** for the specification and verification of properties in multi-seller **diffusion auctions.**

We propose two logics to capture such mechanisls and study their **model checking** and **strategy existence** problems.

THE MODEL

A deterministic n-seller diffusion auction mechanism (n-DAM) is composed of:

- A set of agents with n sellers,
- A symmetric irreflexive friendship relation;
- A non-negative budget, a naming function, and an utility function for each agent;
- An incentive function representing how much a seller is willing to pay to a buyer to invite their friends;
- The allocation and payment function.

THE LOGIC

$\mathcal{L}^n$, the n-seller strategic logic for diffusion incentives:

$$\varphi ::= \alpha \mid (z_1 t_1 + \ldots + z_m t_m) \geqslant z \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box\varphi \mid [\sigma_1 : \beta_1, \ldots, \sigma_n : \beta_n]\varphi \mid \heartsuit\alpha$$

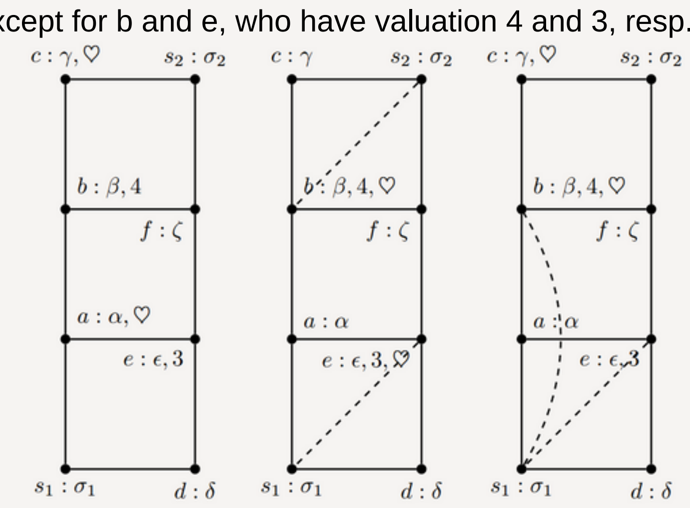
where α is an agent, σ_i is a seller, β_i is a buyer, z_i is an integer, and t_i is a term denoting an agent's utility.

$\Box\varphi$: all friends of the current agent satisfy φ.
$\heartsuit\alpha$: the agent named α currently gets an item.
$[\sigma_1 : \beta_1, \sigma_2 : \beta_2, \ldots, \sigma_n : \beta_n]\varphi$: each seller σ_i incentivizes the buyer β_i to tell her friends about σ_i's auction. After the information diffusion, φ holds.

$\mathcal{SL}^n$ extends the logic with the operator $\langle\!\langle C \rangle\!\rangle\varphi$: there is a strategy for the coalition C to incentivise buyers such that no matter what other sellers do, φ holds.

EXAMPLE

Assume a first-price auction, where the sellers have both budgets 1 and all incentives are 1. Buyers have valuation 1, except for b and e, who have valuation 4 and 3, resp.



$$M \models ut_{\sigma_1} = 2 \wedge ut_{\sigma_2} = 2 \wedge [\sigma_1 : \delta, \sigma_2 : \gamma](ut_{\sigma_1} > 2 \wedge ut_{\sigma_2} > 2)$$

$$M \models [\sigma_1 : \delta, \sigma_2 : \gamma] \bigwedge_{i \in \mathsf{Nom}_M} i \to (\heartsuit\bullet \vee \Diamond\heartsuit\bullet)$$

$$M \models \bigwedge_{i \in \mathsf{Nom}_M \cup \{\bullet\}} [\sigma_1 : \alpha, \sigma_2 : i](ut_{\sigma_1} > ut_{\sigma_2})$$

RESULTS

For the class of finite n-DAMs with polynomially computable functions,

- Model checking $\mathcal{L}^n$ is in PTIME;
- The strategy existence problem (i.e., determine the existence of a finite sequence of concurrent incentivisations) is NP-complete;
- Model checking $\mathcal{SL}^n$ is in PSPACE-complete.

$\mathcal{L}^n$ can express Nash equilibria over a bounded number of steps.
$\mathcal{SL}^n$ is strictly more expressive than $\mathcal{L}^n$. It can capture, for instance, that a coalition of two sellers can preclude another seller from having a better utility than that of the first one: $\langle\!\langle \sigma_1, \sigma_2 \rangle\!\rangle [\![\sigma_3]\!] (ut_{\sigma_1} > ut_{\sigma_3})$

CONCLUSION

We provide logics that can capture various properties of diffusion auctions, like item allocations, utility increase, local properties of the underlying social network, and Nash equilibrium. Our model checking algorithms allow the verification of such properties.

Future work:

- Probabilistic extension;
- Strategic dimension for buyers;
- Axiomatization.

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